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Default Risk and Economic Activity: A Small Open Economy Model with Sovereign Debt and Default*

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Abstract: Empirical evidence shows that sovereign defaults are associated with significant downturns in economic activity in defaulting countries. However, the existing literature on sovereign debt and default mainly analyzes endowment economies and, therefore, does not address the relationship between default risk and macroeconomic dynamics. This paper develops a general equilibrium small open economy model with financial frictions that allows to simultaneously examine the behavior of output, investment and borrowing dynamics and its interaction with sovereign default. When calibrated to match the business cycles properties of an average emerging market economy, the model is able to reproduce the countercyclicality of net exports and sovereign spreads and the negative correlation between investment and net exports observed in the data. Furthermore, when analyzing its behavior around sovereign default, the model successfully captures the declines in output, consumption and investment that are actually associated with these episodes.

Keywords: Sovereign debt, sovereign default, business cycles, small open economy.

JEL Classification: E32, E44, F32, F34.

Resumen: La evidencia empírica muestra que los episodios de impago de deuda soberana están asociados con contracciones significativas en la actividad económica. Sin embargo, la literatura existente sobre deuda soberana e impago analiza principalmente economías de dotación y, por lo tanto, no puede estudiar la relación existente entre el riesgo de impago y la dinámica macroeconómica. Este documento desarrolla un modelo de equilibrio general para una economía pequeña y abierta con fricciones financieras que permite examinar simultáneamente el comportamiento del producto, la inversión y la deuda soberana y su interacción con el riesgo de impago y el impago soberano. Al ser calibrado para reproducir los principales hechos estilizados de una economía emergente promedio, el modelo es capaz de reproducir la contraciclicidad de las exportaciones netas y de los diferenciales de tasas de interés y la correlación negativa entre la inversión y las exportaciones netas observadas en los datos. Asimismo, al analizar su comportamiento alrededor de episodios de impago soberano, el modelo captura exitosamente las disminuciones del producto, consumo e inversión que están de hecho asociadas a dichos episodios.

Palabras Clave: Deuda soberana, impago, ciclo económico, economía pequeña y abierta.

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1 Introduction

Empirical evidence shows that sovereign defaults are generally associated with significant downturns in economic activity in defaulting countries. Figure 1 depicts the evolution of output, consumption and investment in Argentina, Ecuador, Mexico, Philippines and Russia around their default episodes of 2001, 1999, 1982, 1983 and 1998, respectively.¹ All cases are characterized by a decrease in output and consumption, interestingly, however, the largest declines are observed in investment dynamics. Table 1 further looks into this observation by summarizing these variables' deviations from trend around several default episodes within the last 30 years. Average output and consumption deviations from trend range roughly between -6 and -10 percent, respectively, while their investment counterpart varies by as much as 22.5 percent below trend. This evidence emphasizes the need for comprehensive frameworks within which to examine the relationship between sovereign debt crises and macroeconomic dynamics. Yet, the existing literature on sovereign debt and default mainly analyzes endowment economies and, therefore, does not address the interaction between output, investment and borrowing dynamics and default risk nor does it help to understand how sovereign defaults contribute to the decline in economic activity.

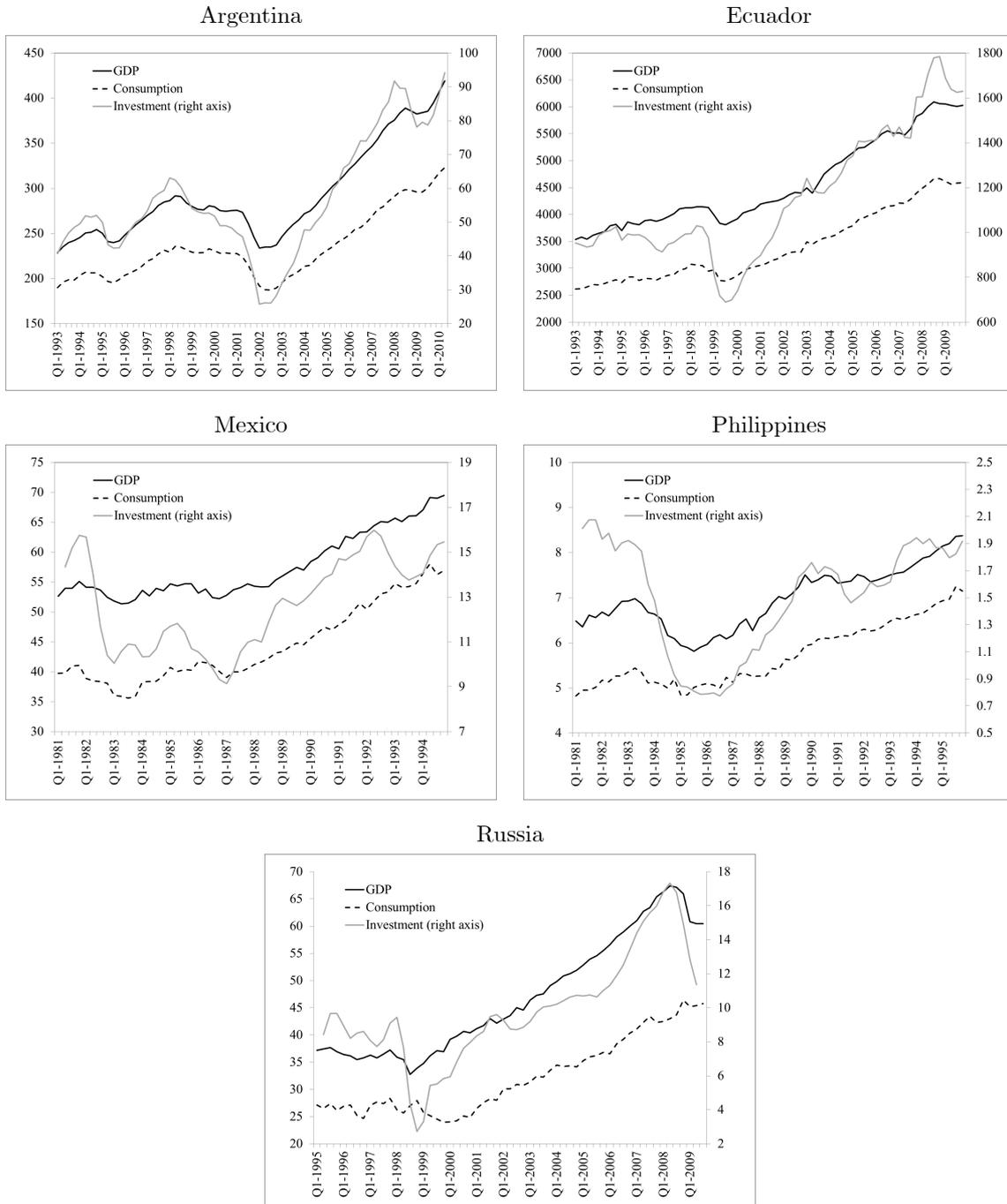
Table 1. Output, consumption and investment dynamics in selected default episodes

Country	Default episode (starting date)	Deviation from trend:		
		Output	Consumption	Investment
Argentina	2001Q4	-11.2	-11.4	-44.2
Ecuador	1999Q3	-6.0	-15.1	-25.1
Indonesia	1998Q2	-7.6	-18.1	-16.3
Mexico	1982Q3	-3.1	-4.9	-21.8
Peru	1980Q1	-3.7	-8.7	-25.7
Peru	1983Q1	-8.4	-10.0	-12.9
Philippines	1983Q4	-7.7	-9.6	-39.1
Russia	1998Q4	-10.8	-10.6	-38.3
South Africa	1985Q3	-2.1	-8.1	-8.2
South Africa	1989Q4	1.5	-1.3	6.4
Average		-5.9	-9.8	-22.5

Notes: The selection of default episodes is guided by data availability. Starting dates of default episodes are from Levy-Yeyati and Panizza (2011). Variables are in logs and detrended using the Hodrick-Prescott filter with a smoothing parameter of 1600. Numbers reported correspond to maximum deviation within the quarter of default and the next four quarters. All deviations are in percentages.

¹Details on data and sources are provided in the Appendix.

Figure 1. Output, consumption and investment dynamics around default episodes



This paper studies sovereign default within a small open economy model with financial frictions in which the interaction between general equilibrium dynamics and the presence of incomplete markets allows output and default risk to be jointly determined. In particular, the fact that in our set-up the economy is able to smooth out consumption both by borrowing resources from the rest of the world and by changing the level of capital stock introduces mechanisms that generate endogenous costs of default, which affect default incentives and, in turn, the economy's default decisions and allocation of resources.

The model's equilibrium results are consistent with the fact that countries are more likely to default in bad times or when facing higher levels of outstanding debt. They also suggest that the maximum level of indebtedness relative to output, i.e. the maximum risky debt to output ratio, that an economy is able to sustain is fairly independent of the level of capital stock. When calibrated to match the business cycles dynamics of an average emerging market economy, the model is able to reproduce the simultaneous countercyclicality of net exports and sovereign spreads and the negative correlation between investment and net exports observed in the data. Furthermore, when analyzing its behavior around sovereign default, the model successfully captures the declines in output, consumption and investment that are actually associated with these episodes. Net exports and sovereign spreads are countercyclical because in the model the risk of default increases when the economy is either more indebted or transiting through periods where productivity is low. As a consequence, in bad times, not only is the economy unable to borrow as much resources from the rest of the world as in good times, but also she does so at a higher cost. The negative correlation between investment and net exports arises from the fact that, as borrowing increases, consumption goes up without the need to reduce capital accumulation, which allows investment to increase as well. Whenever the economy defaults, productivity falls and access to external resources is denied; as a consequence output declines which, in turn, makes consumption and investment go down leading to an overall contraction of economic activity. Our work hence contributes to the literature by providing a DSGE framework within which the behavior of output, investment and borrowing dynamics and its interaction with default risk and sovereign default can be simultaneously examined.

The main anomalies of the model are twofold: first, it displays a low rate of default and, second, it does not produce enough volatility of country spreads. These results are potentially driven by the increased ability of the economy to smooth out consumption. In contrast with other existing models, where the only means to smooth out consumption is by borrowing resources from the rest of world and default arises as a way to provide the environment with a higher degree of state-contingency, in our framework the economy is able to adjust capital stock (and often times prefers to do so) in order to avoid default and the costs associated with it. This, on one hand, decreases default occurrences in equilibrium which translates into low rates of default and, on the other hand, produces little variability in default probabilities which, given their tight relationship with debt prices, significantly reduces country spreads

volatility.

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 presents its quantitative analysis, which include the examination of the equilibrium and business cycles properties of the calibrated model, a sensitivity analysis and the study of the model economy's behavior around default. Section 4 concludes.

2 The model economy

The proposed framework is mainly related to two strands of the international economics literature. First, to the literature on sovereign debt and default in emerging economies which builds on the seminal work of Eaton and Gersovitz (1981), who provide a theoretical analysis of borrowing in international private capital markets with default penalties and who has served as framework for the development of a growing literature of quantitative models of debt and default (see Stähler (2012) for a review of the recent developments in quantitative models of sovereign debt and default). And, second, to the literature on financial frictions which introduces endogenous debt limits (based on Kehoe and Levine (2003)) into fully-fledged RBC models.²

Specifically, consider a small open economy where output is produced using capital and labor and can be transformed one-to-one into consumption and investment goods. The economy is populated by infinitely-lived households, who are risk averse and have preferences over consumption goods and leisure, and by a benevolent government, the sovereign, who is in charge of maximizing households' utility at every point in time.

A shock affecting the economy's productivity is realized at the beginning of each period; however, the economy has access to international credit markets where she can borrow resources from the rest of the world as a way to insure against aggregate risk. Risk insurance in this case is partial though since financial markets are incomplete. On one hand, there is limited spanning in the set of financial assets available, which limits trade to one-period, non-contingent discount bonds. On the other hand, there is limited commitment arising from the lack of perfect enforceability of financial contracts; this allows the sovereign to default upon the economy's outstanding debt whenever it is in her best interest to do so. If the sovereign decides to honor the economy's contract and pay the debt that comes due, the economy keeps her good credit standing and continues having access to international credit markets where she can issue more debt. If, instead, the sovereign decides not to pay the debt, the economy is said to be in default, falls in bad credit standing and faces some costs associated with this decision. Following the existing literature on sovereign debt and default, we assume that the penalties associated to default are

²See Kehoe and Perri (2002) for an example that studies the effects of introducing endogenous debt limits in a fully-fledged RBC model.

twofold: first, the economy is forced into financial autarky where she remains with certain probability until being redeemed and, second, the economy is penalized with a direct output cost while being out of the international credit markets.

After having described the general features of the economy, we now discuss the details of the economic problem. Then, we present the model's equilibrium and comment on the implications of introducing endogenous output dynamics with investment into the existing framework.

2.1 The sovereign

Let $V(a, k, z)$ be the economy's continuation value when initial asseholdings are given by a , the initial level of capital stock is k , and the productivity shock at the beginning of the period is represented by z .³ The sovereign's problem can be hence stated in recursive form in the following way:

$$V(a, k, z) = \max_{d \in \{0,1\}} \{(1-d)V^G(a, k, z) + dV^B(k, z)\} \quad (1)$$

where d is the default decision and $V^G(a, k, z)$ and $V^B(k, z)$ are the economy's continuation values of being in good and bad credit standing, respectively.

If the economy is in good credit standing, the sovereign chooses today's consumption and labor and tomorrow's capital stock and asseholdings in order to maximize the economy's present utility and expected discounted continuation value, given her budget constraint. This is, $V^G(a, k, z)$ is defined by:

$$\begin{aligned} V^G(a, k, z) &= \max_{c, l, k', a'} \{u(c, l) + \beta E[V(a', k', z')|z]\} \\ \text{s.t.} \quad &c + k' - (1 - \delta)k + q(b', k', z)a' = e^z f(k, l) + a \end{aligned} \quad (2)$$

where $u(\cdot)$ and $f(\cdot)$ are the representative household's utility function and the economy's production function, respectively; E is the expectational operator and x' denotes x one period ahead. c and l stand for consumption and labor, respectively, δ is the depreciation rate and $q(a', k', z)$ represents the zero-coupon bond price.

If, instead, the economy is in bad credit standing, no debt is issued and the sovereign chooses today's consumption and labor and tomorrow's capital stock so as to maximize the economy's present utility and expected discounted continuation value, given available resources. Thus, $V^B(k, z)$ is defined by:

$$\begin{aligned} V^B(k, z) &= \max_{c, l, k'} \{u(c, l) + \beta[(1 - \lambda)E[V^B(k', z')|z] + \lambda E[V(0, k', z')|z]]\} \\ \text{s.t.} \quad &c + k' - (1 - \delta)k = y^{aut}(z, f(k, l)) \end{aligned} \quad (3)$$

where y^{aut} represents output in financial autarky, after considering the direct output cost penalties to

³ $a < 0$ implies the issuance of debt. The model will be calibrated so as to ensure that asseholdings are non-positive in equilibrium.

which the economy is subject.⁴ Notice that the second term in the objective function represents the fact that, a period ahead, the economy remains in autarky with probability $(1 - \lambda)$ or is redeemed and regains access to credit markets, with zero asseholdings, with probability λ .⁵

2.2 International credit markets

International credit markets are perfectly competitive and populated by risk neutral investors. Bonds are hence priced so that the zero-profit condition holds and investors' expected returns equal the international interest rate. This implies that:

$$q(a', k', z) = \frac{1}{1 + r^w} E(1 - d') \quad (4)$$

where r^w denotes the world risk-free interest rate.

Notice that, since international investors need to be compensated for the likelihood of default, there is a direct and close relationship between the expected probability of default and the interest rate spread, defined as $q^{-1} - (1 + r^w)$. Specifically, the higher the expected probability of default the lower the price at which investors are willing to purchase sovereign debt and, in consequence, the higher sovereign spreads. Given that default probabilities, in turn, depend on the sovereign's optimal choices on how much debt to issue and how much capital to accumulate, i.e. on a' and k' , the economy internalizes the fact that her actions affect international credit markets' perceptions regarding the degree of riskiness of her debt. Interest rate dynamics are thus endogenous.

2.3 Equilibrium

Definition: *The recursive equilibrium for the economy is defined as a set of value functions $V(a, k, z)$, $V^G(a, k, z)$ and $V^B(k, z)$, a set of sovereign's decision rules for consumption, labor, capital accumulation, asseholdings and default, represented by $c(a, k, z)$, $l(a, k, z)$, $k'(a, k, z)$, $a'(a, k, z)$ and $d(a, k, z)$, respectively, and a bond price function, $q(a', k', z)$; such that, given initial levels for a , k and z and the*

⁴Assuming that the defaulting country is penalized with a direct output cost while in bad credit standing accounts for the output losses incurred when other trade relations are interrupted after sovereign default or as a result of the reduction of aggregate credit available in the economy while the economy is denied access to international credit markets. In the last respect, see Mendoza and Yue (2012) for a model of sovereign debt where the direct output cost of default is a result of the disruption of firms' private financing in a framework where default on public and private foreign obligations occurs simultaneously. Levy-Yeyati and Panizza (2011) argue that evidence supporting the precense of output losses associated with sovereign default suffers from measurement and identifications problems. Identifying and properly measuring direct output losses after default is a relevant issue that remains to be studied in the literature.

⁵Assuming an exogenous probability of redemption with a zero recovery rate after default is a simple alternative to considering a more complex environment where the defaulting country engages into a sovereign debt renegotiation process with international creditors. See Benjamin and Wright (2008) and Yue (2010) for examples of debt renegotiation processes in models with defaultable debt.

exogenous law of motion for z' , $V(a, k, z)$, $V^G(a, k, z)$, $V^B(k, z)$, $c(a, k, z)$, $l(a, k, z)$, $k'(a, k, z)$, $a'(a, k, z)$ and $d(a, k, z)$ solve the sovereign's optimization problem (1) with $q(a', k', z) = q(a'(a, k, z), k'(a, k, z), z)$ satisfying (4).

Due to the complexity of the problem at hand it is difficult to derive analytical results that serve as guidelines on the behavior of the economy. However, it can be proved that the model preserves one of the main features that characterize existing models of sovereign debt and default that examine endowment economies; namely, if at any given level of capital stock k and productivity shock z , the economy is better off defaulting upon debt level a , she is also better off defaulting upon any other higher level of debt. To formalize this statement notice that the sovereign's decision rule for default, $d(a, k, z)$, equals 1 whenever $V^G(a, k, z) < V^B(k, z)$. Let a default set, $D(a, k)$, be defined as the set of shock realizations for which $d(a, k, z) = 1$, i.e. $D(a, k) = \{z : V^G(a, k, z) < V^B(k, z)\}$, then:

Proposition 1: For $0 \geq a_1 \geq a_2$, if default is optimal for a_1 in some combination of states (k, z) then default will be optimal for a_2 in the same combination of states (k, z) . This is $D(a_2, k) \subseteq D(a_1, k)$.

Proof: See Appendix.

2.4 Investment and endogenous output dynamics

The model developed in this paper endogenizes output by embedding investment dynamics into the standard endowment economy model of sovereign debt and default by means of a capitalistic production function. Bai and Zhang (2012) also study a model of sovereign debt and default with capitalistic production. Nevertheless, the focus of their analysis is different from ours in that they aim at studying the effects of financial integration on international risk sharing, whereas our goal is to assess the ability of this framework to reproduce both the real business cycles properties of an economy that suffers from default risk and the macroeconomic dynamics around default episodes.

The economy's ability to accumulate capital stock, both when being in good and bad credit standing, introduces mechanisms that affect default incentives and, in consequence, the sovereign's decisions on whether to default or not and on how much to consume, invest and borrow. In particular, two implications of this are worth noting.

First, in contrast with the endowment economy model, where the only way in which agents can smooth out consumption is by borrowing resources from the rest of the world, through the issuance of sovereign debt, in this framework the economy is also able to self-insure by adjusting her level of capital stock. This adds some degree of state-contingency into the set-up by allowing the economy to adjust consumption without having to default. It also establishes a mechanism by which output dynamics feed back into default risk and vice versa. This is, the decreased need for external financing (brought about

by the possibility to self-insure) increases incentives to default; higher default incentives, however, have a negative effect on the conditions under which the economy has access to international credit markets (if any) which, in turn, affect the pace at which capital stock is accumulated and, ultimately, impact the economy's size and ability to produce future output, thus increasing the gains from being in good credit standing.

Second, by modeling endogenous output dynamics, the value of being in bad credit standing is no longer exogenous.⁶ After defaulting, the economy is penalized by being excluded from the international credit markets and facing direct output costs during financial autarky. The effect of these two penalties in endowment economy models is exogenous by definition: the economy has no control on the income flows that she receives until being redeemed and, in consequence, the value of being in bad credit standing depends on the assumptions about the stochastic process driving these flows and about the form and size of the income losses that she faces. In contrast, in our framework the economy has the ability to self-insure, even in financial autarky, which makes V^B dependent on the economy's policy function for capital stock.

All in all, the introduction of output and investment dynamics into the model modifies the way in which both V^G and V^B are determined and their interaction. The next section analyzes the quantitative implications of these effects on the model's performance.

3 Quantitative analysis

In order to test the numerical properties of the model, first, we describe the data used to contrast our set-up with empirical evidence. Then, we present the calibration and the functional forms used and comment on the solution method. We continue by examining the properties of the calibrated model. Specifically, we analyze its implications for default, debt and debt prices and the business cycles properties that it delivers and present a sensitivity analysis of the results. Finally, we study the model's dynamics around sovereign default.

3.1 The data

The literature on sovereign debt and default has historically focused on contrasting its models with the behavior of emerging market economies. This is mainly driven by two facts: first, because many of the recent episodes of sovereign default have occurred in emerging economies and, second, because empirical evidence suggests that emerging economies' distinguishing features are related to their access to international credit markets (see Neumeyer and Perri (2005), Uribe and Yue (2006) and Oviedo (2005))

⁶In this respect, the proposed framework relates to Mendoza and Yue (2012) who develop an economy model of sovereign debt and default with endogenous output fluctuations but without investment dynamics.

which makes this type of models an ideal framework within which to study their behavior.⁷ For this reason, our quantitative analysis focuses on contrasting the model's results with emerging economies' stylized facts. In contrast with the common approach of using a specific country as a study case, we compare the model's results with the empirical properties of a representative emerging economy. In order to do so, we collect quarterly data on national accounts and sovereign debt and default for a sample of 11 emerging market economies and assume that the representative economy's behavior is characterized by the sample average estimates. The emerging economies considered are Argentina, Brazil, Ecuador, Indonesia, Malaysia, Mexico, Peru, Philippines, Russia, South Africa and Turkey.

Table 2 presents information regarding the default episodes of the country sample for the period 1910-2009. With the exception of Malaysia, all countries in the sample went through at least one default episode. On average, our representative emerging economy defaulted 3 times during the last 100 years, which represents a rate of default of 0.77 percent in quarterly terms (3.13 percent in annual terms).

Table 2. Default episodes on foreign currency debt: 1910-2009

Country	Bond debt	Bank debt	Num. default episodes
Argentina	1989; 2001-2005	1982-1993; 2001-2005	3
Brazil	1914-1919; 1931-1933; 1937-1943	1983-1994	4
Ecuador	1914-1924; 1929-1954; 1999-2000	1982-1995; 2008-2009	5
Indonesia		1998-1999; 2000; 2002	2
Malaysia			0
Mexico	1914-1922; 1928-1942	1982-1990	3
Peru	1931-1951	1976; 1978; 1980; 1984-1997	5
Philippines		1983-1992	1
Russia	1918; 1998-2000	1991-1997	3
South Africa		1985-1987; 1989; 1993	3
Turkey	1915-1928; 1931-1932; 1940-1943	1978-1979; 1982	5

Sources: Beers and Chambers (2006) and Beers, Chambers and Ontko (2010).

⁷ Reinhart and Rogoff (2010) document that the most recent default cycle is the one encompassing the emerging market debt crises of the 1980s and 1990s.

Table 3. Business cycles and sovereign spreads in emerging economies

	(1)	(2)	(3)	(5)	(6)	(7)	(8)
Country	$\sigma(y)$	$\frac{\sigma(c)}{\sigma(y)}$	$\frac{\sigma(i)}{\sigma(y)}$	$\sigma(\frac{nx}{y})$	$\sigma(spr)$	$\rho(\frac{nx}{y}, y)$	$\rho(spr, y)$
Argentina	4.15	1.00	3.41	1.60	6.81	-0.91	-0.63
Brazil	1.97	1.02	2.67	0.86	3.92	-0.05	-0.32
Ecuador	2.18	2.13	3.50	3.70	4.23	-0.47	-0.48
Indonesia	3.12	1.73	2.36	2.29	1.62	-0.49	-0.29
Malaysia	2.82	2.07	4.25	4.40	1.36	-0.60	-0.80
Mexico	3.03	1.28	2.65	2.05	1.71	-0.38	-0.20
Peru	5.03	1.17	2.11	2.36	2.07	-0.41	-0.19
Philippines	2.80	1.51	3.97	2.92	1.39	-0.43	-0.21
Russia	3.46	1.50	2.97	3.83	3.14	0.02	-0.42
South Africa	1.66	2.31	3.69	2.39	0.90	-0.55	-0.18
Turkey	3.64	1.37	2.94	2.33	2.22	-0.61	-0.68
Average	3.08	1.55	3.14	2.61	2.67	-0.44	-0.40
AG	2.74	1.45	3.91	3.22	N.A.	-0.51	N.A.
NP	2.79	1.71	3.29	2.40	2.32	-0.61	-0.55

Notes: Sovereign spreads standard deviation, in column 6, is conditional on being in good credit standing. For all cases but the latter, variables are in logs and detrended using the Hodrick-Prescott filter with a smoothing parameter of 1600. All standard deviations are in percentages. Rows AG and NP are taken from Table 1 in Aguiar and Gopinath (2007) and Table 1 in Neumeyer and Perri (2005), respectively.

Table 3, on the other hand, summarizes the countries' main business cycles statistics and compares them with other representative estimates in the literature.⁸ In general, the business cycles properties of the representative emerging economy examined here are in line with previous findings. Average output standard deviation is 3.08 percent while consumption is 1.55 times more volatile than output. Although output is slightly more volatile in our sample, these estimates are not significantly different to the ones reported by Aguiar and Gopinath (2007) and Neumeyer and Perri (2005), henceforth AG and NP, of 2.74 and 2.79 percent for output and 1.45 and 1.71 times for consumption, respectively. Investment is also more volatile than output in our sample, 3.14 as much. Notice, however, that this value is slightly smaller than AG and NP's estimations; this may be due, in part, to the increased volatility of output

⁸Details on data and sources are provided in the Appendix.

observed in our sample. Conditional on being in good credit standing, sovereign spreads feature an average standard deviation of 2.67 percent which is similar to the one reported by NP.⁹ Finally, the estimations are also consistent with two very well documented stylized facts of emerging economies, namely, the countercyclicality of net exports and of sovereign spreads: the average correlation between net exports and output and between sovereign spreads and output is -0.44 and -0.45, respectively.

3.2 Calibration and functional forms

The model is calibrated on a quarterly basis. The world risk-free interest rate, r^w , is set equal to 1%.

We assume that the period utility function takes the GHH form:

$$u(c_t, l_t) = \frac{\left(c_t - \frac{1}{\psi} l_t^\omega\right)^{1-\sigma}}{1-\sigma}$$

where σ , the coefficient of risk aversion, equals 2; ω , the parameter that regulates the curvature of labor supply, is set equal to 1.455, which implies a labor supply elasticity ($1/(\omega - 1)$) of around 2.2, and ψ is an adjustment factor set to target a labor participation rate of 1/3 in equilibrium. As emphasized by the literature, abstracting labor supply from wealth effects by using this functional form allows the model to better reproduce some business cycle facts that characterize emerging and small open economies.¹⁰

The production function is assumed to be a constant returns to scale Cobb-Douglas:

$$f(k_t, l_t) = k_t^\alpha l_t^{1-\alpha}$$

where the capital share, α , is set equal to 0.36. In order to match an annual depreciation rate of capital of 10%, δ is set equal to 0.025.

We assume that productivity shocks follow an AR(1) process:

$$z_{t+1} = \mu_z + \rho_z z_t + \varepsilon_{t+1}$$

with $\varepsilon \stackrel{iid}{\sim} N(0, \sigma_\varepsilon^2)$. ρ_z and μ_z are set to 0.95 and $-\frac{1}{2}\sigma^2$, respectively, and are consistent with an unconditional mean of e^z of 1. σ_ε^2 is set so as to match the average output standard deviation of our country sample.

So far, all parameter values and functional forms discussed are in line with the RBC literature. Now, we comment on those whose values and forms are closely related to the literature on sovereign debt and default: the discount factor, the probability of redemption after default and the direct output costs of default while in financial autarky. A high degree of impatience is needed in order to take the equilibrium

⁹NP's definition of interest rate is the expected 3-month real interest rate at which firms in a country can borrow. However, for reasons discussed in the paper, they use secondary market prices of emerging market bonds to construct this variable which makes it suitable for our comparison.

¹⁰See, for instance, Mendoza (1991) and Neumeyer and Perri (2005).

of the economy to the region where default does occur, this is why the discount factor, β , is set equal to 0.91.¹¹ As we will see in the sensitivity analysis, the value of this parameter plays an important role in determining the frequency of default in the model. The probability of redemption after default, λ , is set equal to 10% which is consistent with an average stay in autarky of 2.5 years. This number is in line with the estimations of Gelos et al. (2004) and Reinhart and Rogoff (2010) who document an average time of re-entry into the international credit markets of 5.4 years in the 1980s and of roughly 1 year in the 1990s, and a median duration of default spells in the post World-War II period of 3 years, respectively. The effects of changing the value of this parameter are also examined below. Finally, we refer to the direct output cost while the economy is in financial autarky, y^{aut} . One of the novel features of our set-up has to do with the fact that this penalty affects the production capacities of the economy in contrast to other models that assume that the cost affects a set of income draws so, in this sense, there is no guideline to follow when it comes to model its form and to establish its magnitude. We take a simple stance and model it as a constant, small share of output:

$$y^{aut}(z, k, l) = (1 - \gamma)e^z f(k, l)$$

with γ equal to 0.02. The sensitivity analysis will show that the value of this parameter has an important effect on the extension of credit in the model. Table 4 summarizes the values of the parameters used in the benchmark model.

Table 4: The benchmark model: parameter values

Risk aversion	σ	2
Curvature of labor	ω	1.455
Adj. factor for labor	ψ	targets $l = 1/3$
Capital share	α	0.36
Depreciation	δ	0.025
Mean of transitory shock	μ_z	$-\frac{1}{2}\sigma_\varepsilon^2$
Autoregressive coeff. of transitory shock	ρ_z	0.95
Std. deviation of transitory shock	σ_ε	targets $\sigma(y) = 3.08$
World rate interest rate	r^w	1%
Discount factor	β	0.91
Direct loss of output in autarky	γ	2%
Probability of redemption	λ	10%

¹¹ Values of β range from 0.72 to 0.95 in endowment economy models of sovereign debt and default that aim at matching the business cycles properties of Argentina. See Aguiar and Gopinath (2006), Arellano (2008), Cuadra and Saprizza (2008) and Yue (2010), among others.

3.3 Computation

The model is solved numerically using value function iteration and a continuous method to approximate V , V^G and V^B in (1), (2) and (3). In particular, we follow Hatchondo et al. (2010), who use interpolation methods to study the quantitative properties of sovereign debt and default models, and extend their strategy to include capital stock as an additional endogenous state variable. Value functions are approximated using Chebyshev collocation, their expectations are calculated with a numerical integration method and policy functions are solved by means of a derivative-free non-linear optimization routine. Details on the optimization algorithm are provided in the Appendix.

There are several reasons why we choose to use a continuous method rather than the discrete state space approach, DSS, that is commonly used in the existing literature. One of the main challenges associated to the quantitative analysis of models of sovereign debt and default is its curse of dimensionality. Adding capital stock as a third state variable does increase the dimension of the problem under scope, specially as it is desirable that the grid defined for it (when using the DSS technique) is as fine as possible. In this respect, using a continuous method represents an advantage since, once the value functions are approximated, optimal policies can be chosen from continuous sets for a and k , $[\underline{a}, \bar{a}] \times [\underline{k}, \bar{k}]$, in every iteration. Using a continuous method also allows to construct a one-loop algorithm to iterate simultaneously on the value and the bond price functions, which significantly reduces the computation time. Lastly, recent evidence shows that the simulated behavior of sovereign spreads can be very much affected by approximation errors, which are more likely to occur when using coarse grids with the DSS technique, and that the results obtained using the latter converge towards the ones obtained using interpolation methods as the number of grid points increases.¹²

3.4 Results

3.4.1 Equilibrium results: default, the extension of credit and debt prices

We begin the analysis of the equilibrium results by examining the sovereign's default decision. Figure 2 depicts the repayment and default sets of the economy in the a - k space for the mean productivity shock. The shaded region represents all combinations of asset holdings and capital stock levels for which the economy is better off defaulting (the default set), while its complement represents all combinations for which repaying outstanding debt is optimum (the repayment set) in equilibrium. Our results show that the model displays a negative correlation between default risk and asset holdings. This is, for a given level of capital stock (y -axis), the economy is more likely to default the more indebted she is. The model also predicts a negative correlation between default risk and capital stock, as in Bai and Zhang (2012). This is, for a given level of debt (x -axis), default is more likely to occur in economies with lower capital

¹²See Hatchondo et al (2010).

stock, i.e. in poor economies. Figure 2 also speaks for the amount of debt that the economy can borrow. In this respect, notice that the line dividing the repayment set from its default counterpart is negatively sloped meaning that the extension of credit increases the richer the economy is. Nevertheless, when further analyzing the model's implications on the extension of credit we find that, although absolute credit increases with the level of capital stock, credit limits relative to output remain fairly constant. Figure 3 exemplifies this result by depicting the level of both absolute and relative debt capacities across capital levels for the mean productivity shock.

Figure 2. Default and repayment sets for $z = E(z)$

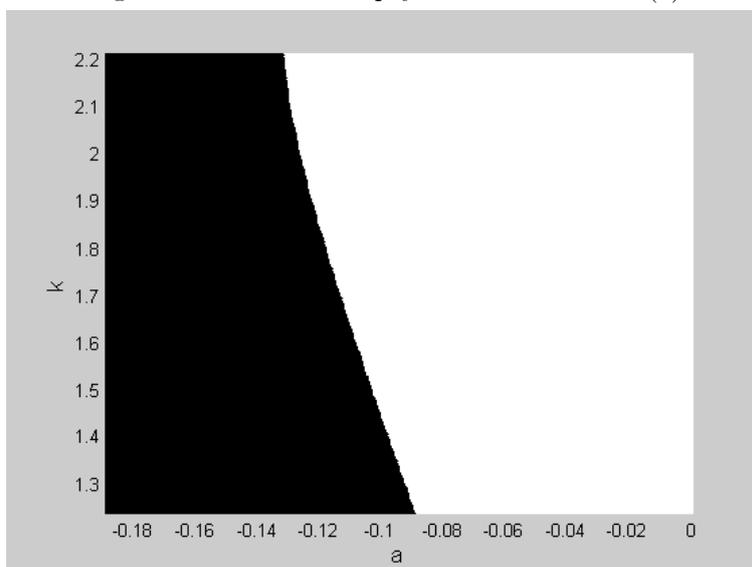
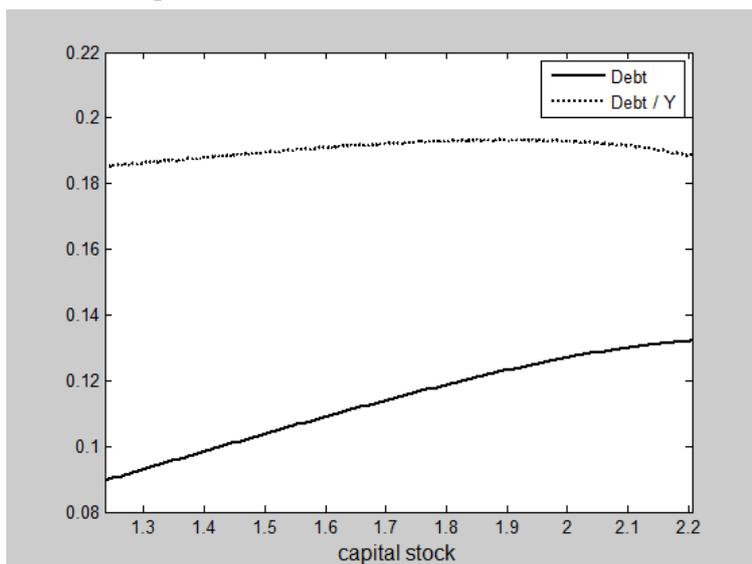
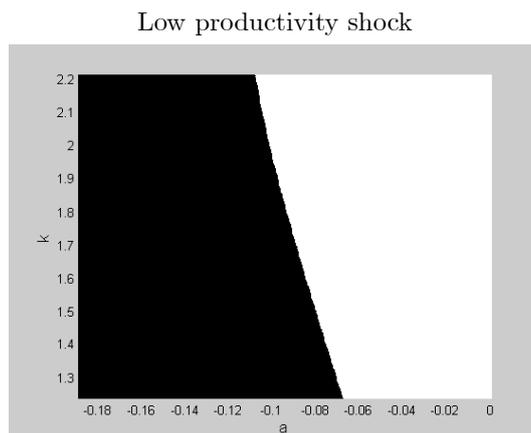


Figure 3. Absolute and relative credit limits



An important implication derived from the simultaneous presence of limited spanning of assets and limited commitment is that default risk and the state of the world are negatively correlated. This property is in line with empirical evidence in Tomz and Wright (2007), who study the relationship between economic output and sovereign default for the period 1820-2004. According to their findings “countries default on loans from private foreign creditors more often in bad times than in good times.”¹³ In the model, paying non-contingent debt is more costly in bad times because it is precisely then when the economy needs more resources to finance consumption and investment the more; the option to default upon outstanding debt adds some degree of contingency to sovereign decisions at the expense of increasing incentives to default, and hence default risk, at those difficult times. Figure 4 displays this property of the model by comparing the default and repayment sets across levels of z . As it can be observed, default sets shrink as z increases which is also translated into larger extensions of credit in absolute terms.¹⁴

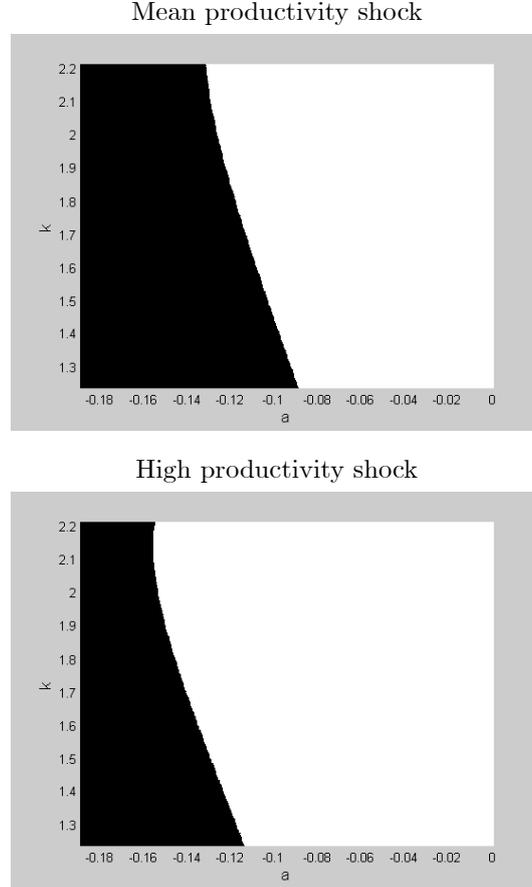
Figure 4. Default and repayment sets across z



¹³See Tomz and Wright (2007), pgs. 352-353. Although they claim the negative relationship between output and default is weak, they do find that the maximum default frequency occurs when output is below trend.

¹⁴As mentioned before, given the complexity of the problem at hand, finding analytical results that serve as guidelines of the behavior of the economy is not straightforward; specially when analyzing the relationship of V^G and V^B with k which is convex in both cases. As can be seen in the graph corresponding to a high productivity shock, the default set appears to shrink as the level of capital stock increases, however, this should not be taken as a general result but rather as a result from the particular combination of parameters used in this example.

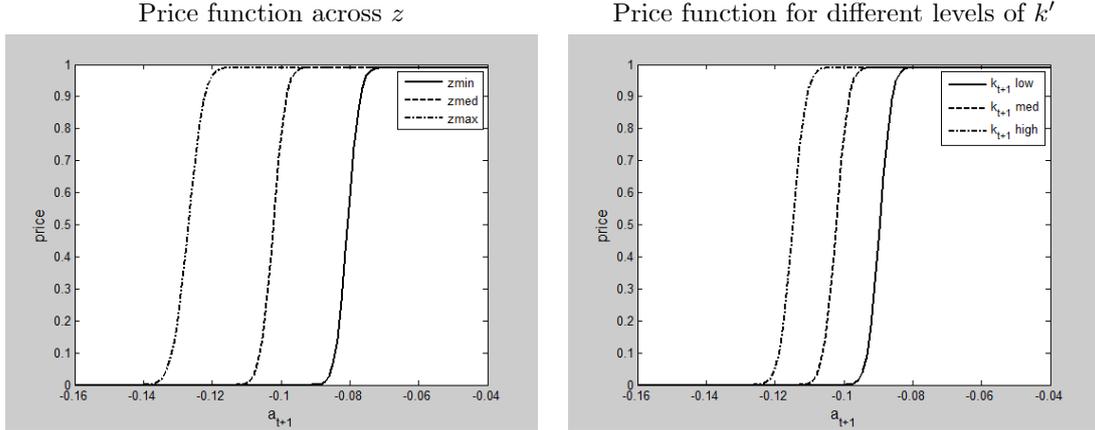
Figure 4. Default and repayment sets across z (contd.)



Debt prices are tightly related to the risk of default. From expression (4) we know that debt issued is priced according to the expected probability of repayment which, in turn, depends on a' , the level of debt issued itself, on tomorrow's production possibilities, determined to a great extent by k' , and on z which provides relevant information as to what the state of the world will be a period ahead when debt comes actually due. Figure 5 depicts several price schedules to examine the relationship between debt prices and their determinants. First, notice that because of the negative correlation between the risk of default and asseholdings, debt price schedules take their maximum value, the inverse of the international interest rate ($1/(1+r^w)$), for small levels of debt which are associated with a zero probability of default. Debt prices thus take positive but smaller values as debt becomes riskier until, eventually, they are equal to zero for high levels of debt upon which the economy will default with certainty. The left panel of Figure 5 depicts debt prices across different states of the world showing that creditors are willing to pay a higher (lower) price for a given level of sovereign debt, and thus ask for lower (higher) returns, in good (bad) times. A similar pattern can be observed with respect to the investment decision of the economy.

The right panel depicts debt prices across different levels of k' . Notice that, for a given level of capital stock, sovereign debt is more expensive (cheaper) the higher (lower) the level of k' , this is, the higher (lower) the investment choice.

Figure 5. Debt price schedules



Notes: The left panel is calculated for an intermediate level of k_{t+1} while the right panel is calculated for the mean level of z .

3.4.2 Real business cycles and debt and default properties

In order to determine the business cycles properties of the proposed framework, we simulate the model economy for 500 periods with shocks drawn from the stationary distribution of z . The results reported correspond to the average over 100 simulations of the last 400 observations of every series. Each set of realizations hence may be interpreted as the evolution of a sample emerging economy over a hundred years.¹⁵

Table 5 reports the results of the model and contrasts them with the data. First, notice that the model matches the countercyclicity of net exports and the countercyclicity of sovereign spreads. This is, it is consistent with the fact that, in bad times, economies borrow less and at a higher cost. Importantly, our framework is able to generate the simultaneous countercyclicalities without having to rely on a specific stochastic process for shocks or on ad hoc output costs of default like several other papers in the literature. Notice also that the model delivers a strong negative correlation between net exports and investment. This accounts for the fact that, as borrowing increases (when net exports

¹⁵All statistics, with the exception of sovereign spreads standard deviation which is computed from the series levels, are computed based on the log-deviations of the Hodrick-Prescott filtered series with a smoothing parameter of 1600.

become more negative), consumption goes up without the need to reduce capital accumulation, which allows investment to increase as well.

Table 5. Business cycles and sovereign debt and default properties: the data vs the model

Business cycles properties:			Sovereign debt properties:		
	Data	Benchmark		Data	Benchmark
$\sigma(y)$	3.08	3.10	Mean($debt/y$)		17.47
$\sigma(c)/\sigma(y)$	1.55	0.87	Default rate	0.77	0.07
$\sigma(l)/\sigma(y)$	N.A.	0.69			
$\sigma(i/y)$	1.56	0.92			
$\sigma(nx/y)$	2.61	0.60			
$\rho(nx/y, y)$	-0.44	-0.28			
$\rho(nx/y, i/y)$	-0.59	-0.84			
$\sigma(spr)$	2.67	0.012			
$\rho(spr, y)$	-0.40	-0.26			

Notes: All standard deviations, the mean debt to output ratio and the default rate are in percentages. The default rate is computed based on all default occurrences in the countries in the sample since 1910.

As mentioned above, the introduction of investment into the model's dynamics provides the economy with an additional means to smooth out consumption. This has the obvious implication on consumption's behavior, namely, a smaller volatility of consumption relative to output. Specifically, in our model, consumption is 0.87 times as volatile as output, which contrasts with the data that shows that the former is actually more volatile than the latter. The variability of the investment to output ratio displayed by the model is also smaller than that in the data, 0.92 vs 1.56, respectively. This is explained by the mere presence of limited commitment which tends to dampen investment flows.¹⁶ As we will observe in the sensitivity analysis, investment becomes more volatile as the effects brought about by the presence of limited commitment are attenuated due to more severe penalties of default.

Finally, notice that the model fails to deliver sufficient variability of sovereign spreads. In regards to this, first, recall that there exists a tight relationship between debt prices and default probabilities. If the model does not produce enough variability in default probabilities it has no chances to deliver sufficient variability of debt prices and, therefore, of sovereign spreads. In our set-up the small variability of default

¹⁶In this respect, Kehoe and Perri (2002) show that adding limited enforcement constraints into a standard complete markets RBC-like model reduces investment volatility relative to output by more than 85 percent (from 25.23 to 3.04 percent).

probabilities is driven by the fact that the increased ability to smooth out consumption allows to increase the degree of state-contingency in the economy without the need to vary the risk of default. Nevertheless, the low variability in default probabilities is common in other quantitative models of sovereign debt and default.^{17,18}

As for the debt and default properties, the mean debt to output ratio delivered by the model is 17.47 percent. Although this number is small when compared to the data, it is significantly larger than the ones produced by other existing models.¹⁹ The relatively higher debt to output ratio obtained by the model is closely related to the increased degree of state-contingency observed and to the endogenous penalties of default introduced when modeling output dynamics in and out financial autarky, both of which allow for a greater extension of credit. Results also show that default is a rare event. The maximum number of defaults observed in our simulated economies is 2 per 100 years, which occurs in 3.2 percent of the cases analyzed (25 percent of the cases present a single default occurrence within the same period of time, while in the rest of the cases default does not occur). This translates into a rate of default of 0.07 percent, which is much lower than the rate of default of our country sample of 0.77 percent.

3.5 Sensitivity analysis

In this section we analyze the effects of changes in the values of β , λ and γ on the business cycles and debt and default properties of the model. These parameters are closely related to sovereign debt and default dynamics, therefore, studying their impact on the model's behavior may shed light on the direction towards which the benchmark calibration should move in order to improve upon the results presented above.

Changing the degree of agents' impatience through changes in β has a clear impact on the rate of default of the economy, as columns 2 and 3 in Table 6 show. In particular, when agents are more

¹⁷Additionally, Hatchondo et al. (2010) document that the discretization of the state variables introduces discontinuities in the bond price schedule that may overestimate the variability of interest rate spreads. When re-solving existing models with the DSS technique but with finer grids and with continuous methods, they find that the volatilities reported in other studies in the literature significantly decrease.

¹⁸As an alternative to introduce mechanisms that increase the variability of default probabilities, the existing literature has proposed different ways to loosen the relationship between these and debt prices in this type of models. These include modeling the debt renegotiation process after default, which makes debt prices also dependent on debt recovery rates (as in Yue (2010)), introducing longer-term debt, which adds a short- vs long-term trade off in the issuance and riskiness of debt into the set-up (as in Hatchondo and Martinez (2009) and Arellano and Ramanarayanan (2012)), and assuming that international creditors are risk averse, which makes them sensitive to changes in their valuation of consumption over time and hence to fluctuations in the world interest rate (as in Arellano's (2008) example).

¹⁹Arellano (2008), for instance, reports a mean debt to output ratio of 5.95 percent in her benchmark model (calibrated to match Argentinian default episode in 2001). This accounts only for the 17 percent of Argentinian average debt to output ratio from 1980 to 2004 (35 percent) reported by Mendoza and Yue (2012).

impatient, i.e. when β equals 0.87, the rate of default increases by more than 70 percent, from 0.07 to 0.12 percent. Lower levels of β reduce the value of future consumption which, in turn, decreases the losses of being outside the international credit markets and having to face direct output costs, thus, increasing incentives to default. As a result of this, the model observes a higher variability in default probabilities that, consequently, translates into the increase of sovereign spreads volatility and a higher correlation between sovereign spreads and output.

Regarding the effects of changing the average time that the economy stays in financial autarky after default, we analyze the cases where λ is equal to 0.044 and 0.25 which imply average stays in autarky of 5.4 and 1 years, respectively. As columns 4 and 5 in Table 6 show, changes in λ have a great impact in both the amount of debt that the model is able to sustain and on borrowing and investment dynamics. In particular, when the economy is outside the international credit markets for a longer period of time after default, i.e. when λ equals 0.044, the debt to output ratio goes up by around 90 percent, from 17.47 to 33.22 percent. Increasing default penalties has two main consequences: first, it reduces default incentives which make international investors willing to lend more resources and, second, it forces the economy to adjust borrowing and investment levels by more in order to avoid the increased penalties of default. Notice also that, as a consequence of the higher levels of debt, the probability of default becomes slightly more volatile and translates into more volatile sovereign spreads. Interestingly, the negative relationship between sovereign spreads and output slightly loosens, suggesting that the overall effect of increasing the average stay in autarky in our calibration is a mild increase in default probabilities.

Finally, columns 6 and 7 in Table 6 show the effects of changing the proportion of output that is lost while the economy remains in financial autarky after default. Increasing the proportion of loss from 0.02 to 0.03, has similar effects than increasing the average stay in financial autarky. This is, the extent of credit increases (this time by more than 50 percent, to 26.51 percent) and investment, borrowing and spreads dynamics are more volatile. In contrast to the case where λ is decreased, the negative relationship between sovereign spreads and output strengthens in this case, which indicates that increasing direct output losses after default is an effective way to decrease default probabilities.

Table 6. Business cycles and sovereign debt and default properties: sensitivity analysis

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Benchmark	$\beta = 0.87$	$\beta = 0.93$	$\lambda = 0.044$	$\lambda = 0.25$	$\gamma = 0.01$	$\gamma = 0.03$
$\sigma(y)$	3.10	3.20	3.04	3.17	3.04	3.04	3.16
$\sigma(c)/\sigma(y)$	0.87	0.92	0.84	0.93	0.84	0.85	0.89
$\sigma(l)/\sigma(y)$	0.69	0.68	0.69	0.69	0.69	0.69	0.68
$\sigma(i/y)$	0.92	0.92	0.93	1.42	0.71	0.73	1.18
$\sigma(nx/y)$	0.60	0.73	0.55	1.34	0.25	0.30	0.96
$\rho(nx/y, y)$	-0.28	-0.24	-0.29	-0.21	-0.29	-0.29	-0.26
$\rho(nx/y, i/y)$	-0.84	-0.89	-0.80	-0.95	-0.66	-0.70	-0.91
$\sigma(spr)$	0.012	0.021	0.014	0.019	0.008	0.011	0.015
$\rho(spr, y)$	-0.26	-0.58	-0.05	-0.10	0.14	-0.06	-0.33
Mean($debt/y$)	17.47	17.70	17.45	33.22	7.65	8.70	26.51
Default rate	0.07	0.12	0.04	0.06	0.06	0.07	0.07

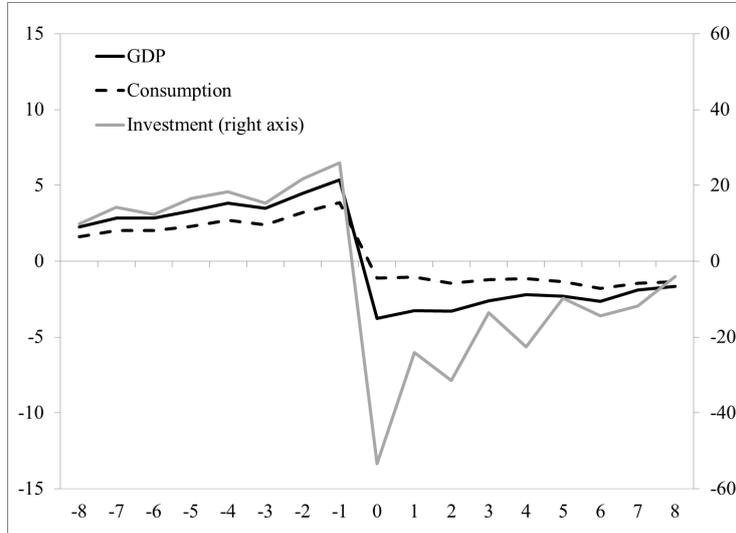
Notes: All standard deviations, the mean debt to output ratio and the default rate are in percentages.

3.6 The model around default episodes

As stated before, sovereign defaults are generally associated with significant declines in economic activity, however, given the nature of the existing models of sovereign debt and default, the literature has been unable to address the interactions between the two of them. This framework constitutes a step towards closing this gap.

In order to analyze the behavior of the model's dynamics around sovereign default, we compute the average deviations of output, consumption and investment from their respective trends across default occurrences in the benchmark model. Figure 6 depicts our results. As it can be observed, the model is able to endogenously account for the declines in the three variables around default episodes. Furthermore, in line with the data, the model produces investment deviations from trend of larger magnitude than those of output and consumption, -52.3 vs -3.8 and -1.4, respectively. Upon default, output drops due to the default penalties to which the economy is exposed. Given agents' impatience and the lack of access to external resources, investment goes down in order to smooth out consumption which, despite the significant decline in savings, decreases as well but in a smaller proportion relative to the drops in output and investment.

Figure 6. Output, consumption and investment deviations from trend around default episodes



Notes: The date of default is normalized to zero. Deviations are in percentages.

4 Conclusions

This paper develops a small open economy model with financial frictions, within which output and default risk are jointly determined, that allows to simultaneously examine the behavior of output, consumption and investment and their interaction with sovereign debt and default. The model is consistent with stylized facts showing that economies are more likely to default in bad times and as their level of outstanding debt increases. When calibrated to match the business cycles properties of an average emerging market economy, it is able to reproduce the countercyclicality of net exports, the countercyclicality of sovereign spreads and the negative correlation between investment and net exports observed in the data. However, it fails to deliver sufficiently high default rates and enough variability in consumption and sovereign spreads due to the economy's increased ability to smooth out consumption (that introducing investment dynamics brings about). These findings suggest that adding mechanisms that reduce the degree of substitutability between borrowing and savings might potentially contribute to improve upon our results. Furthermore, our sensitivity analysis, shows that changing the values of the discount factor, the probability of staying in financial autarky before redemption and the direct output costs of default, which are closely related to debt and default dynamics, could also help in taking the model's implications closer to the empirical evidence.

The framework developed constitutes a step towards closing the current gap between the data, which shows that sovereign default episodes are associated with important declines in economic activity, and the existing literature on sovereign debt and default, which mostly focuses on examining endowment economies and, therefore, is unable to address the interaction between default risk and macroeconomic dynamics. Specifically, when analyzing the model's dynamics around sovereign default, we find that it is able to endogenously produce declines in output, consumption and investment of similar magnitude as the ones observed in the data. This confirms the fact that sovereign defaults contribute to the downturn in economic activity and that a future line of research should focus on the assessment of the quantitative importance of these episodes's effects on macroeconomic dynamics.

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6 Appendix

6.1 Proofs

Proof of Proposition 1:

First, notice that $V^G(a, k, z)$ increases monotonically in assetholdings:

Let $u(c, l)$ be such that, given (1), optimal labor, l^* , is independent of c , i.e. $l^*(k, z)$. Consider any pair of assetholdings levels a_1 and a_2 such that $0 \geq a_1 \geq a_2$, then for combinations (a_1^*, k_1^*) and (a_2^*, k_2^*) were $V^G(a_1, k, z)$ and $V^G(a_2, k, z)$ are attained, respectively:

$$\begin{aligned} V^G(a_1, k, z) &\equiv u(e^z f(k, l(k, z))) + (1 - \delta)k + a_1 - q(a_1^*, k_1^*, z)a_1^* - k_1^*, l(k, z) + \beta E[V(a_1^*, k_1^*, z')] \\ &\geq u(e^z f(k, l(k, z))) + (1 - \delta)k + a_1 - q(a_2^*, k_2^*, z)a_2^* - k_2^*, l(k, z) + \beta E[V(a_2^*, k_2^*, z')] \\ &\geq u(e^z f(k, l(k, z))) + (1 - \delta)k + a_2 - q(a_2^*, k_2^*, z)a_2^* - k_2^*, l(k, z) + \beta E[V(a_2^*, k_2^*, z')] \\ &\equiv V^G(a_2, k, z) \end{aligned}$$

The proof of Proposition 1 is now straightforward:

Given that $V^G(a, k, z)$ increases monotonically in assetholdings and $V^B(k, z)$ is independent of assetholdings, if $V^B(k, z) \geq V^G(a_1, k, z)$, then for $0 \geq a_1 \geq a_2$, $V^B(k, z) \geq V^G(a_1, k, z) \geq V^G(a_2, k, z)$.

QED.

6.2 The data

The representative emerging economy is characterized by the average statistics of a country sample consisting of 11 emerging economies: Argentina, Brazil, Ecuador, Indonesia, Malaysia, Mexico, Peru, Philippines, Russia, South Africa and Turkey. All of these countries belong to the set included in the literature documenting emerging economies' stylized facts (see Neumeyer and Perri (2005) and Aguiar and Gopinath (2007)), are emerging economies that defaulted in the past 15 years (see Beers and Chambers (2006) and Beers, Chambers and Ontko (2010)), or both.

Collecting and standardizing data on national accounts and sovereign debt for emerging economies is not an easy task; the process of gathering and organizing aggregate data within a country is costly and, being the economies in our sample still in a developing stage, it is common that not many resources have been devoted to build up large historical information sets, specially on a quarterly basis. Despite this,

we choose to keep this time frequency because one of our interests is to study the RBC properties of the model. Most of the data thus covers the 1990s and 2000s; however, for each country and variable, our time series go as far back in time as possible. The specifics of the data set are as follows.

National accounts data:

Real GDP, consumption (defined as the sum of private and government consumption, change in inventories and a statistical discrepancy), investment (defined as gross fixed capital formation) and net exports are seasonally adjusted and expressed in domestic currency. For most of the cases the data is from the International Financial Statistics service of the International Monetary Fund (IFS); the exceptions are Argentina and Ecuador whose data was obtained from the Ministry of Finance (Ministerio de Economía) and the Central Bank (Banco Central del Ecuador), respectively. Available periods vary across countries as follows: Argentina 1993Q1-2010Q2, Brazil 1995Q1-2010Q1, Ecuador 1993Q1-2010Q3, Indonesia 1997Q1-2010Q2, Malaysia 1991Q1-2010Q2, Mexico 1981Q1-2010Q2, Peru 1980Q1-2010Q2, Philippines 1981Q1-2010Q1, Russia 1995Q1-2009Q3, South Africa 1980Q1-2010Q2 and Turkey 1987Q1-2010Q1.

Sovereign spreads:

Sovereign spreads are measured by the EMBI+ Government Bond Spread taken from the Global Financial Data database. Available starting dates vary across countries as follows: Argentina 1994Q1, Brazil 1994Q2, Ecuador and South Africa 1995Q1, Turkey 1996Q3, Malaysia 1996Q4, Mexico, Peru, Philippines and Russia 1998Q1 and Indonesia 2004Q1. All series end in 2010Q4.

6.3 Optimization algorithm

The optimization algorithm follows Hatchondo et al. (2010) and extends it to allow for capital stock as an additional endogenous state variable. Specifically, it consists of the following steps:

1. Create grids for asseholdings, capital stock and productivity shocks within the approximation set $[\underline{a}, \bar{a}] \times [\underline{k}, \bar{k}] \times [\underline{z}, \bar{z}]$ using Chebyshev nodes.
2. Guess initial values for $V^{G(0)}$ and $V^{B(0)}$ and use them to determine $V^{(0)}$ for every combination of (a, k, z) .
3. Use $V^{G(0)}$, $V^{B(0)}$ and $V^{(0)}$ to solve for (2) and (3) for every combination of (a, k, z) .

Value functions are approximated using Chebyshev collocation for all (a, k, z) that belong to the approximation set $[\underline{a}, \bar{a}] \times [\underline{k}, \bar{k}] \times [\underline{z}, \bar{z}]$. To get optimal values of a' and k' a derivative-free nonlinear optimization routine is used. The initial guesses $a'^{(0)}$ and $k'^{(0)}$ used to feed the optimization routine are set equal to the solution derived from solving the same problem with the DSS technique.

Notice that whenever the sovereign's objective functions are evaluated within the optimization routine $E[V(a', k', z')|z]$, $E[V^B(k', z')|z]$ and $E[V(0, k', z')|z]$, as well as $q(a', k', z)$, need to be determined. The expectational objects are calculated as numerical integrals using Gauss-Legendre quadrature while

$V(a', k', z')$, $V^B(k', z')$ and $V(0, k', z')$ are approximated with $V^{G(0)}$, $V^{B(0)}$ and $V^{(0)}$. Knowing the expected values of next-period's V^G and V^B allows us to determine the probability of default and hence to evaluate $q(a', k', z)$ using (4). Notice also that, in the search for the optimal level of a' and k' or while determining the possible values of z' given z to evaluate the numerical integrals, the algorithm may want to evaluate the corresponding value functions on values for a , k or z lying outside the approximation set. Whenever this is the case, a first-order Taylor approximation is performed around the closest point in the approximation set to calculate the value functions.

4. Use the results in step 3 to update the value functions $V^{G(1)}$ and $V^{B(1)}$ and to determine $V^{(1)}$.
5. Use $V^{G(i)}$ and $V^{B(i)}$ to repeat step 3 and step 4 until $|V^{(i)} - V^{(-i)}| < \varepsilon$, where i is the number of iteration and $\varepsilon = 5e^{-4}$.