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Spurious Long-Horizon Regression in Econometrics*

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Abstract

This paper extends recent research on the behaviour of the t-statistic in a long-horizon regression (LHR). We assume that the explanatory and dependent variables are generated according to the following models: a linear trend stationary process, a broken trend stationary process, a unit root process, and a process with a double unit root. We show that, both asymptotically and in finite samples, the presence of spurious LHR depends on the assumed model for the variables. We propose an asymptotically correct inferential procedure for testing the null hypothesis of no relationship in a LHR, which works whether the variables have a long-run relationship or not. Our theoretical results are applied to an international data set on money and output in order to test for long-run monetary neutrality. Under our new approach and using bootstrap methods, we find that neutrality holds for all countries.
Keywords: Long-horizon regression, asymptotic theory, deterministic and stochastic trends, unit roots, structural breaks, long-run monetary neutrality.
JEL Classification: C12, C22, E51.

Resumen

Este artículo extiende investigación reciente sobre el comportamiento del estadístico t en una regresión de horizonte largo (RHL). Suponemos que la variable explicativa y la dependiente son generadas de acuerdo a los siguientes modelos: un proceso lineal estacionario en tendencias, uno de tendencia quebrada, uno de raíz unitaria, y uno con doble raíz unitaria. Mostramos tanto asintóticamente como en muestras finitas, que la presencia de RHL espuria depende del modelo asumido para las variables. Proponemos un procedimiento de inferencia asintóticamente correcto para probar la hipótesis nula de no relación en una RHL, el cual funciona ya sea que las variables tengan una relación de largo plazo o no. Nuestros resultados teóricos son aplicados a un conjunto internacional de datos sobre producto y dinero para probar neutralidad monetaria de largo plazo. Bajo nuestro nuevo enfoque, y usando métodos de remuestreo, encontramos que la neutralidad se mantiene para todos los países.
Palabras Clave: Regresión de horizonte largo, teoría asintótica, tendencias deterministas y estocásticas, raíces unitarias, cambios estructurales, neutralidad monetaria de largo plazo.

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1 Introduction

Valkanov (2003) studies the asymptotic behaviour of the t-statistic in a long-horizon regression and finds that, under a local-to-unity framework in an AR process, this statistic diverges, even when both variables are generated independently of each other. A similar divergent behaviour of the t-statistic is reported in Lee (2007) for the case of independent fractionally integrated processes.

This has potentially important implications for statistical inference in areas of economics in which Long-Horizon Regressions (LHRs) have been used; for example in the areas of stock returns predictability, the Fisher effect, monetary neutrality, and the exchange rate and fundamentals.[1] For instance, a popular time-series approach for testing long-run monetary neutrality, put forward by Fisher and Seater (1993), is based on the time series properties of money and output, and on the t-statistic in a long-horizon regression model. In this context, divergence of the t-statistic would indicate, with probability approaching one as the sample size grows to infinity, a long-run relationship between money and output, i.e., rejection of the (long-run) monetary neutrality proposition.

After more than two decades of research on testing for unit roots, there is still no consensus on the source of persistence in the long-run behaviour of many macro variables. There is a variety of empirical models in the literature that support different types of long-run nonstationary behaviour, with linear trends, linear trends with breaks and unit roots being very popular ones.[2]

In this paper, we study the asymptotic behaviour of the t-statistic in a long-horizon regression under different combinations of linear trends, linear trends with breaks and unit root processes among the dependent and explanatory variables. This has not been explored before in the literature of long-horizon regressions, and has important inferential implications in applied work in economics.

Our results show that the asymptotic spurious long-horizon regression phenomenon depends on the assumed data generating process for both the dependent and the explanatory variables. For instance, we corroborate Valkanov's finding when both variables follow a unit root, but we also find that when the explanatory variable follows a unit root while the dependent follows a linear trend, then the t-statistic does not diverge. On the other hand, when both variables follow a broken trend model, the statistic diverges, spuriously rejecting

[1] See Valkanov (2003) for an interesting discussion of the first three areas in the context of LHR, and Chen and Chou (2010) for empirical evidence on the relationship between exchange rates and fundamentals.

[2] There is also a literature on testing for a unit root against some non-linear alternative model, as the Threshold Autoregressive models, Smooth Transition Autoregressive models, and the Markov Switching models (see for instance Caner and Hansen (2001), Kapetanios, et. al. (2003), and Nelson, et. al (2001)). The analysis of these cases is out of the scope of the paper.

the null of no relationship (between independent variables). We arrive at such conclusion from the calculation of the order in probability for the t-statistic in a long-horizon regression. In general, we find that a spurious long-horizon regression problem will arise whenever the dependent variable, the explanatory variable, or both, are hit by a permanent shock, which could be of an stochastic or deterministic nature. In order to alleviate this problem, we introduce a procedure that asymptotically guarantees correct inference, whether the variables have a long-run relationship or not: given a significance level, when the variables are independent, the null of no relationship will not be rejected, while when the variables are cointegrated, the null will be rejected, correctly indicating a long-run relationship.

We also present new evidence on monetary neutrality for the international data set analyzed in Noriega (2004) and Noriega *et. al.* (2008), which comprises money and output data for Australia, Argentina, Brazil, Canada, Italy, Mexico, Sweden, and the UK. Based on our asymptotic results, we test for long-run neutrality by rescaling a bootstrapped t-statistic in a long-horizon regression for each country, using output and money data generated from models identified by Noriega *et. al.* (2008). Our results indicate that, in all cases, monetary neutrality cannot be rejected.

We discuss in section 2 the issue of the trending mechanisms for the variables. Section 3 presents the asymptotic results, which we derive both under the null of no long-run relationship between the variables, and under the alternative of cointegration. The finite sample counterpart of our limit theory is analyzed in section 4 via simulations. Section 5 presents the empirical application of testing for monetary neutrality. The final section concludes.

2 Trending mechanisms for the data

Since the early 1980s, a great deal of effort has been devoted to uncover the trending nature of economic time series.[3] However, the issue has not been resolved yet, and while there are authors who favor the use of stochastic trends for macro data, there are others in favour of deterministic ones. Some have even argued that data can be uninformative as to whether the long-run trend is better described as deterministic or stochastic.[4]

In general, the empirical literature has shown that the long-run behavior of macro data can be well characterized using linear deterministic trends, linear deterministic trends with

[3]Some examples of this can be found in Nelson and Plosser (1982), Perron (1989, 1992, 1997), Murray and Nelson (2000), Cook (2005), Assaf (2008), Maslyuk and Smyth (2008), Rahman and Saadi (2008), and Kim and Perron (2009).

[4]For an analysis on whether the long-run trend function of US real output should be modelled as trend stationary or difference stationary see, among others, Rudebusch (1993), Diebold and Senhadji (1996), Nelson and Murray (2000), Papell and Prodan (2004), Vougas (2007), and Darne (2009).

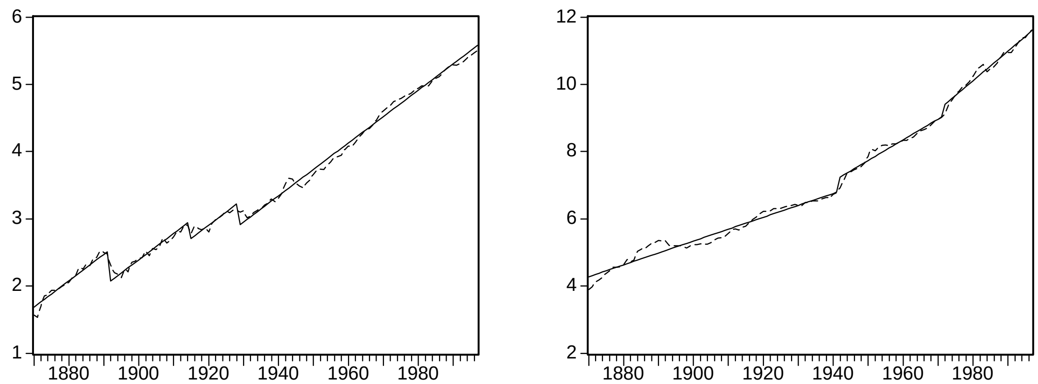
structural breaks, or stochastic trends. As an example of this set of models, Noriega *et. al.* (2008, NSV hereafter) found that for Australia, Sweden and the UK, long-annual data on output and money seem to be well characterized by a Broken Trend Stationary (BTS) model. As an illustration, the case of Australia is reproduced in Figure 1, where the data is shown (y_t is real output and x_t is money) together with its respective fitted (broken) trend.

Figure 1

Long-run trend for Australia output and money

$y_t \sim I(0) + 3$ breaks — 1891 (L), 1914 (L), 1928 (L)

$x_t \sim I(0) + 2$ breaks — 1941 (LT), 1971 (LT)



Note: L (LT) stands for Level (Level and Trend)

NSV also find that for Canada, y_t seems to follow a linear trend model, while x_t a broken trend one. For Argentina and Mexico, y_t follows a BTS model, while x_t a unit root one. For Brazil y_t follows a BTS model, while x_t follows a double unit root process. Furthermore, for the US and Denmark, deviations of x_t and y_t from a linear trend seem to reject a unit root, making y_t and x_t Trend Stationary, or TS. Finally, for Italy, a unit root [or $I(1)$] model was most supported by the data for both x_t and y_t.

From the above examples, and indeed from many other examples in the empirical literature, it seems reasonable to assume that linear trends, broken trends, and stochastic trends are capable of adequately represent the long-run behaviour of macro data. We therefore study inference using Long-Horizon Regressions (LHRs) under four DGPs, described in the following assumption:

Assumption. The DGPs for $z = y, x$ are as follows

Case	Name	Model
1.	TS	$z_t = \mu_z + \beta_z t + u_{zt}$
2.	BTS	$z_t = \mu_z + \theta_z DU_{zt} + \beta_z t + \gamma_z DT_{zt} + u_{zt}$
3.	$I(1)$	$\Delta z_t = \mu_z + u_{zt}$
4.	$I(2)$	$\Delta^2 x_t = u_{xt}$

where u_{yt} and u_{xt} are independent innovations obeying the general level conditions of Assumption 1 in Phillips (1986), and DU_{zt}, DT_{zt} are dummy variables allowing changes in the trend's level and slope respectively, that is, $DU_{zt} = \mathbf{1}(t > T_{b_z})$ and $DT_{zt} = (t - T_{b_z})\mathbf{1}(t > T_{b_z})$, where $\mathbf{1}(\cdot)$ is the indicator function, and T_{b_z} is the unknown date of the break in z. We denote the break fraction as $\lambda_z = (T_{b_z}/T) \in (0, 1)$, where T is the sample size. The DGPs include both deterministic and stochastic trending mechanisms, with 12 possible nonstationary combinations of them among the dependent and explanatory variables.[5] Note that the $I(2)$ case is restricted to the x_t variable only. This is so because of our particular interest on testing for monetary neutrality, which we explore in Section 5. In this regard, empirical evidence suggests that the $I(2)$ case might be relevant for nominal variables in levels, such as the level of monetary aggregates or the price level, but not for data in real terms, or growth rates of nominal data.[6]

Lemma 1 collects useful results on the innovations of the Assumption for subsequent analysis. The proof is given in Appendix 1.

Lemma 1. *For $z = y, x$, let the random variables $\{u_{zt}\}_1^\infty$ obey Assumption 1 in Phillips (1986, p. 313). Define $\sum_{j=1}^{k} u_{zt-j+1} = S_{zt}$, and let the sample size T and the length of the horizon k grow such that $\frac{k}{T} \to \kappa \in (0, 1)$ when both $T \to \infty$ and $k \to \infty$. Then*

a) $T^{-1/2} \sum_{t=k+1}^{T} (u_{zt} - u_{zt-k}) \equiv Su_z = O_p(1)$

b) $T^{-1} \sum_{t=k+1}^{T} (u_{zt} - u_{zt-k})^2 \equiv Su_z 2 = O_p(1)$

c) $T^{-1/2} \sum_{t=k+1}^{T} (u_{xt} - u_{xt-k})(u_{yt} - u_{yt-k}) \equiv Su_{xy} = O_p(1)$

d) $T^{-3/2} \sum_{t=k+1}^{T} S_{zt} \equiv SSu_z = O_p(1)$

e) $T^{-2} \sum_{t=k+1}^{T} S_{zt}^2 \equiv SSu_z 2 = O_p(1)$

f) $T^{-2} \sum_{t=k+1}^{T} S_{xt} S_{yt} \equiv SSu_x Su_y = O_p(1)$

g) $T^{-3/2} \sum_{t=k+1}^{T} (u_{zt} - u_{zt-k})(DT_t - DT_{t-k}) \equiv Su_z DT = O_p(1)$

h) $T^{-1} \sum_{t=k+1}^{T} S_{xt}(u_{yt} - u_{yt-k}) \equiv SS_{xy} = O_p(1)$

i) $T^{-5/2} \sum_{t=k+1}^{T} S_{xt}(DT_t - DT_{t-k}) \equiv SSu_x DT = O_p(1)$

[5] The cases of $I(1)$ processes with fractionally integrated processes is studied in Lee (2007).

[6] See for instance Juselius (1996, 1999), Haldrup (1998), Muscatelli and Spinelli (2000), Coenen and Vega (2001), and Nielsen (2002).

The asymptotic analysis of next section will make use of sample moments of the models in the Assumption. We collect them in Lemma 2 by factoring out descending powers of the sample size. In this way, the orders in probability can be determined by retaining only the asymptotically relevant terms, upon a suitable normalization. We omit the proof since these sample moments can be obtained by (sometimes tedious but) direct calculation, using results in Lemma 1.[7]

LEMMA 2. *If, for $z = y, x$, the random variables $\{u_{zt}\}_1^\infty$ obey Assumption 1 in Phillips (1986, p. 313) and $\frac{k}{T} \to \kappa \in (0,1)$ when both $T \to \infty$ and $k \to \infty$, then the sample moments of $z_t^* \equiv \sum_{j=1}^{k} \Delta^{\langle z \rangle} z_{t-j+1}$, can be written as follows:*

a) For the case of z following a TS process:
i) $\sum_{t=k+1}^{T} z_t^* = \kappa(1-\kappa)\beta_z T^2 + Su_z T^{1/2}$
ii) $\sum_{t=k+1}^{T} z_t^{*2} = \kappa^2(1-\kappa)\beta_z^2 T^3 + 2\beta_z \kappa Su_z T^{3/2} + Su_z 2T$
iii) $\sum_{t=k+1}^{T} x_t^* y_t^* = \kappa^2(1-\kappa)\beta_x\beta_y T^3 + \kappa\left(\beta_x Su_y + \beta_y Su_x\right) T^{3/2} + Su_{xy} T^{1/2}$

b) For the case of z following a BTS process:[8]
i) $\sum_{t=k+1}^{T} z_t^* = \left\{\kappa(1-\kappa)\beta_z + \left[\kappa(1-\kappa) - \frac{1}{2}\lambda_z^2\right]\gamma_z\right\} T^2 + O(T)$
ii) $\sum_{t=k+1}^{T} z_t^{*2} = \left\{(1-\kappa)\kappa^2\beta_z^2 + \kappa\left[2(1-\kappa)\kappa - \lambda_z^2\right]\beta_z\gamma_z + \left[(1-\kappa)\kappa^2 - \kappa\lambda_z^2 + \frac{1}{3}\lambda_z^3\right]\gamma_z^2\right\} T^3$
$+O(T^2)$
iii) $\sum_{t=k+1}^{T} x_t^* y_t^* = [\kappa^2(1-\kappa)\beta_x\beta_y + \kappa\beta_x\gamma_y\left[\kappa(1-\kappa) - \frac{1}{2}\lambda_y^2\right]$
$+\kappa\beta_y\gamma_x\left[\kappa(1-\kappa) - \frac{1}{2}\lambda_x^2\right] + \gamma_x\gamma_y f_{\lambda\kappa}]T^3 + O(T^2)$
where $f_{\lambda\kappa} \equiv \frac{1}{3}(\kappa-\lambda_d)^3 + \frac{1}{2}\lambda_d(\kappa-\lambda_d)^2 + \kappa\lambda_d(\kappa-\lambda_d) - \frac{1}{3}(\kappa-\lambda_y)^3 - \frac{1}{2}\lambda_d(\kappa-\lambda_y)^2$
$+\frac{1}{2}\kappa\lambda_d^2 + (1-\kappa-\lambda_y)\kappa^2$, and $\lambda_d = \lambda_y - \lambda_x$

c) For the case of z following an $I(1)$ process:
i) $\sum_{t=k+1}^{T} z_t^* = \kappa(1-\kappa)\mu_z T^2 + SSu_z T^{3/2}$
ii) $\sum_{t=k+1}^{T} z_t^{*2} = \kappa^2(1-\kappa)\mu_z^2 T^3 + 2\kappa\mu_z SSu_z T^{5/2} + SSu_z 2T^2$
iii) $\sum_{t=k+1}^{T} x_t^* y_t^* = \kappa^2(1-\kappa)\mu_x\mu_y T^3 + \kappa\left[\mu_x SSu_y + \mu_y SSu_x\right] T^{5/2} + SSu_x Su_y T^2$

d) For the combination where y follows a BTS process, while x a TS process (for the reverse case simply interchange the y and x subscripts):

$\sum_{t=k+1}^{T} x_t^* y_t^* = \left\{\beta_x\beta_y\kappa^2(1-\kappa) + \beta_x\gamma\kappa\left[\kappa(1-\kappa) - \frac{1}{2}\lambda^2\right]\right\} T^3 + (\beta_x\kappa\theta\lambda + \frac{1}{2}\beta_x\kappa\lambda\gamma)T^2 + (\beta_x\kappa Su_y + \beta_y\kappa Su_x + \gamma Su_x DT)T^{3/2} + O_p(T)$

e) For the combination where y follows a TS process, while x an $I(1)$ process (for the reverse case simply interchange the y and x subscripts):

[7] Calculations were assisted by the software Mathematica 6.

[8] For the BTS processes we compute the sample moments assuming that $\kappa > \lambda_z$ and $\kappa + \lambda_z < 1$, $z = y, m$. These assumptions imply that the break occurs in the first half of the sample, and does not affect results in terms of order of magnitude. This also applies to cases d) and f) below.

$\sum_{t=k+1}^{T} x_t^* y_t^* = \kappa^2(1-\kappa)\mu_x\beta_y T^3 + \kappa\beta_y SSu_x \cdot T^{5/2} + \kappa\mu_x Su_y \cdot T^{3/2} + SS_{xy}T$

f) For the combination where y follows a BTS process, while x an $I(1)$ process (for the reverse case simply interchange the y and x subscripts):

$\sum_{t=k+1}^{T} x_t^* y_t^* = \left\{\kappa^2(1-\kappa)\mu_x\beta + \mu_x\gamma\kappa\left[\kappa(1-\kappa) - \frac{1}{2}\lambda^2\right]\right\} T^3 + (\kappa\beta SSu_x + \gamma SSu_x DT)T^{5/2} + O_p(T^2)$.

As expected, deterministic trends dominate asymptotically. These moments represent the main input for the asymptotic analysis of next section.

3 Asymptotics for long-horizon regressions

In this section we will be concerned with the asymptotic behaviour of the t-statistic $t_{\widehat{\delta}_k}$ for testing the null hypothesis $H_0 : \delta_k = 0$ in the following estimated long horizon regression:

$$y_t^* = \widehat{\alpha}_k + \widehat{\delta}_k x_t^* + \widehat{\varepsilon}_{kt} \tag{1}$$

where $z_t^* \equiv \sum_{j=1}^{k} \Delta^{\langle z\rangle} z_{t-j+1}$, for $z = y, x$; ε_k is a mean zero random variable, and Δ represents the difference operator ($\Delta^j z_t \equiv (1-L^j)z_t = z_t - z_{t-j}$, where L is the lag operator). The notation $\langle z\rangle$ refers to the order of integration of z, i.e. $\langle z\rangle = 1$ means that z is integrated of order one, $I(1)$.

In the literature of monetary neutrality for example, equation (1) is the vehicle advocated by Fisher and Seater (1993) for testing long-run monetary neutrality (for the case of $\langle z\rangle = 1$) and superneutrality (for the case of $\langle y\rangle = 1$, $\langle x\rangle = 2$).

In the asymptotic analysis of this paper, we allow both the sample size T and the length of the horizon k to grow, but restrict their ratio to converge to a finite constant, i.e., $\frac{k}{T} \to \kappa \in (0,1)$ when both $T \to \infty$ and $k \to \infty$ [this is the approach followed by Richardson and Stock (1989), Valkanov (2003), and Lee (2007)]. Equation (1) can be written in matrix form $y = X\beta_k + \varepsilon$, with y a $T \times 1$ vector of y_t data, with $\sum_{j=1}^{k} \Delta^{\langle y\rangle} y_{t-j+1}$ as its tth element, X a $T \times 2$ matrix comprising a constant term and data on x_t with $\sum_{j=1}^{k} \Delta^{\langle x\rangle} x_{t-j+1}$ as its tth element, and ε a $T \times 1$ vector of zero mean disturbances.

The vector of OLS estimators is defined as:

$$\widehat{\beta}_k = \begin{bmatrix} \widehat{\alpha}_k \\ \widehat{\delta}_k \end{bmatrix} = (X'X)^{-1}X'y,$$

where

$$X'X = \begin{bmatrix} T(1-k) & \Sigma_{t=k+1}^{T} x_t^* \\ \Sigma_{t=k+1}^{T} x_t^* & \Sigma_{t=k+1}^{T} x_t^{*2} \end{bmatrix}, \text{ and } X'y = \begin{bmatrix} \Sigma_{t=k+1}^{T} y_t^* \\ \Sigma_{t=k+1}^{T} x_t^* y_t^* \end{bmatrix}.$$

The t-statistic is defined by

$$t_{\widehat{\delta}_k} = \widehat{\delta}_k \left[\widehat{\sigma}_\varepsilon^2 (X'X)_{22}^{-1}\right]^{-1/2},$$

where $\widehat{\sigma}_\varepsilon^2$ is the estimated regression variance,

$$\widehat{\sigma}_\varepsilon^2 = \frac{\Sigma_{t=k+1}^T (y_t^* - \widehat{\alpha}_k - \widehat{\delta}_k x_t^*)^2}{T(1-k)}$$

and $(X'X)_{22}^{-1}$ denotes the 2^{nd} diagonal element of $(X'X)^{-1}$. As an illustration on the use of this framework, the long-run neutrality (LRN) proposition is verified using $t_{\widehat{\delta}_k}$ via testing the null hypothesis $H_0 : \delta_k = 0$ in (1). Under H_0, LRN holds, while rejection of the null implies failure of monetary neutrality.

We compute the order of magnitude and the asymptotic distribution of $t_{\widehat{\delta}_k}$ with the aid of a *Mathematica 6.0* code. For each combination of DGPs in the Assumption for y and x, we use the following steps, which we accompany with some lines from the code (Appendix 2 presents an example of the complete code).

1. We define the (symmetric) 2×2 matrix $X'X$, with elements
   $a_{11} = T(1-\kappa)$,
   $a_{12} = a_{21} = \Sigma_{t=k+1}^T x_t^*$,
   $a_{22} = \Sigma_{t=k+1}^T x_t^{*2}$,

and the 2×1 vector $X'y$ with elements
   $b_1 = \Sigma_{t=k+1}^T y_t^*$,
   $b_2 = \Sigma_{t=k+1}^T x_t^* y_t^*$.

We also define $c_1 = \Sigma_{t=k+1}^T y_t^{*2}$ for the calculation of the error variance.
In the case of estimating equation (1) using as DGPs the $I(1)$ model: $\Delta z_t = \mu_z + u_{zt}$, $z = y, x$, the matrices in the *Mathematica* code will include the following objects (see part *c)* in Lemma 2):
   $a11 = (1-\kappa) * T;$
   $a12 = a21 = \kappa * (1-\kappa) * \mu_x * T^2 + SSu_x * T^{3/2};$
   $a22 = \kappa^2 * (1-\kappa) * \mu_x^2 * T^3 + 2 * \kappa * \mu_x * SSu_x * T^{5/2} + SSu_x 2 * T^2;$
   $b1 = \kappa * (1-\kappa) * \mu_y * T^2 + SSu_y * T^{3/2};$
   $b2 = \kappa^2 * (1-\kappa) * \mu_x * \mu_y * T^3 + \kappa * (\mu_x * SSu_y + \mu_y * SSu_x) * T^{5/2} + SSu_x Su_y * T^2;$
   $c1 = \kappa^2 * (1-\kappa) * \mu_y^2 * T^3 + 2 * \kappa * \mu_y * SSu_y * T^{5/2} + SSu_y 2 * T^2;$

The asymptotic behavior of SSu_z and $SSu_z 2$, for $z = x, y$, and $SSu_x Su_y$ is presented in Lemma 1.

2. We compute the *OLS* estimators of $\widehat{\alpha}_k$ and $\widehat{\delta}_k$ from the product of $(X'X)^{-1}$ and $X'y$, and call them '*alpha*' and '*delta*' in the code, as follows:

$$XX = \begin{pmatrix} a11 & a12 \\ a21 & a22 \end{pmatrix};$$

$$invXX = Inverse[XX];$$

$$alpha = Factor[invXX[[1,1]] * b1 + invXX[[1,2]] * b2];$$

$$delta = Factor[invXX[[2,1]] * b1 + invXX[[2,2]] * b2];$$

3. Next, we define the numerator and denominator of *alpha* and *delta*, labeled '*alphanum*', '*alphaden*', '*deltanum*', '*deltaden*', and use *Mathematica* to find the maximum exponent of T of the elements of both the numerator and denominator of *alpha* and *delt*a, which we call '*expalphanum'*, '*expalphaden*', '*expdeltanum*' and '*expdeltaden*'. For instance, for the case of $\widehat{\delta}_k$ the code would be:

deltanum=Numerator[delta];

deltaden=Denominator[delta];

expdeltanum=Exponent[deltanum,T];

expdeltaden=Exponent[deltaden,T];

4. We then use *Mathematica* to find the limit of the numerator and denominator of *alpha* and *delta* normalized by T to the corresponding maxima powers '*expalphanum'*, '*expalphaden*', '*expdeltanum*' and '*expdeltaden*'. We call these limits '*numalpha*', '*denalpha*', '*numdelta*' and '*dendelta*'. In this way, we only preserve the asymptotically non-negligable terms of the numerator and denominator of the *OLS* estimators. Again, for the case of $\widehat{\delta}_k$ the code would look like:

numdelta=Limit[Expand[deltanum/ $T^{\text{expdeltanum}}$*],T* $\to \infty$];

dendelta=Limit[Expand[deltaden/ $T^{\text{expdeltaden}}$*],T* $\to \infty$];

5. The asymptotic distribution of the *OLS* estimators is then found by dividing the limits of the numerator and denominator obtained in the previous step. The order of magnitude is found by multiplying these ratios by the ratio of the maximum powers of T. For instance, the order of magnitude for $\widehat{\delta}_k$ (and the asymptotic distribution) comes from the following code line:

deltalim=(numdelta/dendelta)(* $T^{\text{expdeltanum}} / T^{\text{expdeltaden}}$*);*

6. We then follow the same steps for the error variance and the 2^{nd} diagonal element of $(X'X)^{-1}$.

7. From this code, we get the asymptotic behaviour for the three components of the t-statistic, which can be then assembled. For instance, in the example used above with both y and x following an $I(1)$ model, the *Mathematica* output shows that $\widehat{\delta}_k = O_p(1)$, $\widehat{\sigma}^2_\varepsilon = O_p(T)$,

and $(X'X)^{-1}_{22} = O_p(T^{-2})$, from which we can deduce that: $\widehat{\delta}_k \left[T^{-1}\widehat{\sigma}^2_\varepsilon T^2(X'X)^{-1}_{22}\right]^{-1/2} = T^{-1/2}t_{\widehat{\delta}_k}$ converges, implying that $t_{\widehat{\delta}_k}$ diverges at rate $\sqrt{T}$, or in other words, that $t_{\widehat{\delta}_k} = O_p(T^{1/2})$.

Following this procedure for each combination of DGPs in the Assumption, it is straightforward to prove the following theorem, which collects the main results. In the Theorem, a combination of DGPs is indicated by the pair $i - j$, $(i = 1, 2, 3,\ j = 1, 2, 3, 4)$ indicating that y_t is generated by case i, while x_t by case j, both defined in the Assumption. Thus, for instance, the combination $1 - 3$ corresponds to model (1) where y_t is TS (case 1), while x_t is $I(1)$ (case 3).

Theorem 1. *The order in probability of $t_{\widehat{\delta}_k}$ in model (1) depends on the combination of DGPs for y_t and x_t in the Assumption, as follows:*

a) $t_{\widehat{\delta}_k} = O_p(1)$ *for combination of cases* $1 - j$, $j = 1, 2, 3, 4$ *and* $i - 1, i = 1, 2, 3$;

b) $t_{\widehat{\delta}_k} = O_p(T^{1/2})$ *for combination of cases* $i - j, i = 2, 3$ *and* $j = 2, 3, 4$;
where x and y are generated independent from each other.

Theorem 1 shows that, even when y_t and x_t are generated independently from each other, the null hypothesis will be (spuriously) rejected in large samples when both variables follow a BTS process, or a unit root process. Furthermore, this phenomenon will prevail whenever a single or double unit root process interacts with a BTS process. In contrast with results in Hassler (2000), Kim, *et. al.* (2004) and Noriega and Ventosa-Santaulària (2007), the statistic does not diverge when at least one of the variables follows a linear trend.

Table 1 summarizes the above findings. The symmetry of results imply that the order in probability does not depend on the type of nonstationarity among dependent and explanatory variables.

Table 1. Orders in probability of $t_{\widehat{\delta}_k}$

y	x			
	TS	*BTS*	*I(1)*	*I(2)*
TS	1	1	1	1
BTS	1	$T^{1/2}$	$T^{1/2}$	$T^{1/2}$
I(1)	1	$T^{1/2}$	$T^{1/2}$	$T^{1/2}$

Results indicate that, as the sample size grows, it is more likely to spuriously reject the null when both variables follow some type of permanent shock, either deterministic or stochastic. Results also indicate that a spurious rejection is less likely asymptotically when either x_t or y_t (or both) follows a linear trend.[9]

Using results from Noriega and Ventosa-Santaulària (2006, 2007), Table 2 presents the orders in probability of the t-statistic in a regression model like (1), but with no horizon, that is, with $k = 1$. The DGPs are the same as those in Table 1.

Table 2. Orders in probability of $t_{\widehat{\delta}}$, $(k = 1)$

y	x			
	TS	*BTS*	*I(1)*	*I(2)*
TS	$T^{3/2}$	$T^{1/2}$	T	$T^{1/2}$
BTS	$T^{1/2}$	$T^{1/2}$	$T^{1/2}$	$T^{1/2}$
I(1)	T	$T^{1/2}$	T	$T^{1/2}$

A comparison of Tables 1 and 2 makes clear the effect of using a long-horizon framework: in seven of the twelve cases, the orders in probability in the long-horizon regression case are smaller than those corresponding to the regression model with no horizon. Furthermore, in six cases the statistic does not diverge. To a certain extent, the long-horizon framework alleviates the problem of asymptotic spurious regression.

In order to verify our large sample results, we simulated both the asymptotic and empirical distributions of the t-statistic $t_{\widehat{\delta}_k}$. Consider first the asymptotic distribution of the t-statistic $t_{\widehat{\delta}_k}$ in the long-horizon regression (1) under two cases:[10] 1) $t_{\widehat{\delta}_k}^{(1)}$, when y follows a TS process and x an $I(1)$ plus drift process, and 2) $t_{\widehat{\delta}_k}^{(2)}$, when y follows a BTS process and x an $I(1)$ plus drift process.

Corollary 1.

$$t_{\widehat{\delta}_k}^{(1)} \xrightarrow{D} N^{(1)}(W)\left[D^{(1)}(W)\right]^{-1/2}$$

$$T^{-1/2}t_{\widehat{\delta}_k}^{(2)} \xrightarrow{D} \gamma_y N^{(2)}(W)\left[D^{(2)}(W)\right]^{-1/2}$$

where

$$N^{(1)}(W) = \sigma_y\left\{(1-\kappa)\left[\int_\kappa^1 W_x(r)dW_y(r) - \int_\kappa^1 W_x(r)dW_y(r-\kappa)\right] - \int_\kappa^1 W_{x1}(r,\kappa)W_{y1}(r,\kappa)dr\right\}$$

[9]This is not surprising, given that LRN in the Fisher and Seater (1993) test requires the presence of permanent shocks. This is closely related to the approach of Noriega, et. al. (2008), where, under their deterministic notion of LRN, x_t should be long-run neutral when there are structural breaks in x_t (permanent deterministic shocks), while y_t follows a linear deterministic trend (i.e., with no permanent shock, neither stochastic nor deterministic).

[10]Here we present 2 cases, but we have made this exercise for all combinations of DGPs in the Assumption, and obtained qualitatively similar results across combinations.

$$D^{(1)}(W) = 2(\sigma_y^2 - \gamma_k)\left[(1-\kappa)\int_\kappa^1 W_{x1}^2(r,\kappa)dr - \left(\int_\kappa^1 W_{x1}(r,\kappa)dr\right)^2\right]$$

$$N^{(2)}(W) = \left[\kappa(1-\kappa) - \tfrac{1}{2}\gamma_y^2\right]\int_\kappa^1 W_{x1}(r,\kappa)dr - (1-\kappa)$$
$$\times\left[\int_\kappa^1 rW_x(r)dr - \lambda_y\int_\kappa^1 W_x(r)dr - \int_{\lambda_y+\kappa}^1 rW_x(r)dr + (\lambda_y+\kappa)\int_{\lambda_y+\kappa}^1 W_x(r)dr\right]$$

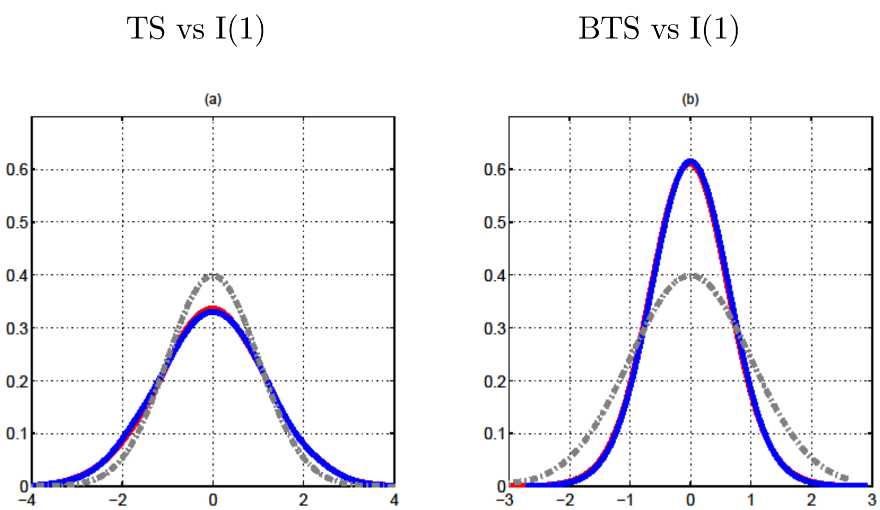
$$D^{(2)}(W) = 2\overline{W}_x\left[\kappa(1-\kappa) - \tfrac{1}{2}\lambda_y^2\right]\int_\kappa^1 W_{x1}(r,\kappa)dr - 2(1-\kappa)\overline{W}_x^2$$
$$-2\left[\kappa^2(1-\kappa) - \lambda_y^2(\kappa - \tfrac{1}{3}\lambda_y)\right]\left[\int_\kappa^1 W_{x1}(r,\kappa)dr\right]^2 + \tfrac{2}{3}(1-\kappa-\tfrac{3}{4}\lambda_y)\lambda_y^3\int_\kappa^1 W_{x1}^2(r,\kappa)dr,$$

and,

$$\overline{W}_x \equiv \int_\kappa^1 rW_x(r)dr - \lambda_y\int_\kappa^1 W_x(r)dr - \int_{\lambda_y+\kappa}^1 rW_x(r)dr + (\lambda_y+\kappa)\int_{\lambda_y+\kappa}^1 W_x(r)dr.$$

We then simulated these formulae for the two t-statistics using 10,000 replications, and present the resulting density functions as the solid graphs in Figure 2.[11] Next, using these same $DGPs$, we ran regression equation (1) 10,000 times using a (very large) sample size of $T = 10,000$ and computed the empirical density functions of each t-statistic (note that the one corresponding to the second case has to be scaled by $T^{1/2}$, as indicated in Theorem 1). The resulting empirical density functions are also depicted in Figure 2. Note that the graphs of the asymptotic and simulated densities are indistinguishable from each other. Finally, a standard normal density function is also presented in Figure 2 as the dashed graph. As can be seen from both panels of Figure 2, the t-statistics for these two models have nonstandard distributions. Therefore, inference using a standard normal distribution could be misleading.

Figure 2



[11]Note that the asymptotic distributions depend on nuisance parameters. Without loss of generality, we assume, for $t_{\widehat{\delta}_k}^{(1)}$, $\kappa = 0.5$, $\gamma_k = 0.2$, and $\sigma_y = 1.0$. For the case of $t_{\widehat{\delta}_k}^{(2)}$, we assume $\kappa = 0.5$, $\gamma_y = 0.07$ and $\lambda_y = 0.20$. We approximate the Wiener processes using standard normal variates with a sample size of 10,000.

From Valkanov (2003), we know that under a near unit root framework for both variables, it could be feasible to simulate the asymptotic distribution of $T^{-1/2}t_{\widehat{\delta}_k}$, in order to carry out correct inference. Our Monte Carlo experiment reveals, however, that whenever y_t or x_t is generated by a BTS process, the asymptotic distribution of the t-ratio (even when correctly scaled) is not free of nuisance parameters, thus complicating the simulation exercise. In this case, the inferential procedure proposed by Valkanov (2003) is no longer feasible.

The phenomenon of spurious regression in a long-horizon regression framework can be further investigated by analyzing the behaviour of the t-statistic in model (1) when x_t and y_t do have a long-run relationship.[12] This will allow us to device an alternative procedure to test for a linear relationship, under different forms of $DGPs$ for x_t and y_t. Following the procedure outlined above, the next Theorem, which presents the asymptotics of this case, can be easily proved.

THEOREM 2. *Let x_t be generated either as:*
i) (unit root) $x_t = X_0 + \mu_x t + \sum_{i=1}^{t} u_{xi}$, or
ii) (broken trend) $x_t = \mu_x + \beta_x t + \gamma_x DT_{xt} + u_{xt}$,
and y_t be generated as:
$y_t = \mu_y + \beta_y x_t + u_{yt}$
with the innovations $\{u_{zt}\}_1^{\infty}$, for $z = y, x$, satisfying the conditions of Lemma 1. Let $\frac{k}{T} \to \kappa \in (0,1)$ when both $T \to \infty$ and $k \to \infty$. Then, the OLS estimator $\widehat{\delta}_k$ from (1) and its t-statistic $t_{\widehat{\delta}_k}$ have the following asymptotic properties:

a) $\widehat{\delta}_k \xrightarrow{p} \beta_y$

b) $t_{\widehat{\delta}_k} = O_p(T^{3/2})$

Note that under case i) of Theorem 2, x_t and y_t are $I(1)$ and cointegrated, $CI(1,1)$, since u_{yt} is assumed stationary. Under these circumstances, as part *b)* of Theorem 2 shows, the t-statistic diverges at the (fast) rate $T^{3/2}$, implying that the null hypothesis $H_0 : \delta_k = 0$ will always be rejected asymptotically, leading to the correct conclusion.

This result, together with results form Theorem 1, allow us to construct the following reasoning: We know from Theorem 1 that the t-statistic will either diverge at rate $T^{1/2}$ when there are permanent shocks to the variables, or not diverge, when there are no permanent shocks in at least one of the variables. On the other hand, when x_t and y_t are CI (Theorem 2), the t-statistic will diverge at rate $T^{3/2}$, delivering correct inference.

Based on this background, we propose to scale the t-statistic by the sample size, T, and define $t^R_{\widehat{\delta}_k} \equiv T^{-1}t_{\widehat{\delta}_k}$. The following Corollary formally states these ideas.

[12] In the context of money and output, this would imply failure of monetary neutrality.

Corollary 2.

1a) $t^R_{\widehat{\delta}_k} = O_p(T^{-1/2})$ for all spurious cases (those of Theorem 1 with a permanent shock)

1b) $t^R_{\widehat{\delta}_k} = O_p(T^{-1})$ for all spurious cases (those of Theorem 1 without a permanent shock)

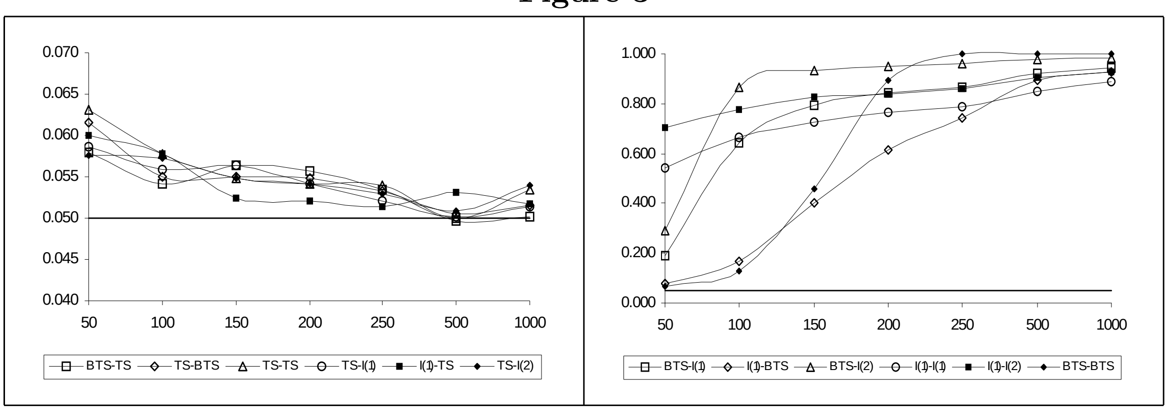
2) $t^R_{\widehat{\delta}_k} = O_p(T^{1/2})$ for all non-spurious cases (those of Theorem 2)

Therefore, dividing by T will asymptotically guarantee correct inference, whether the variables have a long-run relationship or not: when the variables are independent, the scaled t-statistic will converge to zero, thus not rejecting the null of no relationship; on the other hand, when the variables are cointegrated the scaled t-statistic will diverge, correctly indicating a long-run relationship. Similar arguments apply to case *ii)* of Theorem 2, where x_t is hit by a permanent deterministic shock, instead of a stochastic one.

4 Simulation Results

In order to assess the usefulness of our asymptotic results in finite samples, we computed rejection rates for $t_{\widehat{\delta}_k}$ in model (1), using a 1.96 critical value (5% level) for a standard normal distribution, based on simulated data, for various sample sizes and combinations of DGPs in the Assumption. This is obviously incorrect from our point of view, given that both the asymptotic and the finite sample distributions of $t_{\widehat{\delta}_k}$ depend on nuisance parameters, and might deviate from normality. However, for the wide set of parameters chosen for the simulations presented below, the finite sample behaviour tends to support the asymptotic results presented above. Figure 3 presents rejection rates for the 12 possible combinations in Table 1.

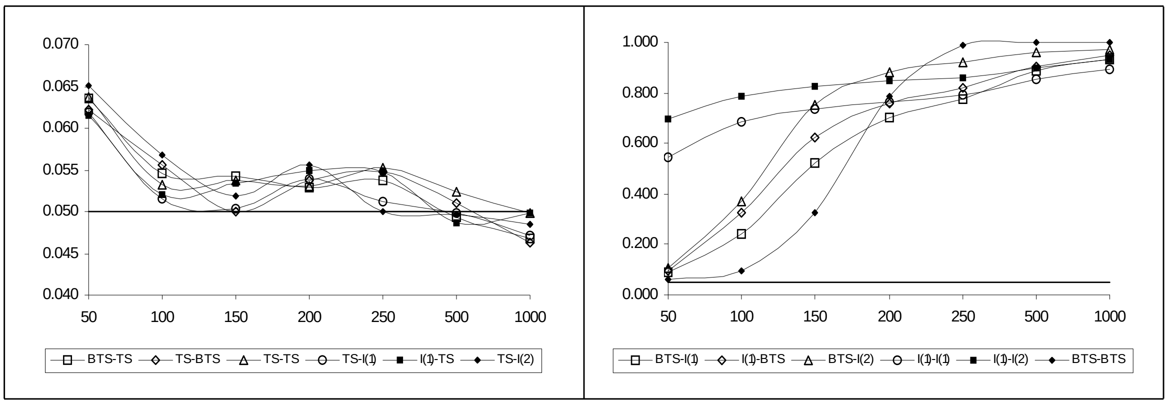
Figure 3



The left hand panel depicts combinations where at least one of the variables follows a TS process. As can be noted, rejection rates tend to converge to the 5% nominal level as the sample size grows; i.e., there is no spurious rejection whenever x_t or y_t are trend stationary, consistent with our asymptotic results. On the other hand, and also in line with the limit results above, the graphs in the right panel show that rejection rates tend to one as the sample size grows, when neither x_t nor y_t is generated by a TS process. As can be seen, for combinations such as $I(1)-I(1)$, $I(1)-I(2)$, $BTS-I(1)$, and $BTS-I(2)$, rejection rates are above 60% for samples as small as $T = 100$.

Results in Figure 3 were calculated assuming $\lambda_x < \lambda_y < \kappa$, and $\lambda_y + \kappa < 1$. On the other hand, Figure 4 assumes that $\kappa < \lambda_x < \lambda_y$, and $\lambda_y + \kappa > 1$. The similar behaviour of rejection rates from both Figures implies that results do not depend on whether the breaks in the variables occur before or after the end of the horizon.[13]

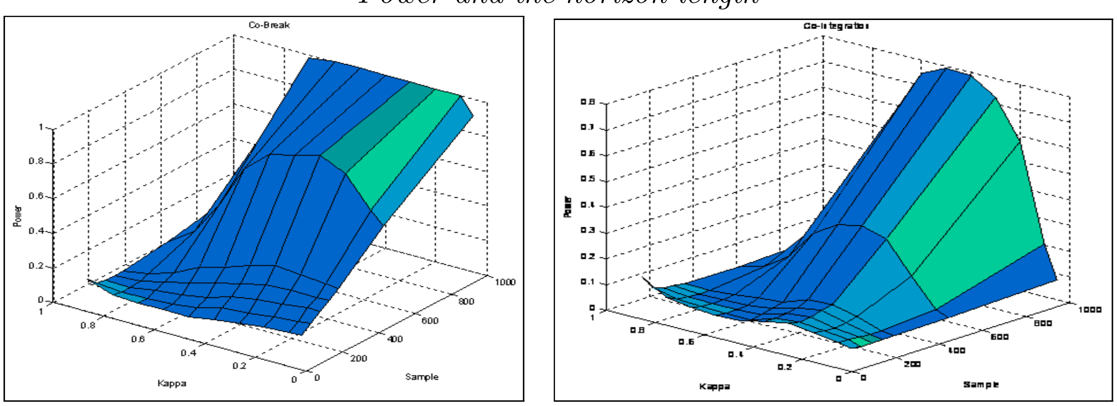
Figure 4



Turning to power considerations, Figures 5-9 show how power is related to several parameters in the DGPs of Theorem 2, like the size of the horizon, or the size of the break. For instance, Figure 5 depicts power against κ, the horizon size, for different sample sizes.

Figure 5

Power and the horizon length



[13]In particular, we assumed, for results in Figure 3, that : $\lambda_y = 0.40$, $\lambda_m = 0.25$, and $\kappa = 0.50$, while for Figure 4: $\lambda_y = 0.80$, $\lambda_m = 0.65$, and $\kappa = 0.50$. For the rest of the parameters we used, $\beta_y = 0.09$, $\beta_m = 0.06$, $\theta_y = \theta_m = 0$, $\gamma_y = 0.07$, $\gamma_m = 0.04$, $\sigma_y = \sigma_m = 1$. Finally, for processes with deterministic trends: $\mu_y = 0.20$, $\mu_m = 0.10$, for processes with stochastic trends: $\mu_y = 0.09$, $\mu_m = 0.06$. We experimented with many different combinations of parameter values and obtained qualitatively similar results.

As in this case the null of no relationship is false, we simulated two possibilities: the variables cobreak[14] (left panel), or cointegrate (right panel). In the case of cobreaking, power tends to one uniformly in κ, as the sample grows (with the exception of values of κ close to zero or one). In the case of cointegration, however, power is maximized for values of κ close to 0.5, as the sample gets large.[15]

Figure 6 shows power for different values of the linear trend in the explanatory variable (β_x in Theorem 2). As can be seen, power increases uniformly in β_x.[16]

Figure 6

Power and the size of the linear trend

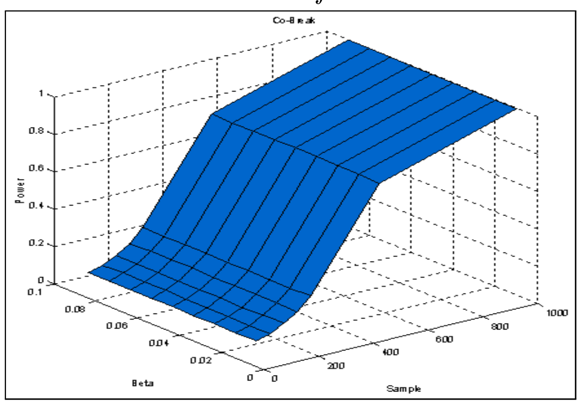
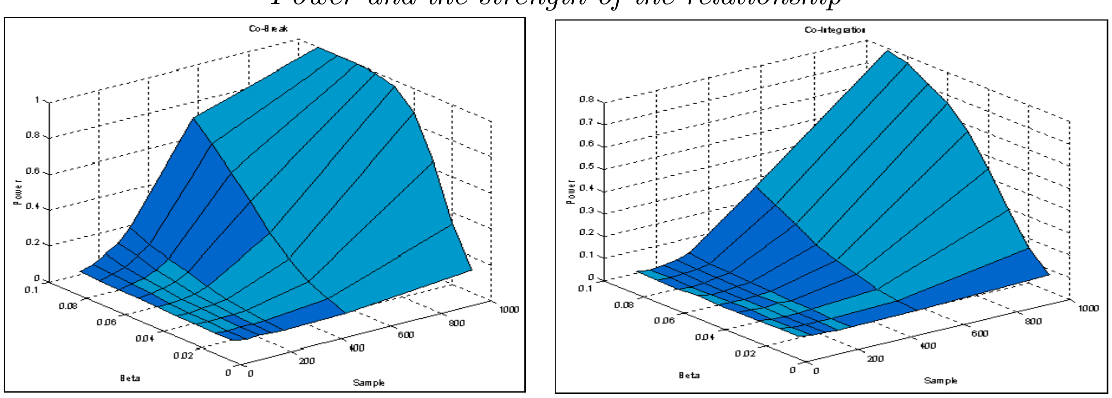


Figure 7 shows how power is increasing in β_y, as expected, since large values of this parameter are associated to a significant relationship between x and y (at a faster rate for the case of cobreak). Similar results are obtained for the effect on power of the size of the break, as depicted in Figure 8. Again, power is increasing in the size of the break.

Figure 7

Power and the strength of the relationship



[14] We use the term 'cobreak' to indicate a linear relationship between two non-independent broken-trend-stationary variables.

[15] This result can be used as a guide in applied work, if no guidance exists on horizon-length determination.

[16] Values for β_x are based on estimated values obtained from the empirical application with international annual data on money and output presented in the next section.

Finally, Figure 9 shows that power reaches a maximum when the break date parameter approaches 0.5.

Figure 8

Power and the size of the trend-break

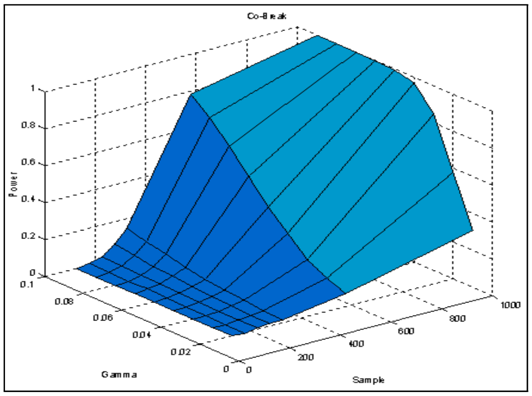
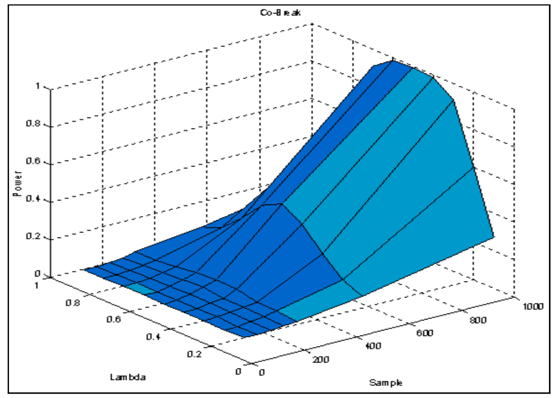


Figure 9

Power and the location of the trend-break



Whilst it is true that the above experiments might suggest the presence of low power in small samples, the main aim of these experiments, however, was to shed some light on the relationship between some key parameters, sample sizes and power.[17]

5 Empirical Application: Long-horizon regressions and monetary neutrality

A central tenet of macroeconomics is the monetary neutrality proposition, which states that there should be no long-run real effects of an unanticipated, permanent change in the level of money. One of the main tools for testing this key macro proposition is the non-structural test devised by Fisher and Seater (1993, FS in what follows). This test depends on the time series properties of both x_t and y_t, and uses as the testing vehicle a long-horizon regression:

$$\left[\sum_{j=1}^{k} \Delta^{\langle y \rangle} y_{t-j+1}\right] = \alpha_k + \delta_k \left[\sum_{j=1}^{k} \Delta^{\langle x \rangle} x_{t-j+1}\right] + \varepsilon_{kt} \tag{2}$$

where y and x stand for real output, and (exogenous) money, respectively. Theoretically, $\lim_{k\to\infty} \delta_k \equiv \delta$, gives an estimate of the long-run derivative of real output with respect to a permanent stochastic exogenous shock in the level of money.

It is common practice to assume that in the long-horizon regression above the potential nonstationarity in the variables takes the form of a unit root. Typically, in empirical applications, the time series properties of x_t and y_t data are investigated via unit root tests, such

[17] We have to bear in mind that power is sensitive to the chosen parameter values. The issue of low power in long-horizon regression is discussed in Coe and Nason (2003, 2004).

as the Dickey-Fuller, Phillips-Perron, KPSS, or Ng-Perron.[18] Noriega (2004), for instance, presents results on LRN using an international data set while discussing the use of these tests for uncovering the time series properties of the data. He finds that LRN does not hold for several countries in his sample. More recently, NSV find that conclusions on monetary neutrality are sensitive to the presence of structural breaks in the trend function of both x_t and y_t.

Given this evidence, we estimate equation (2) by allowing not only $\langle z \rangle = 1$, but also $\langle z \rangle = 0$, in the form of linear trends and linear trends with breaks, and $\langle x \rangle = 2$. Hence, the DGP for $z = y, x$ could be not only an $I(1)$ process, as the FS test requires, but also a combination of linear trends, broken trends and stochastic trends.

The neutrality of money is measured by FS through the long-run elasticity, or Long-Run Derivative (LRD) of y_t with respect to permanent stochastic exogenous changes in x_t:

$$LRD_{y,x} \equiv \lim_{k \to \infty} \frac{\partial y_{t+k} / \partial u_t}{\partial x_{t+k} / \partial u_t}$$

where u_t is a random variable that represents the shock to x_t. The limit of the ratio measures the ultimate effect of a (stochastic) monetary disturbance on real output relative to that disturbance's ultimate effect on the monetary variable. Theoretically, $\lim_{k \to \infty} \delta_k \equiv \delta$, gives an estimate of the LRD, where δ_k is the coefficient from the OLS regression in (2).

As explained by FS, to carry out a test for Long-Run Neutrality (LRN) the time series properties of y_t and x_t should obey certain restrictions. For instance, the money variable should contain a permanent shock (identified in the form of a unit root) for LRN to make sense, otherwise there are no stochastic permanent changes in money that can affect real output. Under this assumption, and if y_t is stationary, then LRN holds, since permanent changes in x_t cannot be associated to non-existent permanent changes in y_t. On the other hand, when both y_t and x_t follow a unit root process, then LRN is testable, and Fisher and Seater (1993) propose a very simple test based on the t-statistic of the slope parameter in the long-horizon regression (2), $t_{\widehat{\delta}_k}$.

As an application of some of the theoretical results derived in this paper, we utilize the international data on money and output of NSV, to calculate the t-statistic for testing long-run neutrality of money, using the long-horizon regression model (2), as in FS. The data consist of annual observations of the logarithms of real GDP and a money aggregate, for Australia (1870-1997), Argentina (1884-1996), Brazil (1912-1995), Canada (1870-2000), Italy (1870-1997), Mexico (1932-2000), Sweden (1871-1988) and the UK (1871-2000).[19]

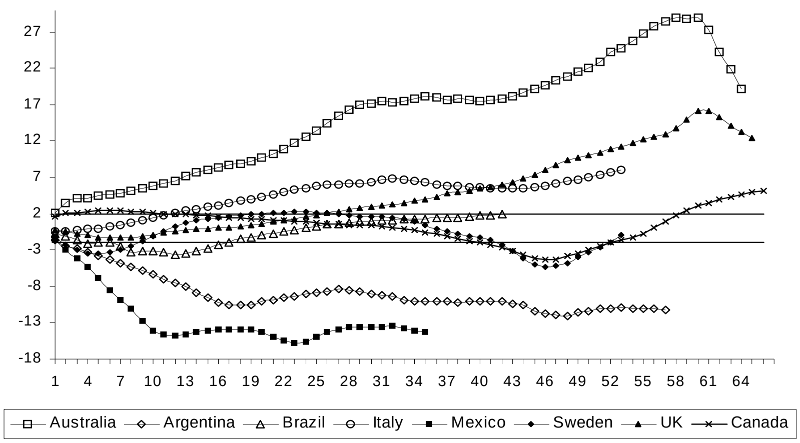
[18] See, respectively, Said and Dickey (1984), Phillips and Perron (1988), Kwiatkowski, et. al. (1992) and Ng and Perron (2001).

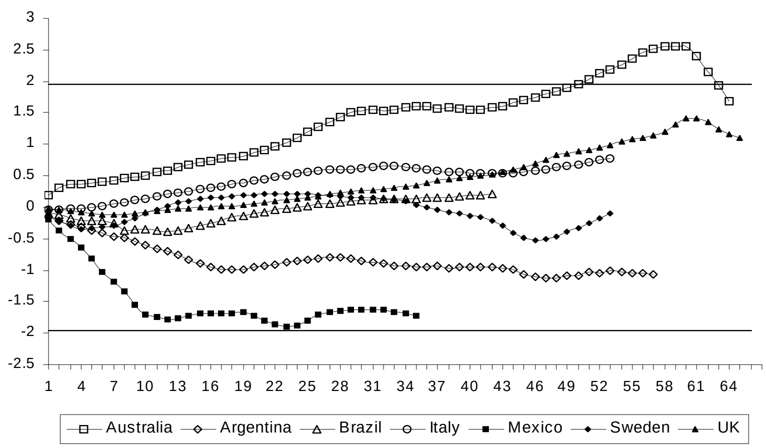
[19] Two other countries, Denmark and the US, are left out of the analysis since money and output for these

NSV find that for Australia, Sweden and the UK both y_t and x_t follow a BTS model. For Canada, y_t follows a TS model, while x_t a BTS one. For Argentina and Mexico y_t follows a BTS model while x_t an $I(1)$ one. For Brazil y_t follows a BTS model while x_t an $I(2)$ one. Finally, for Italy both variables appear to be $I(1)$. Since Canadian output is stationary, LRN holds by construction, as discussed in NSV. For the rest of countries, the t-statistic should be, according to our results in Theorem 1, normalized by $T^{1/2}$, to prevent the statistic from diverging, and thus asymptotically indicate a spurious rejection of LRN.

Figure 10 shows the behaviour of the t-statistic $t_{\widehat{\delta}_k}$ for testing the null hypothesis $H_0 : \delta_k = 0$ in the Long-horizon regression (2) for values of $k = 1, 2, ..., \frac{1}{2}T$.[20]

Figure 10





The upper panel in Figure 10 shows the behaviour of the non-normalized statistic, while the lower panel the behaviour of the normalized statistic. The effect of normalization is

two countries are found to be $I(0)$, making LRN not addressable. See Noriega et. al (2008) for details. The data set is available from the authors upon request.

[20] Our selection of $\kappa = 1/2$ is based on our simulation results, which show that power can be maximized when the length of the horizon is half the sample size.

quite clear: for all countries in the sample, monetary neutrality is not rejected. If we had normalized by T instead of by $T^{1/2}$, as discussed above, the sequences of the t-statistics would had all been even closer to zero, indicating a clearer non-rejection of monetary neutrality. Note however, that critical values used in Figure 10 assume normality and therefore serve only an illustrative purpose.

In order to carry out inference on monetary neutrality using appropriate critical values, we use bootstrap methods to simulate the distribution of the t-statistic in the LHR (2).[21] In particular, for each country, we simulated $t_{\widehat{\delta}_k}$ using output and money data generated from the models identified by NSV, described above in the Assumption (see Section 2).

We use the case of Australia to exemplify the steps involved in the simulations.[22] NSV found that output follows a BTS model with three level breaks, while money a BTS model with two level-and-trend breaks. Hence, for Australia's output (y_t) and money (m_t) time series, we estimate by OLS the following BTS models:[23]

$$\Delta y_t = \mu_y + \beta_y t + \sum_{i=1}^{3} \theta_{yi} DU_{it} + \alpha_y Y_{t-1} + \sum_{i=1}^{4} a_{yi} \Delta y_{t-i} + \varepsilon_{yt},$$

$$\Delta m_t = \mu_m + \beta_m t + \sum_{i=1}^{2} \theta_{mi} DU_{it} + \sum_{i=1}^{2} \gamma_{mi} DT_{it} + \alpha_m m_{t-1} + \sum_{i=1}^{1} a_{mi} \Delta m_{t-i} + \varepsilon_{mt},$$

We then use the estimated parameters from these models to generate 10,000 samples of Δy_t, $t = 6, ..., T$, and Δm_t, $t = 3, ..., T$, with randomly selected residuals (with replacement).[24] For each generated sample, the long-horizon regression equation (2) is estimated, and the corresponding 10,000 values of $t_{\widehat{\delta}_k}$ are used to construct the empirical density function of this statistic.[25] Table 3 report critical values from these distributions, together with the calculated $T^{1/2}$-rescaled t-statistic.[26]

As can be seen, for Australia, Italy and Mexico, the null hypothesis of long-run neutrality can be rejected at the 10% level, but not at the 5% or lower.

[21] For an exposition of the bootstrap method see Davidson and G. MacKinnon (2004).

[22] Table A1 in Appendix 3 sumarizes the models used to simulate the LHR t-statistic for each country, based on the identified models in NSV.

[23] See Figure 1 for a graphical representation of these series.

[24] Note that for output, we use the first 5 observations as initial conditions, while for money the first 2 observations as initial conditions.

[25] The 10,000 estimates of the LHR equation (2) use a value of $k = \frac{1}{2}T$, since this choice seems to maximize power, as found in our simulation experiments.

[26] Note that the t-statistic for Canada was not rescaled, since for this country output was found to be trend-stationary. All calculations were carried out in Matlab 7.0. Data and codes available at http://dl.dropbox.com/u/1307356/Arxius%20en%20la%20web/LHR%20code/Programs.zip

Table 3. Results of Long-run monetary neutrality tests

Country	Critical Values						Calculated
	1%	5%	10%	90%	95%	99%	t-statistic
Argentina	-2.009	-1.656	-1.451	1.467	1.605	2.025	-1.061
Australia	-0.385	-1.102	0.042	1.652	1.840	2.232	1.689*
Brazil	-0.637	-0.472	-0.397	0.388	0.459	0.612	0.293
Canada	-0.992	-0.709	-0.586	0.562	0.686	0.938	0.425
Italy	-2.015	-1.339	-1.099	1.109	1.392	2.011	1.358*
Mexico	-2.840	-2.038	-1.670	1.661	2.005	2.702	-1.723*
Sweden	0.288	0.383	0.429	1.010	1.081	1.219	0.525
U.K.	0.423	0.564	0.637	1.639	1.773	2.006	1.094

*Indicates rejection at the 10% level

Therefore, using a 5% level, it is not possible to reject long-run monetary neutrality for any of the countries analyzed. As a final remark, note that the same qualitative results are obtained if the t-statistic is scaled by T (as discussed in Section 3) instead of by $T^{1/2}$, i.e., neutrality results are unaffected.

Conclusions

We show that the presence of spurious long-horizon regression is highly likely when both x_t and y_t are hit by either a deterministic or stochastic permanent shock, both asymptotically and in finite samples. In other words, when x_t and y_t are generated independently from each other, and follow any combination of broken trends and unit roots, the t-statistic for a linear relationship in a long-horizon regression, will diverge to infinity (at rate $T^{1/2}$), indicating a spurious relationship asymptotically. On the other hand, when one of the variables follows a trend stationary process, our results indicate that the t-statistic does not diverge. Our large sample results are confirmed by simulations.

We also analyzed the case when x_t and y_t are cointegrated. In this case our results indicate that divergence still occurs, but at a much faster rate ($T^{3/2}$). The difference in divergence rates allowed us to propose an asymptotically correct inferential procedure, which works whether the variables have a long-run relationship or not.

As an application of our results, we reanalyzed results of NSV and found that long-run neutrality seems to hold for all countries in the sample.

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Appendix 1

Proof of **Lemma 1**.

To prove part a) note that $\sum_{t=k+1}^{T}(u_{zt}-u_{zt-k}) = \sum_{j=1}^{k} u_{zT-j+1} - \sum_{j=1}^{k} u_{zj}$, where $\sum_{j=1}^{k} u_{zT-j+1} = \sum_{j=1}^{T} u_{zj} - \sum_{j=1}^{T-k} u_{zj}$.

Hence, $\sum_{t=k+1}^{T}(u_{zt} - u_{zt-k}) = \sum_{j=1}^{T} u_{zj} - \sum_{j=1}^{T-k} u_{zj} - \sum_{j=1}^{k} u_{zj}$

The asymptotic behaviour of these sums can be studied by first transforming each of them as follows [see for instance Phillips (1987)],

$T^{-1/2} \sum_{j=1}^{[Tr]} u_{zj} \equiv X_{zT}(r)$

$T^{-1/2} \sum_{j=1}^{[Tr]-[T\kappa]} u_{zj} \equiv X_{zT}(r-\kappa)$

$T^{-1/2} \sum_{j=1}^{[T\kappa]} u_{zj} \equiv X_{zT}(\kappa)$

where $r, \kappa \in [0,1]\,, \kappa < r$, $[Tr]$ denotes the integer part of Tr, and then applying a functional central limit theorem due to Herrndorf (1984),

$X_{zT}(r) \xrightarrow{D} \sigma_z W_z(r)$

$X_{zT}(r-\kappa) \xrightarrow{D} \sigma_z W_z(r-\kappa)$

$X_{zT}(\kappa) \xrightarrow{D} \sigma_z W_z(\kappa)$

where all limits are taken as $T \to \infty$, $\xrightarrow{D}$ signifies convergence in distribution, and W_z is a standard Wiener process. For instance, $W_z(r)$ is normally distributed for every r in $[0,1]$; that is $W_z(r) \sim N(0,r)$. It is assumed that $\sigma_z^2 = \lim_{T\to\infty} T^{-1}E(\Sigma_{j=1}^{T} u_{zj})^2$ exists and is strictly positive. Hence, $Su_z \xrightarrow{D} \sigma_z[W_z(r) - W_z(r-\kappa) - W_z(\kappa)] \equiv \sigma_z W_{z2}(r,\kappa)$, where $W_{z2}(r,\kappa) \equiv W_z(r) - W_z(r-\kappa) - W_z(\kappa)$.

To prove part b) note that $T^{-1}\sum_{t=k+1}^{T} u_{zt}^2 \xrightarrow{p} \sigma_z^2$, and $T^{-1}\sum_{t=k+1}^{T} u_{zt-k}^2 \xrightarrow{p} \sigma_z^2$, using the Strong Law of Large Numbers of McLeish (1975), while $T^{-1}\sum_{t=k+1}^{T} u_{zt}u_{zt-k} \xrightarrow{p} \gamma_k$ (see Hamilton (1994), p. 506). Hence, $Su_z2 \xrightarrow{p} 2(\sigma_z^2 - \gamma_k)$.

To prove part c) note that $E(\sum_{t=k+1}^{T} u_{xt}u_{yt}) = 0$, and $V(\sum_{t=k+1}^{T} u_{xt}u_{yt}) = (1-\kappa)\sigma_x^2\sigma_y^2 T$. A Central Limit Theorem can be used to show that $T^{-1/2}\sum_{t=k+1}^{T} u_{xt}u_{yt} \xrightarrow{D} N(0,(1-\kappa)\sigma_x^2\sigma_y^2)$. These same arguments can be applied to the remaining products.

To prove parts d) and e) note that $S_{zt} = \sum_{j=1}^{k} u_{zt-j+1} = \sum_{j=1}^{t} u_{zj} - \sum_{j=1}^{t-k} u_{zj}$. We can then apply a similar transformation as the one in part a) above: $T^{-1/2}(\sum_{j=1}^{[Tr]} u_{zj} - \sum_{j=1}^{[Tr]-[T\kappa]} u_{zj}) \xrightarrow{D} \sigma_z\,[W_z(r) - W_z(r-\kappa)] \equiv \sigma_z W_{z1}(r,\kappa)$, where $W_{z1}(r,\kappa) \equiv W_z(r) - W_z(r-\kappa)$. Finally, using analogous arguments as those of the proof of Lemma 1 in Phillips (1986):

$T^{-3/2}\sum_{t=k+1}^{T} S_{zt} \xrightarrow{D} \sigma_z \int_\kappa^1 W_{z1}(r,\kappa)dr$

$T^{-2}\sum_{t=k+1}^{T} S_{zt}^2 \xrightarrow{D} \sigma_z^2 \int_\kappa^1 W_{z1}^2(r,\kappa)dr.$

To prove part f) note that, in analogy to Phillips (1986, p.315),

$T^{-2}\sum_{t=k+1}^{T}(\sum_{j=1}^{k} u_{xt-j+1})(\sum_{j=1}^{k} u_{yt-j+1}) \xrightarrow{D} \sigma_x\sigma_y \int_\kappa^1 W_{x1}(r,\kappa)W_{y1}(r,\kappa)dr.$

The proof of part g) will assume that the sample moment is computed over the whole sample, instead of over $t = k+1, ..., T$. The reason is that computations are much easier, and the orders of probability are not affected. Hence, note that the first element of the sum,

$$\sum_{t=1}^{T} u_{zt} DT_t = \sum_{t=T_b+1}^{T} (t-T_b) u_{zt} = \left(\sum_{t=1}^{T} t u_{zt} - \sum_{t=1}^{T_b} t u_{zt}\right) - \lambda T \left(\sum_{t=1}^{T} u_{zt} - \sum_{t=1}^{T_b} u_{zt}\right).$$

Now, from Lemma A.1 in Perron (1989),

$$T^{-3/2} \sum_{t=1}^{T} u_{zt} DT_t \xrightarrow{D} \sigma_z \left\{ (1-\lambda) \left[W_z(1) - W_z(\lambda)\right] - \left[\int_0^1 W_z(r) - \int_0^{\lambda} W_z(r)\right] \right\}.$$

The same arguments can be applied to the remaining elements.

To prove part h), we follow Hamilton (1994, p. 547) to obtain:

$$T^{-1} \left(\sum_{t=k+1}^{T} S_{xt} u_{yt} - \sum_{t=k+1}^{T} S_{xt} u_{yt-k} \right) \xrightarrow{D} \sigma_x \sigma_y \left[\int_{\kappa}^{1} W_x(r) dW_y(r) - \int_{\kappa}^{1} W_x(r) dW_y(r-\kappa) \right].$$

To prove part i) note that the first element of the product can be written as:

$\sum_{t=k+1}^{T} S_{xt} DT_t = \sum_{t=1}^{T} S_{xt} DT_t - \sum_{t=1}^{k} S_{xt} DT_t$. Now,

$$\sum_{t=1}^{T} S_{xt} DT_t = \sum_{t=T_b+1}^{T} (t - T_b) S_{xt} = \sum_{t=T_b+1}^{T} t S_{xt} - \lambda T \sum_{t=T_b+1}^{T} S_{xt}.$$

Using Lemma A.1 in Perron (1989):

$$T^{-5/2} \sum_{t=k+1}^{T} S_{xt} DT_t \xrightarrow{D} \sigma_x \left[\int_{\kappa}^{1} r W_x(r) dr - \lambda \int_{\kappa}^{1} W_x(r) dr \right].$$

The second element of the product can be written as:

$\sum_{t=k+1}^{T} S_{xt} DT_{t-k} = \sum_{t=1}^{T} S_{xt} DT_{t-k}$, since $\sum_{t=1}^{k} S_{xt} DT_{t-k} = 0$. Now,

$$\sum_{t=1}^{T} S_{xt} DT_{t-k} = \sum_{t=T_b+1+k}^{T} (t - T_b - k) S_{xt} = \sum_{t=T_b+1+k}^{T} t S_{xt} - \lambda T \sum_{t=T_b+1+k}^{T} S_{xt} - \kappa T \sum_{t=T_b+1+k}^{T} S_{xt}.$$

Again, using Lemma A.1 in Perron (1989):

$$T^{-5/2} \sum_{t=k+1}^{T} S_{xt} DT_{t-k} \xrightarrow{D} \sigma_x \left[\int_{\lambda+\kappa}^{1} r W_x(r) dr - (\lambda + \kappa) \int_{\lambda+\kappa}^{1} W_x(r) dr \right].$$

Appendix 2

Example of a *Mathematica 6* code for the case of $z \sim I(1)$, $z = y, x$, that is, $\Delta z_t = \mu_z + u_{zt}$.

1. The code

a11=(1-κ)*T;
a12=κ*(1-κ)*μ_x*T^2+SSUx*$T^{3/2}$;
a21=κ*(1-κ)*μ_x*T^2+SSUx*$T^{3/2}$;
a22=κ^2*(1-κ)*μ_x^2*T^3+2*κ*μ_x*SSUx*$T^{5/2}$+SSUx2*T^2;
b1=κ*(1-κ)*μ_y*T^2+SSUy*$T^{3/2}$;
b2=κ^2*(1-κ)*μ_x*μ_y*T^3+κ*(μ_x*SSUy+μ_y*SSUx)*$T^{5/2}$+SSUxSUy*T^2;
c1=κ^2*(1-κ)*μ_y^2*T^3+2*κ*μ_y*SSUy*$T^{5/2}$+SSUy2*T^2;
A=$\begin{bmatrix} a11 & a12 \\ a21 & a22 \end{bmatrix}$;
invA=Inverse[A];
alfa=Factor[invA[[1,1]]*b1+invA[[1,2]]*b2]; delta=Factor[invA[[2,1]]*b1+invA[[2,2]]*b2];
alfanum=Numerator[alfa]; alfaden=Denominator[alfa];
deltanum=Numerator[delta]; deltaden=Denominator[delta];
expalfanum=Exponent[alfanum,T]; expalfaden=Exponent[alfaden,T];
expdeltanum=Exponent[deltanum,T]; expdeltaden=Exponent[deltaden,T];

numalfa=Limit[Expand[alfanum/$T^{\text{expalfanum}}$],$T \to \infty$];
denalfa=Limit[Expand[alfaden/$T^{\text{expalfaden}}$],$T \to \infty$];
numdelta=Limit[Expand[deltanum/$T^{\text{expdeltanum}}$],$T \to \infty$];
dendelta=Limit[Expand[deltaden/$T^{\text{expdeltaden}}$],$T \to \infty$];
alfalim =(numalfa/denalfa)*($T^{\text{expalfanum}}/T^{\text{expalfaden}}$);
deltalim=(numdelta/dendelta)*($T^{\text{expdeltanum}}/T^{\text{expdeltaden}}$)
sigmac=Factor[(T*(1-κ))$^{-1}$*(c1+alfa^2*T*(1-κ)+delta^2*a22-2*alfa*b1-2*delta*b2+
2*alfa*delta*a12)];
s1num=Numerator[sigmac]; s1den=Denominator[sigmac];
exps1num=Exponent[s1num,T]; exps1den=Exponent[s1den,T];
numf=Limit[Expand[s1num/T^{exps1num}],$T \to \infty$];
denf=Limit[Expand[s1den/T^{exps1den}],$T \to \infty$];
σ2=FullSimplify[(numf/denf)]*($T^{\text{exps1num}}/T^{\text{exps1den}}$)
numexplist=Exponent[s1num,T,List]; denexplist=Exponent[s1den,T,List];
xx22=Factor[invA[[2,2]]];
xx22num=Numerator[xx22]; xx22den=Denominator[xx22];

```
expxx22num=Exponent[xx22num,T];   expxx22den=Exponent[xx22den,T];
numxx22=Limit[Expand[xx22num/T^expxx22num],T → ∞];
denxx22=Limit[Expand[xx22den/T^expxx22den],T → ∞];
x2=(numxx22/denxx22)*(T^expxx22num/T^expxx22den)
```

2. The output

The previous code yields three items (which correspond to the three lines in the code not finishing with a semicolon): the asymptotic expressions for the estimated parameter, the error variance, and the second diagonal element of the inverted moments matrix.

i) $\widehat{\delta}_k$ =(SSUx SSUy+SSUxSUy(-1+κ))/(SSUx^2)+SSUx2 (-1+κ))

ii) $\widehat{\sigma}_{\varepsilon}^2 = T$(2 SSUx SSUxSUy SSUy-SSUx^2 SSUy2-SSUx2 (SSUy^2+SSUy2 (-1+κ))+ SSUxSUy^2(-1+κ))/((SSUx^2+SSUx2 (-1+κ))(-1+κ))

iii) $(X'X)^{-1}_{22}$ =(-1+κ)/(T^2(SSUx^2+SSUx2 (-1+κ)))

From this output, both the order in probability and the asymptotic distribution of the t-statistic can be derived by simple algebra.

Appendix 3

Although details can be found in NSV, the following table sumarizes the models used to simulate the LHR t-statistic for each country.

Table A1
Summary of models used to bootstrap the
long-horizon regression t-statistic

Country/Sample	Variable	Model	Break 1	Break 2	Break 3	Lag length
Argentina	Y	BTS	1912 (T)	1917 (LT)	1980 (L)	5
1884 - 1996	M2	$I(1)$	–	–	–	–
Australia	Y	BTS	1891 (L)	1914 (L)	1928 (L)	4
1870 - 1997	M2	BTS	1941 (LT)	1971 (LT)	–	1
Brazil	Y	BTS	1928 (L)	1970 (LT)	–	3
1912 - 1995	M2	$I(2)$	–	–	–	–
Canada	Y	TS	–	–	–	–
1870 - 2000	M2	BTS	1920 (L)	–	–	1
Italy	Y	$I(1)$	–	–	–	–
1870 - 1997	M2	$I(1)$	–	–	–	–
Mexico	Y	BTS	1953 (T)	1981 (T)	1994 (LT)	5
1932 - 2000	M1	$I(1)$	–	–	–	–
Sweden	Y	BTS	1916 (LT)	1930 (LT)	1975 (LT)	5
1871 - 1988	M2	BTS	1912 (LT)	1918 (LT)	1970 (L)	1
U.K.	Y	BTS	1918 (LT)	–	–	1
1871 - 2000	M4	BTS	1939 (L)	1970 (LT)	–	1

L, T and LT stand for level, trend, and level and trend, respectively.