Banco de México

Documentos de Investigación

Banco de México

Working Papers

N° 2010-16

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December 2010

Documento de Investigación 2010-16

Working Paper 2010-16

Strategic Spending in Voting Competitions with Social Networks*

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Abstract: This paper proposes a model of voting competitions (political campaigns and strategic lobbying) where voters are influenced by the opinion of their neighbors on a social network. In the unique pure strategy nash equilibrium, resources are targeted toward individuals with an influential position in the network. This finding contrasts with previous theories of strategic spending which predict that parties (or lobbies) should spend more on individuals who have a higher probability of being pivotal for the vote. The paper then tests the model using data on campaign contributions by interests groups in the US. House of Representatives. The estimations show that both network influence and pivotality are significant predictors of campaign contributions.
Keywords: Network games, strategic spending, Colonel Blotto games, counteractive lobbying, Bonacich centrality.
JEL Classification: D85, D72.

Resumen: Este documento propone un modelo de competencias electorales, que incluyen las campañas políticas y el cabildeo estratégico, donde los votantes están influenciados por la opinión de sus vecinos en una red social. En el único equilibrio de nash en estrategias puras del modelo, los gastos de campaña se concentran en los votantes que tienen una posición influyente en la red. Esta predicción contrasta con lo encontrado por los modelos anteriores de gasto estratégico, donde se había encontrado que el gasto debía concentrarse en los votantes que tienen una probabilidad más alta de ser pivotales para la elección. Después de resolver el modelo, el documento prueba las predicciones empíricamente, usando datos sobre contribuciones de campaña hechas por grupos de interés en la Cámara de Representantes de los Estados Unidos de América. Las estimaciones indican que ambas variables, la influencia de red y la probabilidad de ser pivotal, son predictores significativos de los gastos de cabildeo.
Palabras Clave: Juegos en redes, gastos estratégicos de campaña, juegos del Coronel Blotto, cabildeo estratégico, centralidad de Bonacich.

*I thank the William and Sonja Davidow fellowship fund and the Stanford Institute for Economic Policy Research for their funding. I am greatly in debt to my advisors, Matt Jackson, Doug Bernheim and Manuel Amador, for their high quality advice and feedback. I also thank Jon Levin, Andy Skrzypacz, Giacomo DeGiorgi, Bob Hall, Matthew Harding, Paul Milgrom, Monika Piazzesi, Martin Schneider, Michele Tertilt, BobWilson, Aaron Bodoh-Creed, Matt Elliot, Ben Golub, Alexander Hirsch, Marcello Miccoli, Juuso Toikka, members of the microeconomic theory lunch, the macroeconomics lunch, the networks working group, the political economy working group and the gradlloquium for their useful feedback. I am also very grateful to James Fowler; Keith Poole and Howard Rosenthal; Charlie Stewart; and the Center for Responsive Politics for making their data available online.

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1 Introduction

When people are deciding how to vote or which product to buy, they discuss their decision with people in their social environment. Studying the pattern of social relationships is important to understand how individuals are influenced directly and indirectly by the opinion of others. This paper studies political competitions when voters influence each others' opinion. Current models of political competitions do not take these effects into account.

Using techniques from social network analysis I propose a model where two *persuaders* strategically assign resources across voters based on their position on a social network and I solve for the unique pure-strategy nash equilibrium. My model allows a rich structure of influence between individuals. For example, I allow for influence to be asymmetric and I put no restriction on the number of connections in the network.

Previous papers on strategic spending in voting competitions have found that resources should be targeted toward voters who have a higher probability of casting a pivotal vote.[1] In contrast, I find that when network effects are strong, persuaders target their resources toward voters who have influential positions in the network, where influence is measured by eigenvector centrality. This measure is frequently used in the sociology literature[2] and lies behind Google's PageRank, the algorithm to sort websites. When the network effects are weak I find that persuaders spend on each voter according to his pivotality and a weighted sum of the pivotality of his neighbors.

The shift away from pivotal voters is surprising. With or without the network, these voters have the highest marginal impact on the outcome of the election. Under perfect targeting, spending resources to change a vote that isn't pivotal is a waste of resources. The change in spending patterns happens because the network prevents resources from being targeted in an effective way. As the network effects become very strong, it becomes impossible to persuade voters in isolation. Persuaders react by moving resources away from pivotal

[1] See Shubik and Weber (1981); Snyder (1989); Lever (2010).
[2] See Wasserman and Faust (1994); Bonacich (1987); Bonacich and Lloyd (2001).

voters and focusing on influential voters.

To test the model I match data on campaign contributions by interest groups with data on cosponsorships networks in the US. House of Representatives. Since I can observe each legislator repeatedly across electoral cycles, I can control for unobservable legislator characteristics. The effect of the network is identified by measuring how year-to-year changes in network influence predict the changes in campaign contributions.

I find that both pivotality and strong network influence are significant predictors of campaign contributions, even after controlling for several confounds. I find that increasing strong network influence by one standard deviation increases the campaign contributions of the average legislator by 26,000 US. dollars, which is 6% of the contributions. ($p = 0.03$) Increasing pivotality by one standard deviation increases the contributions by 39,000 dollars, or 9% of the average contributions. ($p = 0.00$) The pivotality of the neighbors on the network is not a significant predictor, neither statistically nor substantively, and has the wrong sign.

My paper brings together two strands of research: political competitions and social networks. In the research on political competitions, there is a literature on counter-active lobbying[3] and on strategic spending in presidential elections,[4] but these papers do not allow for voters to influence one other.

In the social networks literature there has been much work on identifying the influential members of a network, but very little work has been done on how this information would be used in a competition. There exists a vast number of measures of network influence. My model predicts that eigenvector centrality is the correct one.[5] This measure is closely related to the inter-centrality measure found in the model of Ballester, Calvó-Armengol and Zenou (2006). In their model, inter-centrality identifies the members of a crime network that should be targeted for removal.

The only previous papers on political competitions with network effects are Galeotti

[3]Austen-Smith and Wright (1994, 1996).

[4]Merolla, Munger and Tofias (2005).

[5]See Jackson (2008); Wasserman and Faust (1994) for the many measures of influence and centrality on networks.

and Mattozzi (Forthcoming) and Gröenert (2009). Galeotti and Matozzi build a model of information disclosure when voters inform themselves through a social network. Their work focuses on the amount of information revealed when political parties have an incentive to hide their platforms. They also study how the network alters which candidates run for office. Their work puts much less emphasis on the structure of the network. Gröenert studies the problem of a single lobbyist who wishes to persuade legislators that follow a simple behavioral voting rule: they vote in favor of a proposal if the fraction of their neighbors favoring the proposal exceeds an idiosyncratic threshold. She finds that the optimal strategy for threshold networks consists of successively targeting the legislators with the most connections. She also proves this strategy cannot be guaranteed to be optimal for non threshold networks.

Social networks will be increasingly important for future political campaigns because of the growth in social networking sites. We now have more micro-level data on the structure of social networks than ever before. This will allow a level of targeting that would have been inconceivable a decade ago. Furthermore, younger voters are receiving a proportionally larger amount of information through these sites. In a survey by the Pew Research Center on the 2008 presidential election, 27% of people under 30 reported getting information on the campaign through such sites. The number rose to 37% if you consider only those between 18 and 24 years. This drastically differed from the 4% of people in their 30s and the less than 1% of people above 40 who reported getting any information this way. As younger voters get older, the influence of these sites is likely to increase.

The paper is structured as follows: Section 2 sets up the model; Section 3 solves the model; Section 4 tests the model with data on campaign contributions by lobbies in the US. House; and Section 5 concludes. I present two extensions in appendices. For most of the paper I assume persuaders have a fixed amount of resources, but in Appendix C I solve the model when persuaders have to raise their resources at a cost. Appendix D extends the model for competitions in proportional representation systems, where persuaders maximize their share of votes.

2 The Model

2.1 The voters.

There is a finite number N of voters that select between two options, A and B. These options can be two candidates, in the case of a general election, or the option to pass a bill vs. upholding the status quo, in the case of a legislature. A subscript i denotes voter i. All voters have to chose A or B, so turnout is not an issue.

Each voter will have an opinion p_i of the relative value of A vs. B. A larger p_i will be more favorable to A. These opinions are a summary statistic of the relevant information required to chose between A and B. For example, p_i could capture the the difference in the candidates' ability to deal with a financial crisis; or the perception on which candidate is more determined to carry out difficult reforms; or the difference in charisma between the candidates.

Frequently, games of strategic spending only have equilibria in complicated mixed strategies.[6] Characterizing these equilibria is hard and solving for them along with network influence would be intractable. To avoid it I will assume that voters chose probabilistically. Increasing the opinion p_i will only increase the probability that voter i choses A over B.

I will assume that increasing p_i smoothly increases the probability of voting for A. The easiest way to model this is to reparametrize opinions so that p_i represents the probability i choses A. To be concrete, I assume that votes are cast to maximize the following utility function.

$$U(\textit{voting for } A) - U(\textit{voting for } B) = p_i - \eta_i$$

Where $p_i \in (0, 1)$. Each η_i is distributed Uniform$[0, 1]$ and drawn independently across voters. Voter i choses A if p_i is greater than η_i; this occurs with probability p_i. Voter i

[6]The most well known reference are the Colonel Blotto games. In the most basic games, the Colonel Blotto and his opponent must assign their limited resources to N different battlefields. Whoever assigns more resources to a field wins the battle, and whoever wins most battles wins the war. See Roberson (2006) for a great reference.

choses B with probability $(1 - p_i)$. For technical reasons, I assume everybody has a positive probability of choosing both candidates, although the probability a voter swings his vote can be arbitrarily small.

The stochastic element η_i represents uncertainty about the elements that determine a vote. This variable need not be random from the point of view of the voter, it only matters that it's unknown by the persuaders. There can be all sorts of elements that make voters have a change of heart. For example, a voter might decide to change his vote because he happened to shake hands with a candidate at a rally. It has also been reported that bad weather can change the outcome of an election by influencing turnout differently for Democrats than for Republicans.[7] From the persuaders' perspective, these elements are hard to forecast or control.

2.2 The persuaders.

I will assume there are two persuaders, one associated with each option A and B. The persuaders have to decide how to spend resources over voters. The persuaders can be thought of as political parties or competing lobbies. Interpreting A and B as political parties is straightforward: the parties have to convince voters to chose them and whoever gets a majority wins.

To interpret the persuaders as lobbies, A and B are assumed to be fighting over a bill in Congress. One lobby wants the bill to pass and the other wants it to fail. The lobbies target their resources over different legislators to convince them to vote in their preferred direction. Without loss I will assume A wants the bill to pass while B prefers the status quo.

Every persuader has a fixed amount of resources to spend denoted by R_A, R_B. These finite resources could be money for advertising budgets, money for campaign contributions or time. Let (a_i, b_i) be, respectively, the percentage of resources that persuader A and persuader B spend on voter i; so $(a_i R_A, b_i R_B)$ are the amounts in units of resources. In Appendix C,

[7] Gomez, Hansford and Krause (2008).

I solve the model when persuaders have to raise resources at a cost.

I will assume that A and B only care about winning the election. They do not care by how many votes. In the model there will be uncertainty on the votes, so A and B will seek to maximize their probability of winning. Since some bills need a qualified majority of votes to pass, I will also solve the model for supermajority rules. In these cases, I will assume without loss that A needs a qualified majority and that B wins whenever A fails to obtain it. Let $\bar{N}$ be the minimum amount of votes that A needs to win.

Let $\delta_i \in \{0, 1\}$ represent the final decision of voter i. Persuader A wants to maximize $\pi(p_1, \ldots, p_N) = Prob(\sum \delta_i \geqslant \bar{N})$. Persuader B wants to maximize $1 - \pi$, which is the same as minimizing π.

Appendix D solves a model where A and B wish to maximize the percentage of votes they receive. This is important for political systems with proportional representation where the number of seats in congress depends on the share of the vote. The results are similar.

2.3 The timing of the game.

For tractability I will separate strategic spending from network influence into different stages of the game. Inside each stage there are periods which repeat similar actions. The timing of the game is as follows.

Let p_i^t represent the opinion of voter i at period t.

- **The initial stage:** (Period 0) Voters begin with an opinion p_i^0. The network is fixed and known.
- **The persuasion stage:** (Period 1) Persuaders simultaneously spend resources to influence the decision of the voters. (Section 2.4.)
- **The network stage:** (Periods 2 through T) After persuaders spend all their budget, voters update p_i^t through the social network. (Section 2.5.)
- **Final stage.** The η_i's are realized. Voters pick A with probability p_i^T.

2.4 The persuasion stage.

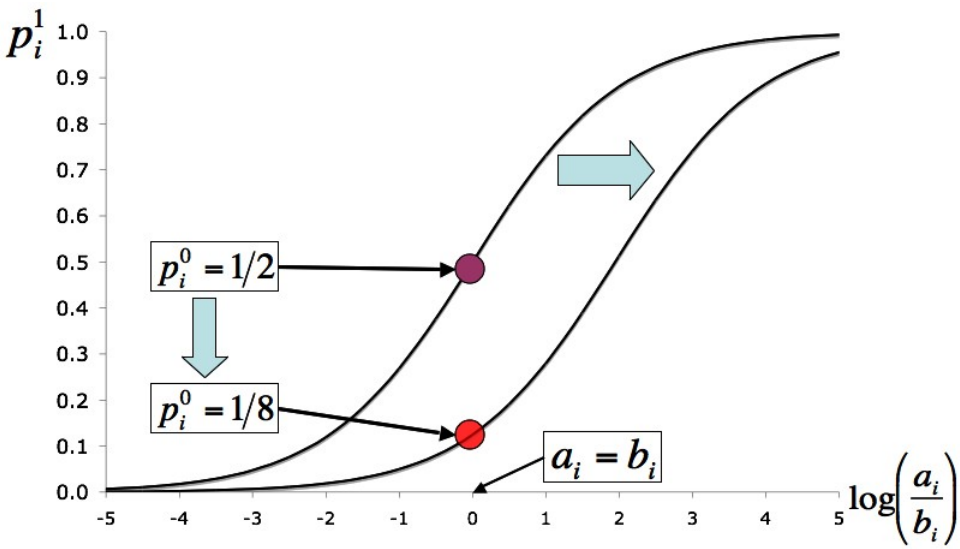


Figure 1: The contest success function depends on the ratio of resources and is "S shaped" in log units. The picture shows two potential functions with a different p_i^0 parameter. The picture assumes $R_A = R_B$.

During the persuasion stage, persuaders simultaneously spend resources to influence opinions. I assume persuaders can change opinions through the following contest success function. (See Figure 1.)

$$p_i^1(0,0) = p_i^0$$

$$p_i^1(a_i R_A, b_i R_B) = \frac{p_i^0 (a_i R_A)^\gamma}{p_i^0 (a_i R_A)^\gamma + (1 - p_i^0)(b_i R_B)^\gamma}; \ \gamma > 0$$

I chose this functional form for tractability. Below are it's main characteristics.

- It takes values in $[0, 1]$ and varies smoothly with the amount of resources each persuader spends.

- If $b_i > 0$, persuader A can only completely convince i by spending and infinite amount of resources, and vice-versa: p_i^1 monotonically tends to 1 as $a_i R_A \to \infty$ if $b_i > 0$; and p_i^1 monotonically tends to 0 as $b_i R_B \to \infty$ if $a_i > 0$.

- If both persuaders spend the same amount of resources, $a_i R_A = b_i R_B$, the opinion of voter i doesn't change: $p_i^1 = p_i^0$.

- If both persuaders scale the amount they are spending on voter i by any positive factor, the opinion p_i^1 is left unaffected.

$$p_i^1(\lambda a, \lambda b) = p_i^1(a, b); \forall \lambda > 0$$

  This happens because the contest success function only depends on the ratio of resources spent on each voter: $(a_i R_A)/(b_i R_B)$.

- The marginal persuadability decreases when A and B scale up their resources. This is crucial to get equilibria in pure strategies.

$$\frac{\partial p_i^1}{\partial a}(\lambda a, \lambda b) < \frac{\partial p_i^1}{\partial a}(a, b); \forall \lambda > 1$$

Contest success functions have been used in the economics literature to study strategic spending in tournaments, arms races and competitions.[8] Skaperdas provides axiomatizations for this and other contest success functions.[9]

Shubik and Weber use this contest success function to solve a smooth Colonel Blotto game.[10] Snyder uses a slightly more general function that does not depend on the ratio of resources but has all the other characteristics above.[11] The results are qualitatively similar but the ratio formula gives convenient analytical solutions that depend on percentages of resources. My model is different from theirs in that it allows influence through the network.

The parameter γ determines the impact of resources on opinions. For a large γ, a small difference in the level of spending between A and B dramatically swings opinions in one

[8] See Hirshleifer (1991); Skaperdas (1992) and Siegel (2009, Forthcoming).
[9] Skaperdas (1996).
[10] Shubik and Weber (1981).
[11] Snyder (1989).

direction or the other. As γ tends to infinity, the game becomes a standard Colonel Blotto game.

2.5 The network stage.

After persuaders have spent all their budget, voters update their opinion by taking a weighted average of the opinion their neighbors on a social network. The network is exogenous and common knowledge by the persuaders.

Each voter has a unit of attention he divides between the opinions of his neighbors and his own. Every round of updating, the opinion p_i^t evolves according to

$$p_i^{t+1} = \sum_{j=1}^{N} M_{ij} p_j^t; \text{ with } \sum_{j=1}^{N} M_{ij} = 1 \text{ and } M_{i,j} \geqslant 0$$

The parameter M_{ij} represents the weight voter i puts on voter j's opinion; M_{ii} represents the weight he puts on his previous opinion. The weights are non negative.

These weights characterize the network. A network is then a row-stochastic matrix M with non-negative entries whose rows sum up to 1. It summarizes all the information on how voters influence each other and who listens to whom.

Voters can have asymmetric weights on each other's opinion, i.e. M_{ij} can be different than M_{ji}. It can even be that voter i influences j but j does not influence i. For example, bloggers can influence the opinion of their readers without having to follow their readers' twits.

The biggest challenge of models with network influence is keeping track of the evolution of opinions when the structure of the network is complex. This is even more complicated for strategic spending in majoritarian-voting competitions because it's hard to calculate how opinions change the probability of winning.

By assuming the process above I can set up the evolution of opinions as a linear transition system. That allows me to apply powerful tools from linear algebra and markov-chain theory

to study the problem.

Let $\mathbf{p^t}$ be the vector of opinions at time t. This vector evolves according to:

$$\mathbf{p^{t+1}} = M\mathbf{p^t} = M^t\mathbf{p^1}$$

There are two ways of interpreting network influence. It can be interpreted as a model of information processing or as a model of social preferences.

In the information interpretation, there is a common value p that captures the true difference in value between A and B, but voters do not know it. Instead, they have disaggregated information, or opinions, and they try to update their assessment of p through the opinion of their neighbors. As people update their opinion, the information gets disseminated through society. Voters update their opinion many times using their neighbors' opinions to incorporate the new information that propagates through the network.

Updating opinions through a linear process is not the optimal bayesian way of processing information, but it can be justified as a simple heuristic for voters who are boundedly rational. In general, the optimal bayesian information processing can be quite cumbersome to solve, while myopic linear updating provides a consistent estimate of the true p for electorates with large numbers of individuals as long as the structure of the network has some reasonable assumptions. The required conditions ensure that the influence of any individual and of any finite group of individuals is not bounded away from zero.[12]

Linear updating would be optimal in a world with a normal prior on the true p and with signals p_i that are normally distributed, although the M_{ij} weights might have to be adjusted between periods.[13]

If we interpret the network as a model of social preferences, there is no true parameter p. Instead, voters have a stochastic preference for choosing A over B and they are positively influenced by the preferences of their social neighbors. The value of p_i captures the intensity

[12]See Golub and Jackson (2010*b*).

[13]See DeMarzo, Vayanos and Zwiebel (2003); Golub and Jackson (2010*b*); Acemoglu, Ozdaglar and Parandeh-Gheibi (2010) for more on these issues.

of the preference and determines the final choice probability. Since voters have a preference to vote like their neighbors, they incorporate the intensity of their neighbors preferences into their choice.[14] The problem is that different voters want to imitate different people, so voters have to continuously update their preferences to match their neighbors. The updating process assumes individuals are myopic in doing so.

For now I will assume that the network can completely change the opinion of each voter. In section 3.5, I will extend the model so that voters have a private ideology that cannot be changed. The final choice will then depend on the interaction between the private ideology and the social preference.

2.6 Some network definitions.

I will refer to the voters as the nodes of the network and will say there is a link from i to j if $M_{ij} > 0$. A network is *directed* if there can be a link from node i to node j without a link from j to i. A directed network is *path-connected* if for every pair of nodes i, j there is a directed path from i to j and a directed path back. That is, either i is connected to j or there exists a series of nodes $\{k_1, \ldots, k_n\}$ such that $\{M_{i,k_1}, M_{k_1,k_2}, \ldots, M_{k_{n-1},k_n}, M_{k_n,j}\} > 0$.

I also need to assume that the network is aperiodic. Aperiodicity is a technical condition that is verified if at least one voter places a positive weight on his previous opinion. I will assume this throughout.[15]

This completes the set-up of the model. I now proceed to solve for the equilibria.

[14] You can call this altruism in the sense that the utility of a voter is a weighted average of his utility p_i and the expected utility of his neighbors.

[15] See Jackson (2008) for more details on the definitions.

3 Solving for equilibria.

I will solve for equilibria of the model in three situations: (1) $T = 1$ and there is no network influence. (2) $T = 2$, there is only one round of network updating, I call this weak network effects. And (3) $T = \infty$, where network influence is arbitrarily large, which I call strong network effects.

In each case, persuader A solves

$$\max_{(a_1,\ldots,a_N)} \pi\big(p_1^T(a_1R_A, b_1R_B), \ldots, p_N^T(a_NR_A, b_NR_B)\big)$$

$$\text{s.t.} \ \sum a_1 = 1$$

While persuader B solves

$$\min_{(b_1,\ldots,b_N)} \pi\big(p_1^T(a_1R_A, b_1R_B), \ldots, p_N^T(a_NR_A, b_NR_B)\big)$$

$$\text{s.t.} \ \sum b_1 = 1$$

In all cases, persuaders will spend on each voter to equate the marginal benefit of changing p_i^1 to the marginal cost of changing it. This simple fact, along with the scale-free contest success function, allows me to pin down the strategies for any pure-strategy nash equilibrium. Proposition 1 shows that in any pure-strategy equilibrium, if there exists one, the persuaders chose a_i and b_i to be proportional to the marginal benefit of p_i^1 multiplied the marginal cost of changing it. This implies that in equilibrium, both persuaders spend the same percentage of resources on each voter ($a_i = b_i$), although the percentage might be different across voters.

If B has less resources than A, she will note be able to prevent A from increasing her probability of winning. But this does not guarantee that A will win, since votes are uncertain. Persuader B still benefits from spending her resources, because it reduces the probability A wins. Furthermore, persuader B prefers to be in an election with $R_A = 1 + \varepsilon + K$ and

$R_B = 1 + \varepsilon$, than in an election with $R_A = \varepsilon + K$ and $R_B = \varepsilon$, because by spending more resources she decreases the marginal impact of A's excess resources.

If the persuaders have the same amount of resources, in equilibrium the spending does not change the probability either will win. If the persuaders have a preference for keeping their resources, they would benefit from signing a contract that forbade both of them from campaigning. Of course, functioning democracies have checks and balances to prevent political parties from splitting the positions of power without running open campaigns. For example, some democracies have a legal mandate on the number of debates the candidates have to attend to.

Unfortunately, the model is not well suited to evaluate what happens to the welfare of the voters, so we can't compare the social welfare for different levels of campaign resources. Even if campaign resources do not change the expected outcome of an election, they might increase the voters' welfare by reducing the uncertainty about the candidates and by giving them the option value to vote against their expected preference when the characteristics of the candidates warrant it so. Personally, I subscribe to the view that one of the main advantages of a democracy is that it forces to "govern by discussion".

Proposition 1 gives a necessary but not a sufficient condition to find a pure-strategy nash equilibrium. This is complemented by Proposition 2 which uses a concavity condition (γ has to be small enough) to show that the stated strategies are indeed an equilibrium. Furthermore, under this condition I can show the equilibrium is unique.

For a larger γ the stated strategies might still be an equilibrium, but there might be other equilibria as well. From Proposition 1 we know these would necessarily be in mixed strategies. Since this is a zero-sum game, from the minimax theorem we know that all equilibria would be payoff equivalent.[16] As $\gamma \to \infty$, the stated strategies cannot continue be an equilibrium, because the game approaches a standard Colonel Blotto game which has no pure-strategy equilibria. (And the equilibrium correspondence as $\gamma \to \infty$ is upper-hemicontinuous).

[16] See the minimax theorem in Mas-Colell, Whinston and Green (1995).

Proposition 1 (On the structure of equilibria)**.** *The strategies below constitute the unique pure-strategy nash equilibrium, if there exists one.*

$$a_i^* = b_i^* = \frac{(\partial\pi/\partial p_i^1)p_i^1(R_A, R_B)\big(1 - p_i^1(R_A, R_B)\big)}{\sum(\partial\pi/\partial p_j^1)p_j^1(R_A, R_B)\big(1 - p_j^1(R_A, R_B)\big)}$$

Proof. See Appendix A. □

Proposition 2 (Existence and uniqueness)**.** *There exists $\bar{\gamma} > 0$ such that for all $\gamma < \bar{\gamma}$, the strategies stated in Proposition 1 are the unique equilibrium of the game.*

Proof. See Appendix B. □

3.1 Solving the model without network updating.

Without the network, persuaders target the pivotal voters. A voter is *pivotal* for the election if, conditional on the realized votes of the others, changing his vote changes the outcome. Because the votes are uncertain, persuaders target the voters with the highest probability of being pivotal. Let q_i represent the *probability voter i is pivotal* under $\mathbf{p^T}$. This is given by

$$q_i = \sum_{\substack{S \subset N \setminus \{i\} \\ |S| = N-1}} \prod_{j \in S} p_j^T \prod_{\substack{j' \notin S \\ j' \neq i}} \left(1 - p_{j'}^T\right)$$

Pivotal voters are important because persuaders only care about winning, which means influencing pivotal voters has the highest expected marginal benefit. Spending money to change a vote that is not pivotal is a waste of resources.

In equilibrium we have

$$a_i^* = b_i^* = \frac{q_i p_i^1 (1 - p_i^1)}{\sum q_j p_j^1 (1 - p_j^1)}$$

Where q_i and p_i^1 are calculated as if the persuaders spend (R_A, R_B) on each voter.

3.2 Solving the model with weak network effects.

With one round of network updating we get a network multiplier that averages the pivotality of a voter with the pivotality of his neighbors. Since the network changes the voting probabilities, it also changes the pivotality of each voter. Measuring pivotality with the network is analytically difficult because pivot probabilities are complicated objects. Conceptually, though, it is straightforward. Let q_i denote the probability voter i is pivotal under $\mathbf{p^2}(R_A, R_B)$. That is, we calculate p_i^1 as if persuaders spend (R_A, R_B) on each voter, then do one round of network updating and calculate the pivot probabilities. Since the lobbies know the network, they have enough information to do this calculation. From here we get

$$\frac{\partial \pi}{\partial p_i^1} = \sum_k \frac{\partial \pi}{\partial p_k^2} \cdot \frac{\partial p_k^2}{\partial p_i^1} = M_{ii} q_i + \sum_{k \neq i} q_k M_{ki}$$

Remember that M_{ki} is how much voter k listens to voter i. In equilibrium persuaders A and B spend according to

$$a_i^* = b_i^* = \frac{(M_{ii} q_i + \sum_{k \neq i} q_k M_{ki}) p_i^1 (1 - p_i^1)}{\sum_j (M_{jj} q_j + \sum_{k \neq j} q_k M_{kj}) p_j^1 (1 - p_j^1)}$$

3.3 Solving the model with strong network effects and consensus.

I could repeat the calculation in the previous subsection for any finite T. Each time I would adjust the calculation of q_i. The cumbersome part would be calculating all the direct and indirect influences after $T - 1$ rounds of updating. Instead, in this section I will focus on the limit as $T \to \infty$ to understand what happens when the network effects are strong. In the limit I get a surprising result: the pivotality of a voter does not matter at all, only his network influence matters. Furthermore, network influence only depends on structure of the network, not on the initial voting probabilities.

The following result by DeGroot (1974) is necessary to solve for equilibria. Under mild conditions on the network, all opinions converge to a consensus in the long run. This con-

sensus is a weighted average of the initial opinion of every voter. The influence of a voter's initial opinion on the consensus is given by the DeGroot weight of the voter. These weights are defined below.

Definition 3 (**The DeGroot Weights**). *Let M be a directed weighted network which is row-stochastic. Suppose the network is path-connected and aperiodic. Define the* ***DeGroot weights of network influence****, or simply the* ***DeGroot weights****, as the unique left eigenvector of matrix M that corresponds to the eigenvalue 1 and whose entries have been normalized to one. I denote it by* $\mathbf{w}$*. In math,* $\mathbf{w}$ *is the unique vector such that*

$$\mathbf{w}M = \mathbf{w} \text{ with } \sum w_i = 1$$

Theorem 4 (DeGroot 1974). *Let M be a path-connected, aperiodic network which is row-stochastic. For any initial vector of opinions $\mathbf{p^1} \in \mathbb{R}^N$ we have*

$$\lim_{t\to\infty} M^t \mathbf{p} = p^* \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} \text{ with } p^* = \sum w_i p_i^1$$

From here I get that in equilibrium:

$$a_i^* = b_i^* = \frac{w_i p_i^1 (1 - p_i^1)}{\sum_j w_j p_j^1 (1 - p_j^1)}$$

It's important to note that the long-run consensus of opinions is not an artifact of the myopic updating. Rational agents who share information cannot "agree to disagree" and must converge to a common posterior in a finite number of steps. DeMarzo, Vayanos and Zwiebels show that rational individuals on a network sharing posteriors that were derived from a normal prior and normal posteriors converge to the optimal bayesian consensus belief

in at most N^2 steps. Furthermore, the work by Acemoglu, Ozdaglar, and Ali Parandeh-Gheibi show that even if network influence is random, in the sense that it depends on the probability each pair of individuals meet each other and on the probability the persuade each other, the long-run opinions converge almost surely to a consensus, although the value of the consensus depends on the realized pattern of influence. In their model, the expected value of the consensus is the weighted sum of the initial beliefs calculated using the DeGroot weights of the expected value of the network.[17]

When voting probabilities reach a consensus all voters are equally likely to be pivotal, so it might seem that pivotal voters don't matter simply because opinions converge. As is show in Section 3.5, this is a wrong interpretation. Network influence displaces pivotality even without a consensus. The shift from pivotal to influential voters happens because the network prevents resources from being targeted. As $T \to \infty$, persuaders cannot independently change the opinion of any single voter, because their opinions mix with the opinion of their neighbors. Persuaders respond by spending to change the DeGroot consensus. The most effective way to do this is to target the influential voters.

Is there a systematic relationship between pivotal and influential voters? Theoretically no, these two concepts are orthogonal. One can always construct a network where pivotal voters are the same as influential voters and one can construct a network where influential and pivotal voters are completely different. This is a consequence of linear updating. Under linear updating, influence is independent of opinions, but the probability of being pivotal crucially depends on them. This is shown in the following example.

[17] DeMarzo, Vayanos and Zwiebel (2003); Acemoglu, Ozdaglar and Parandeh-Gheibi (2010).

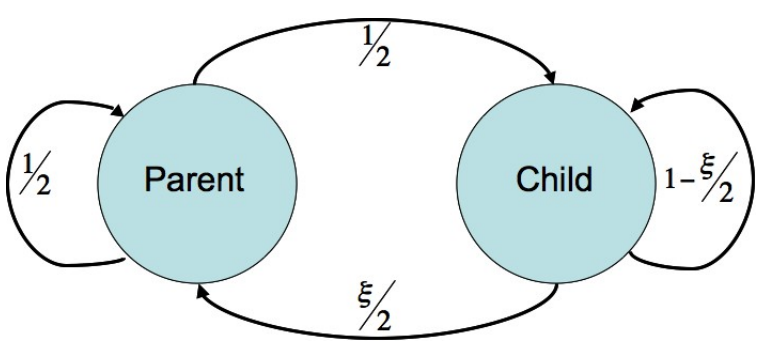
3.4 A parent and child example.

Two voters, a parent and a child, have to decide between two almost identical products: A and B. An important difference between the products is that A is sponsored by a popular cartoon character. Initially, the child is very much convinced that A is better than B: $p^0_{child} \approx 1$. The parent is of the opposite state of mind. For symmetry, assume $p^0_{parent} = 1 - p^0_{child} = 1 - p$.

To decide which product they want, the parent and the child are going to take a vote. Product B is the status quo object. Product A is only chosen if both the parent and the child vote for it. Suppose the persuaders, firms A and B, have the same amount of resources to spend on advertising.

Because of the unanimity rule, a voter is pivotal only if the other voter choses A. Without network influence, the parent will be pivotal with probability p and the child with probability $1 - p$. It's much more likely that the parent's vote will be decisive for the election. Firms react rationally by heavily targeting the parent. In equilibrium, both firms spend a fraction p of their budget on persuading the parent and a fraction $1 - p$ on persuading the child.

Suppose instead that before taking the decision the parent and the child will deliberate. The parent feels it's important to give an equal weight to his child's opinion. The child, being a childish, pays very little attention to the parent. She places $\xi/2 \approx 0$ weight on the parent's opinion and $1 - \xi/2$ on her own.



The matrix representation of the network is

$$M = \begin{pmatrix} M_{parent,parent} & M_{parent,child} \\ M_{child,parent} & M_{child,child} \end{pmatrix} = \begin{pmatrix} 1/2 & 1/2 \\ \xi/2 & 1-\xi/2 \end{pmatrix}$$

The corresponding DeGroot weights are

$$\mathbf{w} = \begin{pmatrix} w_{parent} \\ w_{child} \end{pmatrix} = \begin{pmatrix} \frac{\xi}{1+\xi} \\ \frac{1}{1+\xi} \end{pmatrix}$$

Given this, if the parent and the child talk for long enough, the opinion of the child will almost completely prevail. Knowing this, the firms would spend a large fraction of their resources on the child: $1/(1+\xi) \approx 1$.

Which is the right model? Different products might have different levels of communication. The parent might not be willing to discuss with the child what is the right type clothes for playing in the snow. On the other hand, the car drive from San Francisco to LA will give the child ample time to convince the parent they should go to Disneyland instead of the LA Museum of Contemporary Art.

3.5 Solving the model without consensus: adding ideology.

I will add ideology to the model to show that convergence in voting probabilities is not crucial to have network influence crowd out spending on pivotal voters. Let $\theta_i \in [0,1]$ be an ideology parameter for voter i. Voters with a θ_i closer to 1 are more inclined to support A. The ideologies are common knowledge. Voters maximize the following utility function.

$$U(voting\ for\ A) = u(p_i; \theta_i) - \eta_i$$

$$U(voting\ for\ B) = 0$$

Where $u : [0,1]^2 \to (0,1)$ is continuous function which is strictly increasing in p_i and in θ_i.

With this parametrization the probability that voter i choses A is $u(p_i, \theta_i)$. For example, u could be the function below.

$$u(p_i; \theta_i) = \frac{p_i + \theta_i}{2}$$

Everything else remains as before. Persuaders can spend to change p_i through the contest success function and voters update p_i through the network. The ideology is fixed cannot be changed by spending nor by the opinions of other people.

In the information interpretation of the model, θ_i represents preferences while p_i represents information. The underlying assumption is that when voters interact through the network they are able to separate information from ideologies.

In the social preference interpretation of the model, θ_i represents the private aspects of choice, those that cannot be influenced by other people, while p_i represents the social dimensions of choice, those aspects that voters wish to match with their neighbors.

With $T = \infty$, the pure-strategy equilibria with or without ideology are identical. This happens because persuaders cannot change ideologies, so they focus on influencing the DeGroot consensus. This is spelled out in detail in Remark 1.

Remark 1. ***In a pure-strategy equilibrium with*** $T = \infty$***, persuader*** A ***maximizes*** $\sum w_i p_i^T$ ***while persuader*** B ***minimizes it.*** *To see this note that if all probabilities are between zero and one, the probability persuader A wins the election is strictly increasing in p_i for any i. Now let p^*, p^{**} be two possible values for the DeGroot consensus such that $p^* > p^{**}$. Since the probability voter i choses A is strictly increasing in p_i, the distribution of votes under a consensus of p^* first-order stochastically dominates the distribution under p^{**}. Therefore the probability A wins is a monotone transformation of the DeGroot consensus and they share maximizers. From here the unique pure-strategy equilibrium is as stated in Proposition 1.*[18]

[18]This argument is only true for pure-strategy equilibria.

4 Testing the model with data on lobby contributions in the US. House of Representatives.

Even if myopic linear updating is a stringent assumption, a linear process can be a good local approximation to any smooth updating process. To test if the model is a good enough approximation I take it to the data. To do so I will use campaign contributions by lobbies in the U.S. House of Representatives. My main aim is to test the broadest prediction of the model: do lobbyists spend more on legislators who have more network influence?

The estimation proceeds in three steps. First I construct a measure of the bilateral influence between each pair of legislators, the weight of the links, and I construct a measure of the pivotality. Next I measure the strong network influence of each legislator by calculating his DeGroot weight and I measure the weak network influence by calculating the weighted sum of the pivotality of his neighbors. Finally, I regress campaign contributions on network influence and on pivotality to see which is a better predictor. In the regression I will control for several confounding variables directly and I will use the panel structure of the data to control for unobservable characteristics of the legislators.

4.1 The contributions data.

To measure lobbying expenditures I use the campaign contributions by Political Action Committees (PACs) using data from the Federal Elections Committee (FEC) from 1990 to 2006. The data is made available by the Center for Responsive Politics.[19] The unit of observation is the contributions of a given PAC to a given legislator during a given electoral cycle. Of the 919 total Representatives in my sample, 912 received contributions from PACs in every electoral cycle.

I completed the FEC data by adding a zero if a PAC did not contribute to a legislator in a year where the PAC was active.[20] This allows me to observe corner solutions in the data,

[19] http://www.opensecrets.org

[20] I also sometimes added zeros in a year where a PAC did not contribute to any legislator. I did this

which turn out to be a significant percentage of the total possible contributions. The average active PAC only gave contributions to 8 percent of the legislators in the chamber. Therefore corner solutions are a first order concern that I will deal with in the specification.[21]

For computational reasons, I dropped PACs that did not contribute to at least 8 legislators in at least one electoral year. This corresponds to roughly 50% of the PACs in my sample, but these PACs contribute about 95 percent of the contributions, so explaining their behavior goes a long way into explaining lobbying expenditures in the House. Also, I only kept PACs classified by the FEC as belonging to business or labor groups. There were initially 7,171 PACs in my data, at the end I was left with 2,966 PACs of which 140 are labor PACs and 2,826 are business PACs.

It's generally considered that PACs donate for two reasons: to influence legislation and to help elect and re-elect members. Therefore PAC contributions do not correspond exactly to the lobbying expenditures in my model, but this shouldn't bias my results because it's an extraneous source of variation in the dependent variable that is not be correlated with the variables of strategic spending. Instead, it will increase the standard errors in my regression making it harder to test if network influence is a significant predictor.

In addition to this, PAC contributions imperfectly measure lobbying expenditures because they are not the only way interest groups spend resources on legislators. Interest groups can also hire full time lobbyists. By law, businesses and unions are required to disclose how much they spend on full-time lobbyists, but they are not required to disclose on what bills or over which legislators the lobbyists focus their efforts, so I cannot use that data to measure targeting.

The predictions of the model refer to the total resources lobbies spend on a legislator,

whenever I had both a previous year and a later year where the PAC was active. This gives me a panel structure that has no gaps, although the panel remains unbalanced.

[21] The model ruled out corner solutions by assuming that the marginal effect when contributions are small is very large. This is convenient to get analytical solutions, but the model can be easily modified to allow for corner solutions. For example the contest success function could be replaced by $p_i^1(a+1, b+1)$, so we still have an approximately scale-free function for large expenditures but the legislator's opinion becomes unresponsive for small levels of contributions.

both through PAC contributions and through lobbyists. As long as these two expenditures are positively correlated, PAC contributions work as a proxy for total lobby expenditures.

Representatives also receive contributions from individuals. PAC contributions represent about 30% of the total contributions, while contributions from individuals represent the other 70%. Since the contributions of each individual are a much smaller fraction of the total contributions, I assume their expenditures are not strategic and that they are orthogonal to PAC spending.

In each electoral year, many bills are presented and many different lobbies compete over separate issues. I interpret each bill as an independent realization of my model, with an interest group on each side of the issue that spends according to the network influence or the pivot probabilities.

Even if there are many lobbies influencing a single bill, once the content of the bill is fixed there are only two sides to the issue. Groups of lobbies spending in a coordinated matter should spend as in my model.

Do lobbies really target the same legislators? Do they even spend on legislators with opposing views? Unfortunately I cannot test this directly, because I cannot match PAC contributions to specific bills. There is evidence that some PACs spend very broadly across the political spectrum. In the 2006 electoral cycle, the top contributing PAC was the National Association of Realtors which gave to 49% of it's contributions to Democratic candidates and 51% to Republican. The research on counteractive lobbying has also found that legislators get lobbied by interest groups on both sides of the issue.[22]

4.2 Measuring network influence.

To build the network I use data on the cosponsorship of bills in the U.S. Congress.[23] The data has all the bills, resolutions and amendments between 1972 and 2006, from the 93rd to the 109th Congress.

[22] Austen-Smith and Wright (1996, 1994).

[23] The data was collected and made available by Fowler (2006*a*,*b*).

Every time a bill is presented in Congress it must have a sponsoring legislator and other legislators can sign up as cosponsors. I will use the frequency of cosponsorship as a proxy for bilateral influence. Every time legislator j cosponsors one of legislator i's bills, I interpret that i has some influence over j. This data is very convenient because it has a direction of influence (from cosponsor to sponsor) and because legislators cosponsor together many times within and across legislatures. This allows me to build a weight for each link.

To construct the network for the electoral year t, I take each pair of legislators i, j who served in year t and measure the weight j puts on i's opinion by counting the number of times j cosponsored a bill sponsored by i in any congress where they both served together up to year t. I do the corresponding thing to measure the weight i puts on j.

After counting all the cosponsorships, I divide the cosponsorships from j to i by the total number of times j cosponsored with anybody else. This makes the matrix row-stochastic. The influence of i over j is then just the frequency with which j cosponsors i's bills relative to how often j cosponsors with anybody else. Legislators become more influential as they get more cosponsors on their bills except if those legislators also cosponsor a lot of other bills.

Links in the network accumulate over time for legislators that remain in Congress. Since this strongly biases the DeGroot weights in favor of more senior legislators, I control for seniority in the regression.

A problem with the data is that some bills are cosponsored by almost everybody in the chamber. This probably has to do with the content of the bill rather than the influence of the sponsoring legislator. Most bills have one or no cosponsors. The number of bills with n coauthors diminishes exponentially as n grows, except for n equal to half of the chamber, where the distribution spikes up. Bills with a number of cosponsors larger than half the chamber probably involve signaling by the majority party instead of influence by the sponsoring legislator.

To deal with this I do two things: I drop the bills that were cosponsored by more than

half the House. This drops about 10% of the bills. Next, I weigh down the links between cosponsors and sponsors by the number of cosponsors on each bill. If j cosponsored i's bill along with 9 other legislators, I assign a weight of 1/10 from j to i. Running the regression without these adjustments yields similar coefficients but higher standard errors.

Since I do not observe self links in the data, I need to make some assumption on the weight each legislator puts on his previous opinion to identify the DeGroot weights. I assume that all legislators put the same weight on themselves. As long as this weight is positive, it doesn't matter which value I chose because the DeGroot weights will be the same.[24]

4.3 Measuring pivotality.

To compare the network model to previous models on strategic spending, I include a measure of pivotality in the regressions. To measure pivotality, I estimate the probability each legislator was pivotal on each bill presented in a legislature. Since I can't match contributions to specific bills, I average the pivot probabilities over all bills. This averaging will generate an attenuation bias in the pivotality coefficient, but there is no clear solution to the problem.

Even though we can observe only one realization of votes for each bill, in principle we can still use the realized votes to get a consistent estimator of the average pivot probability in a legislature. Unfortunately, bills are almost never passed by a single vote, so the estimation would depend on an absurdly low number of observations. Given the number of legislators in the US. Congress, pivotal votes are necessarily low probability events.[25]

To get around this I simulate the votes on each bill using the dw-nominate scores, a popular model in political science that gives a parsimonious theory to rationalize the votes

[24] To see this let $\alpha > 0$ be the weight each legislator puts on himself and M be the network matrix whose main diagonal is zero and whose rows sum to one. The true network would be $\alpha I + (1-\alpha)M$. The largest left eigenvector of $\alpha I + (1-\alpha)M$ is also the largest left eigenvector of M.

[25] In the simulation I describe below, about 75% of the bills did not have a single pivotal vote after 5,000 simulations. A further 12% of bills had a probability smaller than 0.001 of having a pivotal vote, which does not do much to distinguish the pivot probabilities across legislators. Only 13% of the bills in the sample had a significant probability of passing or failing by a pivotal vote. For these bills the average probability of a pivotal vote was 0.057.

of legislators in Congress.[26] The structural assumptions in the theory allow me to calculate the *ex-ante* voting probability for each legislator on each bill. I then simulated a vote on each bill thousands of times using independent realizations of the vote for each legislator.[27] Through this I can estimate how often a legislator would have been pivotal. After this I averaged the pivot probabilities across all bills to measure the average pivot probability for each legislator in each electoral year.

To calculate the *neighbor pivot* in the regression, for each legislator I did a weighted average of the pivotality of his neighbors in an electoral year. Consistent with the model, the weights are given by the strength of the link between from the cosponsor to the sponsoring legislators.

As noted before, the theory makes no prediction on the relationship between pivot probabilities and DeGroot weights. Empirically, pivotality is not very correlated with the DeGroot weights. The correlation is 0.04. The correlation between the pivotality of a legislator and the pivotality of his neighbors is also not very strong: 0.1.

4.4 The specification.

Corner solutions are a first order concern to describe the data. The average PAC only gave to 8 percent of the legislators in a chamber. Lobbies spend on influential voters because they optimize by equating the marginal benefit and the marginal costs across legislators. This does not hold for corner solutions. Suppose that for a given spending strategy legislator i has a larger influence than legislator j relative to their marginal costs. Then the lobbies would want to reassign resources in the margin, decreasing the contributions of legislator j to increase the contributions of legislator i. If the lobbies are spending zero on j, there is nothing to reassign and the difference in the marginal benefit does not translate in a change in contributions.

[26] See 'www.voteview.com' for more information.

[27] I first ran a simulation of 5,000 votes and deleted 87 percent of the bills that did not show a significant probability of having a pivotal vote. I then ran 100,000 simulations on the remaining bills.

Therefore the theory predicts a very different relationship between network influence and campaign contributions depending on whether the lobbies donate a positive amount or not. The same applies for pivotality. Running a linear specification will not capture this.[28] Instead, I will run a censored regression model by running a tobit specification.

A second important concern is that unobserved characteristics of the legislators might bias my estimates of network influence and pivotality. For example, assume each legislator has an unobserved ability to raise resources that is correlated with his ability to get cosponsors. The DeGroot weights would be a proxy for the *persuasiveness* of a legislator instead of measuring the effect of a legislator's position in the network.

To control for this I will focus on a regression with legislator random-effects. This specification exploits the multiple observations I get for each legislator over different electoral years. Assuming a legislator's intrinsic persuasiveness is constant from one electoral year to the next, the random-effects control for the bias. Identification on the effect of network influence comes from comparing the changes in influence from one year to the next with the changes in campaign contributions.[29]

It's important to note that the unobserved characteristics will only bias my results if they are correlated with my variables of interest. To control for them I will use a Chamberlain style random effect estimator. Let α_i be an intercept for each legislator. To be able to remain within the tobit framework, I assumes the intercepts are normally distributed. I allow them to be correlated with the mean of the explanatory variables.

$$\alpha_i = \gamma_{\mathbf{X}} \bar{\mathbf{X}}_i + \nu_i$$

Where $\bar{X}$ is the per legislator average of the explanatory variables and ν_i is i.i.d. $Normal(0, \sigma_\alpha)$.

The specification is as follows. For a PAC k, a legislator i and an electoral year t, there

[28]The data also has a second source of censoring, because by law PACs cannot give more than 10,000 dollars to a legislator during a given electoral year. This source of censoring is much less important than the first one. It only affect 0.3 percent of the contributions in my sample.

[29]The regression also includes seniority and seniority squared which allow for quadratic time trends.

is a latent variable for contributions denoted by $y^*_{k,i,t}$ with the following specification:

$$y^*_{k,i,t} = \alpha_i + \beta_1 DeGroot_{i,t} + \beta_2 Pivotality_{i,t} + \beta_3 NeighborPivot_{i,t} + \beta_{\tilde{\mathbf{X}}}\tilde{\mathbf{X}}_{i,t} + e_{k,i,t}. \quad (1)$$
$$= \beta_1 DeGroot_{i,t} + \beta_2 Pivotality_{i,t} + \beta_3 NeighborPivot_{i,t} + \beta_{\tilde{\mathbf{X}}}\tilde{\mathbf{X}}_{i,t} + \gamma_{\mathbf{X}}\bar{\mathbf{X}}_i + \nu_i + e_{k,i,t}.$$

The mean-zero errors $e_{i,t,l}$ are assumed to be i.i.d Normal across legislators, but I will allow correlation within each legislator by using clustered standard errors. The matrix $\tilde{X}_{i,t}$ is a group of controls that includes the following variables:

1. **Seniority and seniority squared.** Measured from the first time a legislator entered the House.[30] It's particularly important to control for seniority because the measure of network influence accumulates over time, so network influence is strongly correlated with seniority.

2. **Leadership dummies.** I include dummies for the House Speaker, the Majority and the Minority leaders and whips.

3. **Committess dummies.** I include dummies for members of influential committees: Appropriation; Ways and Means; Energy and Commerce; and Banking and Finance.

4. **Majority dummy.** Previous work by Cox and Magar had found that being in the majority party is a significant predictor of campaign contributions. It's also significantly correlated with pivotality.[31]

5. **Party.** When PACs spend to get legislators re-elected, Republicans and Democrats receive funding from different sources. I add this dummy to reduce the noise in the data.

6. **Ideological distance to the center.** Measured by the absolute value of the first dimension dw-nominate score. As long as legislators with more extreme ideologies

[30] This is almost identical as a number of years a legislator has served. In general, legislators leave Congress only once.

[31] See Cox and Magar (1999).

are harder to persuade, this measure might be a good proxy for heterogeneity in the marginal cost of a legislator. I also include it to guard against the possibility that pivotality or network influence proxy for ideological moderation.

7. **Electoral year dummies.** My model does not predict the total amount lobbies would spend, only the relative amount on each legislator. In the data I observe a lot of year-to-year variation in contributions. The average year-to-year standard deviation per legislator is 35 per cent of the average contributions. Adding the dummies helps reduce the noise from these variations.

The researcher observes the censored contributions $y_{k,i,t}$ which are equal to

$$y_{k,i,t} = \max\{0, \min\{y^*_{k,i,t}, \bar{y}_t\}\}$$

Where $\bar{y}_t$ is the per PAC contribution limit for the electoral year t expressed in real dollars.

The DeGroot weights of a legislator change over time for two reasons: legislators sponsor and cosponsor new bills, and the previous cosponsors of a given legislator might leave congress. Since legislators get a higher DeGroot weight if their cosponsors have a high DeGroot weight, the influence of a legislator significantly decreases when an influential cosponsor leaves. For example, during the 1994 Republican take-over of Congress, the Democrat Richard Gephardt lost 28% of his DeGroot influence, falling 16 places in the ranking of legislators by DeGroot influence. (He also stopped being the Majority Leader.)

Pivot probabilities also change from year to year. This happens because the composition of the chamber changes and because different bills are included in the agenda. During the Republican takeover of 1994 the Democrats lost pivotality by a large amount.

4.5 The results.

Table 1 presents the results. After controlling for potential confounds, both the DeGroot weights and the pivot probabilities are a statistically significant predictors of campaign contributions. (Respectively $p = 0.03$ and $p = 0.00$.) The pivotality of the neighbors is not a statistically significant predictor. It also has the wrong sign but the magnitude is not substantively significant: 1% of the average campaign contributions versus a 17% increase per unit of pivotality.

As can be seen by comparing column (2) and column (4), the most important channel by which the DeGroot weights influence campaign contributions is by increasing the number of PACs that contribute, rather than increasing the contributions from PACs that are already donating. Since $E(y|\mathbf{X}) = Prob(y > 0|\mathbf{X}) \cdot E(y|y > 0, \mathbf{X})$, the percentage increase in campaign contributions equals the percentage increase of the campaign contributions of those PACs that are already giving plus the percentage increase in the number of PACs that donate. Increasing the DeGroot weights by one unit increases the campaign contributions of the average legislator by 27% but only increases the contributions of PACs that were already donating by 4.3%. Properly accounting for the corner solutions in the data is very important to distinguish between these effects.

Since the DeGroot units are hard to compare with other variables, I also calculate the effect of increasing each variable by one standard deviation. (Table 2.) My specification predicts that such an increase in the DeGroot weight of a legislator is associated with an increase of about 26,000 dollars, or 6% of the average campaign contributions in my sample.

In standard deviation units, the coefficient of the DeGroot weights is not statistically different from the effect of becoming the House Speaker (11%). The effect of becoming the Speaker is huge (921,000 dollars!) but it happens to very few legislators, so it doesn't explain as much variation in the data. Increasing the DeGroot weights has a larger effect than becoming the Majority Leader (3%) or joining the powerful Ways and Means Committee (3%), although I also can't reject that these magnitudes are equal.

Pivotality has a larger effect per unit of standard deviation (9% vs 6%) but the difference is not statistically significant. Both variables might be significant because different bills have different amounts of consultation between legislators. When legislators influence each other a lot, network influence matters more. When legislators vote independently pivotal legislators are the most important.

The results are suggestive of the role of informal networks of relationships in granting influence in congress. Network influence seems to translate into bigger campaign contributions. Unfortunately, reverse causality cannot be ruled out, since legislators who are better fund-raisers might be able to leverage their connections with PACs to develop relationships with other legislators. More work is needed to disentangle these effects.

5 Conclusion

I proposed a model of strategic persuasion over social networks. This is one of the first models to address the role of influence between voters in electoral competitions. The model is tractable and allows me to solve for the equilibrium spending across voters in the network.

In equilibrium, the expenditure on a voter is proportional to his network influence. This contrasts with previous findings on strategic spending for majoritarian-voting competitions, which had found that equilibrium spending targets voters who are more likely to be pivotal for the outcome of an election.

Network influence displaces pivotality because the network effects hamper targeting. When opinions are frequently updated through a social network, it's impossible to change the opinion of a voter in isolation of his neighbors. Persuaders react by spending on voters with influential positions on the network.

The model predicts that the relevant measure of network influence is an eigenvector measures, the DeGroot weights. Eigenvector measures of influence are self-referential: individuals are influential if influential individuals listen to them. This measure highlights the

quality rather than the quantity of connections.

For political campaigns, the model proposes a way to process the highly detailed information about the networks of the electorate that's being generated by social-networking sites. Instead of spending resources on traditional local leaders, the model suggests political campaigns should look at the structure of social relationships to identify who holds influential positions (as measured by the DeGroot weights).

To test my model I put together data on lobbying expenditures by Political Action Committees with data on cosponsorship networks in the US. Congress for the electoral cycles from 1990 to 2006. After controlling for several confounding variables, I find that network influence is a significant predictor of campaign contributions for House of Representatives.

An increase of network influence by one standard deviation predicts an increase of 26,000 dollars ($p = 0.03$) in the campaign contributions of a Representative. This amount corresponds to 6% of the average campaign contributions. An increase in the pivotality of a legislator predicts an increase of 39,000 dollars (9% of the average, $p = 0.00$). The difference is not statistically significant.

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Appendix A Proof for Proposition 1.

Proof. This proof is an adaptation of the Shubik & Weber proof.

I first prove that a pure-strategy equilibrium must be in the interior. I do this by the contrapositive. Suppose that $b_i = 0$. Then A can spend an arbitrarily small quantity on i to obtain $p_i^1 = 1$. Since $p_j^1(\cdot, b_j)$ is continuous for $a_j > 0$ and A must be spending somewhere, A can decrease p_j^1 for some j by an arbitrarily small amount to make $p_i^1 = 1$. This increases π by a discrete amount. Since this is true for an arbitrarily small change in a_j, persuader A has no best-response and the strategies cannot constitute an equilibrium.

Next I show that persuaders spend the same percentage on each voter. From the First Order Conditions (FOCs) for A, I obtain

$$\frac{\partial p_i^1/\partial a_i}{\partial p_j^1/\partial a_j} = \frac{\partial \pi/\partial p_j^1}{\partial \pi/\partial p_i^1}$$

Here the fact that p_j^T is strictly between 0 and 1 for all j and $\partial p_j^T/\partial p_j^1$ is strictly positive guarantees that $\partial \pi/\partial p_i^1$ is positive and the above expression makes sense.

From the FOCs for B we get

$$\frac{\partial p_i^1/\partial b_i}{\partial p_j^1/\partial b_j} = \frac{\partial \pi/\partial p_j^1}{\partial \pi/\partial p_i^1}$$

$$\Rightarrow \frac{\partial p_i^1/\partial a_i}{\partial p_i^1/\partial b_i} = \frac{\partial p_j^1/\partial a_j}{\partial p_j^1/\partial b_j}; \forall i, j$$

On the other hand, by the homogeneity of p^1 I apply Euler's law to get

$$\begin{aligned} a_i \frac{\partial p_i^1}{\partial a_i} + b_i \frac{\partial p_i^1}{\partial b_i} &= 0 \\ -\frac{\partial p_i^1/\partial a_i}{\partial p_i^1/\partial b_i} &= \frac{b_i}{a_i} \end{aligned}$$

From the FOCs we know that the left hand side must be the constant across i. Therefore a_i/b_i must be constant for all voters. This means both A and B must be spending the same fraction of their resources on each voter: $a_i^* = b_i^*$.

Now we need to solve what this percentage is. To find out I first do some manipulation on $(\partial p_i^1/\partial a_i)$.

$$\begin{aligned}
\frac{\partial p^1}{\partial a_i}(a_i^* R_A, b_i^* R_B) &= \frac{\partial p_i^1}{\partial a_i}(b_i^* R_A, b_i^* R_B) && \text{(Plugging in } b^*\text{)} \\
&= \frac{1}{b_i^*}\frac{\partial p_i^1}{\partial a_i}(R_A, R_B)\Big|_{a_i=1} && \text{(By homogeneity of degree -1)} \\
&= \frac{\gamma}{b_i^*} p_i^1(R_A, R_B)\big(1 - p_i^1(R_A, R_B)\big)
\end{aligned}$$

I now substitute this in the first order condition for A.

$$\frac{b_j^*}{b_i^*} = \frac{(\partial\pi/\partial p_j^1)p_j^1(R_A, R_B)\big(1 - p_j^1(R_A, R_B)\big)}{(\partial\pi/\partial p_i^1)p_i^1(R_A, R_B)\big(1 - p_i^1(R_A, R_B)\big)}$$

Since this is true for any two voters and $\sum a_i = \sum b_i = 1$, I conclude that

$$a_i^* = b_i^* = \frac{(\partial\pi/\partial p_i^1)p_i^1(R_A, R_B)\big(1 - p_i^1(R_A, R_B)\big)}{\sum(\partial\pi/\partial p_j^1)p_j^1(R_A, R_B)\big(1 - p_j^1(R_A, R_B)\big)}$$

□

Appendix B Proof for Proposition 2.

Proof. This proof is a strengthening of Shubik & Weber's proof, who could only show that the stated strategies were a local equilibria. The proof works by showing that for a small enough γ the objective function $\pi(\cdot, b^*)$ is strictly concave in the relevant parameter space. From here the FOCs characterize the unique best response to b^*.

Let $\mathbf{a}$ be a spending strategy that potentially is optimal. As seen in the proof for Proposition 1 we can assume $a_i > 0$ for all i. To uncluttered things, in what follows p_i^1 represents $p_i^1(a_i R_A, b_i^* R_B)$. Let H denote the Hessian matrix at $\mathbf{a}$. The entries of H are as follows:

$$\begin{aligned}
H_{ii} =& \frac{\partial}{\partial a_i}\left(\frac{\partial \pi}{\partial p_i^1} \cdot \frac{\partial p_i^1}{\partial a_i}\right) \\
=& \frac{\partial^2 \pi}{\partial^2 p_i^1} \cdot \left(\frac{\partial p_i^1}{\partial a_i}\right)^2 + \frac{\partial \pi}{\partial p_i^1} \cdot \frac{\partial^2 p_i^1}{\partial^2 a_i} \\
=& \frac{\partial^2 \pi}{\partial^2 p_i^1} \cdot \left(\frac{\gamma}{a_i}\right)^2 \left(p_i^1\left(1 - p_i^1\right)\right)^2 \\
&+ \frac{\partial \pi}{\partial p_i^1} \cdot \left(\frac{\gamma}{a_i}\right)^2 p_i^1\left(1 - p_i^1\right)\left(1 - 2p_i^1 - \frac{1}{\gamma}\right)
\end{aligned}$$

$$\begin{aligned}
H_{ij} =& \frac{\partial^2 \pi}{\partial p_i^1 \partial p_j^1} \cdot \frac{\partial p_i^1}{\partial a_i} \cdot \frac{\partial p_j^1}{\partial a_j} \\
=& \frac{\partial^2 \pi}{\partial p_i^1 \partial p_j^1} \cdot \left(\frac{\gamma}{a_i}\right) p_i^1\left(1 - p_i^1\right)\left(\frac{\gamma}{a_j}\right) p_j^1\left(1 - p_j^1\right)
\end{aligned}$$

To verify if H is negative definite we can delete the common elements of each rows and each column. The simplified entries of H become

$$\begin{aligned}
H_{ii} =& \frac{\partial^2 \pi}{\partial^2 p_i^1} + \frac{\partial \pi}{\partial p_i^1} \cdot \frac{1 - 2p_i^1 - \frac{1}{\gamma}}{p_i^1\left(1 - p_i^1\right)} \\
H_{ij} =& \frac{\partial^2 \pi}{\partial p_i^1 \partial p_j^1}
\end{aligned}$$

By Gershgorin's theorem, the eigenvalues of H are at the union of the disks with center at H_{ii} and diameter $\sum_{j \neq i} |H_{ij}|$. The rest of the proof works to show that $\partial^2 \pi / \partial^2 p_i^1$ and

$\partial^2\pi/\partial p_i^1 \partial p_j^1$ are finite while $\partial\pi/\partial p_i^1$ is bounded away from zero. This implies that the elements of the main diagonal of H tend to $-\infty$ as γ tends to 0. Therefore all eigenvalues of H must be negative and the matrix is negative definite.

I proceed case by case:

Case I: $T = 1$. (No rounds of network updating). In this case $\partial\pi/\partial p_i^1$ is q_i. From here, $\partial^2\pi/\partial^2 p_i^1$ is 0 and $\partial^2\pi/\partial p_i^1 \partial p_j^1$ is $Prob(\sum \delta_k = \bar{N} - 2) - Prob(\sum \delta_k = \bar{N} - 1)$ for $k \notin \{i, j\}$, which is bounded between -1 and 1.

Proving that q_i is bounded away from zero is more challenging. In fact, it's not true over all the domain of $\mathbf{a}$. Instead, I will have to restrict the domain by deleting $\mathbf{a}$'s that yield a smaller π than $\pi(\mathbf{a}^*, \mathbf{b}^*)$. To this effect define the following:

$$p_{max} = \max\{p_i^1(R_A, b_i^* R_B)\}_{i \in N}$$

$$p_{min} = 1 - \left(1 - \pi^*\right)^{(N-\bar{N}+1)^{-1}}$$

Where π^* is $\pi(\mathbf{a}^*, \mathbf{b}^*)$. The interpretation of p_{max} is very simple. It's the maximum probability that can be achieved by spending all of A's resources on a single voter. Since $b_i^* > 0$ we have that $p_{max} < 1$.

I will now show that we can restrict ourselves to consider only strategies $\mathbf{a}$ such that at least $\bar{N}$ voters have a probability greater or equal to p_{min}. Take any $\mathbf{a}$ where this is not true and relabel the voters such that $p_1^1 \leqslant p_2^1 \leqslant \ldots \leqslant p_N^1$. We know $p_{N-\bar{N}+1}^1 < p_{min}$. From here

$$\begin{aligned}
\pi(p_1^1, \ldots, p_{N-\bar{N}+1}^1, p_{N-\bar{N}+2}^1, \ldots, p_N^1) &< \pi(p_{min}, \ldots, p_{min}, 1, \ldots, 1) \\
&= 1 - (1 - p_{min})^{N-\bar{N}+1} \\
&= \pi^*
\end{aligned}$$

Now, to show that q_i is bounded away from zero, note that

$$q_i = Prob\Big(\sum_{k\neq i} \delta_k = \bar{N} - 1\Big) \geqslant (p_{min})^{\bar{N}-1}(1 - p_{max})^{N-\bar{N}+1} > 0$$

Case II: $T = 2$. (One round of network updating). Here $\partial\pi/\partial p_i^1$ is $q_i + \sum_{j\neq i} M_{ji}q_j$. The proof is analogous to Case I.

Case III: $T = \infty$. Here π is a monotone transformation of $\tilde{\pi} = \sum w_i p_i^1$. The derivative of $\partial\tilde{\pi}/\partial p_i^1$ is w_i, which only depends on the network, and all the second derivatives and cross-derivatives are zero. Therefore π is strictly quasi-concave whenever $\gamma < 1$.

After solving all three cases, we conclude that a^* is the unique best-response to b^*. Mutato mutandis, b^* is the unique best-response to a^*. This shows that the strategies are indeed an equilibrium.

Uniqueness follows because equilibria for zero-sum games are interchangeable. To show this take any equilibrium of the game: (σ_a, σ_b). These are potentially mixed-strategies. It must be that (σ_a, b^*) and (a^*, σ_b) are also equilibria. Since a^* is the unique best-response to b^*, and vice-versa, we conclude that $(\sigma_a, \sigma_b) = (a^*, b^*)$. $\square$

Appendix C Competition with fundraising.

Until now I have assumed that the amount of resources was exogenous. In this section I analyze the possibility that persuaders have to raise resources at a cost.

I find that in equilibrium the ratio of the resources raised by the persuaders only depends on the relative costs of raising resources. The ratio does not depend on network influence, pivotality, the specific campaign rules nor even on the distribution of initial opinions. The absolute level of resources raised does depend on these things, in ways that are hard to characterize.

I assume persuader j has to pay a cost $c_j \cdot (R_j)^k$ to raise resources R_j. The parameter k is greater than one and the parameter c_j is greater than zero.

In the first stage of the game, persuaders simultaneously collect resources and the amounts they raise become common knowledge. In the second stage, persuaders decide where spend it. By backward induction the spending patterns in the second stage have to be the same as in Proposition (1).

The second stage pay-offs only depend on the ratio of resources collected. Let $r = {}^{R_A}/_{R_B}$ be such ratio and let $\pi(r)$ be the second stage pay-off for persuader A. To solve for the equilibrium r^*, I write persuader A's maximization problem as one that only depends on r.

$$\max_{R_A} \pi(r) - c_A R_A^k = \max_r \pi(r) - c_A r^k R_B^k$$

I rewrite persuader B's problem as follows.

$$\max_{R_B} \big(1 - \pi(r)\big) - c_B R_B^k = \max_r (1 - \pi(r)) - c_B \left(\frac{R_A}{r}\right)^k$$

The FOCs for the problem are

$$\frac{d\pi}{dr} - k c_A r^{k-1} R_B^k = 0$$

$$-\frac{d\pi}{dr} + k c_B r^{-k-1} R_A^k = 0$$

Solving this yields a solution that is independent of π.

$$r^* = \left(\frac{c_B}{c_A}\right)^{1/k}$$

If $c_A = c_B$ both persuaders will raise the same amount of resources and their probability of winning will not change from that determined by the initial opinion of voters plus the network updating.

Since marginal benefit only depends on r^* we can find the absolute level of resources by equating the marginal cost to the marginal benefit in the FOCs above. From this I can derive two easy comparative statics.

- Everything else constant, if voters are less persuadable (γ decreases) the total amount of resources raised by each persuader decreases.
- Suppose the marginal cost parameters increase proportionally. That is, (c_A, c_B) changes to $(\lambda c_A, \lambda c_B)$ with $\lambda > 1$. Then the total amount of resources raised by each persuader decreases.

For majoritarian elections the marginal benefit of resources increases with the probability the election will be decided by a pivotal vote. Persuaders spend more money on elections that are likely to be close.

The network has an ambiguous effect on campaign spending because it can make the election more or less close. For example, if everybody is very likely to choose for A except for one very influential voter, the competition with the network will be more close than without it. On the contrary, one very influential voter can tilt a large number of undecided voters, making the competition less close.

Appendix D Competitions in proportional representation systems.

In this section I solve for equilibria when persuaders want to maximize the share of voters who select them. This model is especially relevant for electoral systems with proportional representation, because parties get seats in parliament in proportion to the share of votes they get in the election.

The main result is qualitatively the same as before: persuaders spend over voters in proportion to an eigenvector-based measure of network influence: Bonacich centrality. Pivotal voters do not matter because the persuaders do not have a threshold number of votes they wish to achieve.

I will assume T follows a geometric distribution. This allows me find a relationship between equilibrium spending and Bonacich centrality, which is an important influence measure in the sociology literature.[32]

Fix $\rho \in (0,1)$. The random variable T follows a geometric distribution if the probability the game ends in $T > t$ conditional on reaching round t is a constant ρ for all t.

$$Prob(T > t | T \geqslant t) = \rho; \forall t$$

Conditional on B's strategy, persuader A solves

$$\max_{(a_1,\ldots,a_N)} E_T\Big[\sum p_i^T\Big] = \max_{(a_1,\ldots,a_N)} (1-\rho)\sum_{t=1}^{\infty}\rho^{t-1}\sum_i p_i^t$$

Definition 5. *Fix $\rho \in (0,1)$. The vector $\hat{\mathbf{w}}$ of **Bonacich influence weights** for a matrix M is*

$$\hat{\mathbf{w}} = (1-\rho)(1/N, \ldots, 1/N)[I - \rho M]^{-1}$$

[32]See Ballester, Calvó-Armengol and Zenou (2006); Bramoullé, Kranton and D'Amours (2010) for the relationship between Bonacich influence and Nash equilibria in games with linear best responses.

Proposition 6. *Suppose each persuader wants to maximize the percentage of voters that selects him. Then the unique pure-strategy nash equilibrium, if it exists, is:*

$$a_i^* = b_i^* = \frac{\hat{w}_i p_i^1(R_A, R_B)\big(1 - p_i^1(R_A, R_B)\big)}{\sum \hat{w}_j p_j^1(R_A, R_B)(1 - p_j^1(R_A, R_B))}$$

If $\gamma < 1$, this is the unique equilibrium of the game.

Proof. Take $(\mathbf{a}, \mathbf{b}) \in (0,1)^n$. I simply show that the objective function of each persuader is equal to $\sum \hat{w}_i p_i^1$. The rest of the proof follows the logic in Proposition (1) and Proposition (2) . Setting up persuader A's maximization problem we have

$$\begin{aligned}
\max_{a_1,\ldots,a_N} (1-\rho) \sum_{t=1}^{\infty} \rho^{t-1} \sum_i p_i^t &= \max_{a_1,\ldots,a_N} (1-\rho) \sum_{t=1}^{\infty} \rho^{t-1} (1,\ldots,1) M^{t-1} \mathbf{p}^1 \\
&= \max_{a_1,\ldots,a_N} (1-\rho)(1,\ldots,1)[I - \rho M]^{-1} \mathbf{p}^1 \\
&\sim \max_{a_1,\ldots,a_N} \hat{\mathbf{w}} \cdot \mathbf{p}^1
\end{aligned}$$

□

Finally, let me point out a well known result of markov chains. In the limit as $T \to \infty$, Bonacich weights converge to DeGroot weights. So in the limit as $\rho \to 1$, strategic spending in proportional representation systems converges to the equilibrium for majoritarian systems.

Summary Statistics for the House of Representatives				
Variable	Mean	Std. Dev.	Min	Max
Campaign contributions*	427	352	-6	4,480
DeGroot Weights (sum=100)	0.23	0.24	0.0	1.8
Pivot Probability (times 100)	0.28	0.55	0.2	3.8
Seniority	10.8	8	2	52
Bills Sponsored	17.6	14	0	158
Number of Cosponsors	205	103	0	429
PACs per Legislator	166	93	0	677
Legislators per PAC	37	49	1	402
*In thousands of 2006 dollars				

Random-effects tobit estimation on PAC contributions.
Marginal effects at the mean.

VARIABLES	(1) MFX on $E(y)$	(2) As % of $E(y)$	(3) MFX on $E(y\|y>0)$	(4) As % of $E(y\|y>0)$
Pivot probability	71 000***	17%	135 000***	2.7%
(Times 100)	(19 000)	(4.6%)	(35 000)	(0.7%)
Pivotality of neighbors	−1 500	−0.4%	−3 000	−0.07%
(Times 100)	(1 600)	(0.4%)	(3 000)	(0.07%)
DeGroot weights	112 000**	27%	211 000**	4.3%
(Times 100)	(52 000)	(12.7%)	(98 000)	(2.0%)
DW-Nominate1	−474 000***	−115%	−899 000***	−18.3%
(Abs value)	(139 000)	(33.8%)	(262 000)	(5.3%)
Majority dummy	38 000***	9%	71 000***	1.4%
	(11 000)	(2.7%)	(22 000)	(0.4%)
Party dummy	−26 000	−6%	−49 000	−1.0%
(Republican=1)	(75 000)	(18.3%)	(142 000)	(2.9%)
Seniority	−7 000*	−2%	−13 000*	−0.3%
(Including seniority squared)	(4 000)	(1%)	(7 000)	(0.1%)
House Speaker	921 000***	224%	1 210 000***	24.6%
	(139 000)	(33.8%)	(142 000)	(2.9%)
Majority Leader	291 000***	70.1%	468 000***	9.5%
	(113 000)	(27.5%)	(159 000)	(3.2%)
Minority Leader	384 000***	93.5%	595 000***	12.1%
	(67 000)	(16.3%)	(88 000)	(1.8%)
Majority Whip	280 000***	68.2%	453 000***	9.2%
	(68 000)	(16.6%)	(96 000)	(2.0%)
Minority Whip	275 000***	70%	446 000***	9.1%
	(55 000)	(13.4%)	(78 000)	(1.6%)
Appropriations	30 000	7%	56 000	1.1%
	(21 000)	(5.1%)	(38 000)	(0.8%)
Ways and Means	183 000***	45%	315 000***	6.4%
	(36 000)	(8.8%)	(56 000)	(1.1%)
Energy and Commerce	71 000***	17%	129 000***	2.6%
	(24 000)	(5.8%)	(42 000)	(0.9%)
Banking	47 000**	11%	87 000**	1.8%
	(23 000)	(5.6%)	(41 000)	(0.8%)
Observations	7,166,690	7,166,690	7,166,690	7,166,690

Clustered standard errors in parentheses. *** p<0.01 ** p<0.05 * p<0.1

Table 1: Campaign contributions in thousands of 2006 dollars.

Marginal effects per unit of standard deviation.

VARIABLES	(1) MFX per std. dev. on $E(y)$	(2) As % of $E(y)$	(3) MFX per std. dev. on $E(y\|y>0)$	(4) As % of $E(y\|y>0)$
DeGroot weights	26 000	6%	50 000	1.0%
(Times 100)	(12 000)	(2.9%)	(23 000)	(0.5%)
Pivot probability	39 000	9%	74 000	1.5%
(Times 100)	(10 000)	(2.4%)	(19 000)	(0.4%)
DW-Nominate1	−81 000	−20%	−153 000	−3.1%
(Abs value)	(24 000)	(5.8%)	(45 000)	(0.9%)
Majority dummy	19 000	5%	35 000	0.7%
	(5 000)	(1.2%)	(11 000)	(0.2%)
House Speaker	45 000	11%	59 000	1.2%
	(7 000)	(1.7%)	(7 000)	(0.1%)
Majority Leader	14 000	3%	23 000	0.5%
	(6 000)	(1.5%)	(8 000)	(0.2%)
Ways and Means	13 000	3%	23 000	0.5%
	(2 000)	(0.6%)	(4 000)	(0.1%)
Observations	7,166,690	7,166,690	7,166,690	7,166,690

Clustered standard errors in parentheses.

Table 2: Marginal effect at the mean multiplied by the standard deviation of each variable. Campaign contributions in thousands of 2006 dollars.