Banco de México

Documentos de Investigación

Banco de México

Working Papers

N° 2018-15

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August 2018

Documento de Investigación
2018-15

Working Paper
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On the Role of Financial Aid in a Default Episode*

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Abstract: We develop a dynamic stochastic quantitative model of sovereign default featuring fiscal policy, endogenous financial aid and risk-averse foreign lenders, in order to explore the role of financial aid in a default episode. After calibrating the model, we feed output shocks into the model to show that it captures some of the most salient features of the fiscal and debt situation in Argentina during the 1998-2002 period. This underscores the economic nature of the decision to default and the role that official aid could have taken in avoiding such an event. In effect, given the economic challenges endured by Argentina, a full-fledged default took place. In addition, we discuss a number of policy implications associated with financial aid programs aimed at preventing sovereign default episodes.

Keywords: Sovereign default, fiscal policy, financial aid.

JEL Classification: E62, F34.

Resumen: Desarrollamos un modelo dinámico estocástico con política fiscal, ayuda financiera endógena y prestamistas internacionales adversos al riesgo, para explorar el papel de la ayuda financiera en un episodio de default. Después de calibrar el modelo, alimentamos el modelo con choques del producto para mostrar que captura algunas de las más sobresalientes características de la situación fiscal y de deuda en Argentina en el periodo 1998-2002. Esto recalca la naturaleza económica de la decisión de default y el papel que la ayuda oficial pudo haber tenido para evitar dicho evento. En efecto, dados los retos económicos enfrentados por Argentina, un default completo tomó lugar. Adicionalmente, discutimos algunas implicaciones de política asociadas a los programas de ayuda financiera dirigidos a prevenir episodios de default soberano.

Palabras Clave: Default Soberano, Política Fiscal, Ayuda Financiera.

*The opinions in this paper are those of the authors and do not necessarily reflect those of Banco de México.

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I. Introduction

In the academic literature, there is some consensus on the negative effects of sovereign default episodes. For instance, De Paoli, Hoggarth and Saporta (2011), and Levy and Panizza (2011) have documented that sovereign defaults have an adverse impact on economic growth. Borensztein and Panizza (2009) find that the cost of borrowing is higher immediately after a default episode. Arteta and Hale (2008) document that foreign credit to the private sector declines after a sovereign default episode. Cruces and Trebesch (2013), and Richmond and Diaz (2009) find that debt restructuring can have a significant impact on access to foreign financing. In short, sovereign defaults are costly.

However, for a government, there are benefits related to defaulting on its debt. The literature has highlighted such benefits, particularly so in papers that model sovereign default events based on a willingness to pay approach (Eaton and Gersovitz, 1981). In this context, an important issue for the international financial community, particularly so for multilateral institutions, has been the implementation of financial aid programs aimed at altering the incentives to default, which a country could encounter. Such an implementation has as an objective to prevent a sovereign default event. More generally, some economies have had (implicit or explicit) multilateral support when they have found themselves in financial complications. Some other economies have had to live with the uncertainty of whether they could obtain such a support. As an important instance of the latter situation, we have Argentina in the early 2000s. In such a case, given the substantial debt burden and the slowdown of economic activity, a full-fledged default took place. In essence, the choice to default is largely the dire consequence of an economic cost-benefit analysis, which depends on the macroeconomic conditions and constraints faced by each country, particularly so, its debt level and economic growth prospects.

To gain a better understanding of the key features explaining the decision to opt for financial aid or default by a sovereign, this paper proposes a dynamic stochastic quantitative model of a small open economy with endogenous fiscal policy, financial aid, and sovereign default. This model illustrates the main trade-offs that a government faces when considering

such a decision. Relatedly, we explore the extent to which financial support (official aid) by a multilateral institution to Argentina could have had prevented the referred default episode.

Literature Review

Our paper builds on Cuadra, Sánchez, and Sapriza (2010), who develop a model of sovereign debt and fiscal policy to explain the pro-cyclical nature of fiscal policy in emerging market economies, and that of Arellano (2008) and Lizarazo (2013), who propose a stochastic discount factor to account for risk-averse foreign lenders.

Some studies in the literature have incorporated conditional financial support into sovereign default models. For instance, Boz (2011) considers a model in which the government has access to credit both from the private sector and from international financial institutions, which we refer to as multilateral institution. Each type of creditor offers a different debt contract. He then goes on to analyze how the government allocates its borrowing needs between these two types of creditors, and if and when it is optimal to default on its debt to the private sector. On their part, Fink and Scholl (2016) propose a model in which the government has access to loans from a multilateral institution. However, this institution imposes fiscal conditions that constrain public expenditure. Since public expenditure and private consumption are substitutes, the government responds reducing the tax rate. This, in turn, increases private consumption. Thus, financial support leads to a private consumption rise.[1] In contrast, in our model, a multilateral institution provides financial aid directly and imposes fiscal conditions that restrict the amount of debt the government can take. Hence, in our case, domestic absorption unambiguously falls, in line with the data.

More generally, several contributions have used a mix of theoretical and empirical methods to assess the extent to which a highly indebted country needs to adjust its fiscal policy to stabilize its debt to GDP ratio. Berrittella and Zhang (2015) use a dynamic stochastic general equilibrium (DSGE) model to estimate the tax rate necessary to stabilize public debt in several countries. They find that, on average, it is close to 50%. They find that tax

[1] The effect on absorption is ambiguous since public expenditure declines but private consumption increases.

adjustments are important to mitigate short-term risks arising from high debt, while economic growth is necessary to ensure long-term debt sustainability. Trachanas and Katrakilidis (2013) present evidence that governments have a tendency to increase taxes after they take spending decisions. In other words, they find evidence on the "spend and tax", hypothesis rather than on the "tax and spend" one. Tielens et al. (2014) use a vector autoregressive model to assess the impact that Eurobonds have on debt dynamics in Portugal, Ireland, and Greece. They argue that Eurobonds improve budgetary sustainability by absorbing macroeconomic shocks and by decreasing uncertainty over debt forecasts. In turn, the theoretical model of Cheng, Dai, and Dufourt (2017) shows that, absent the role of lender of last resort for the central bank in a monetary union, banking regulation and government deposit guarantees may not be able to avert the occurrence of twin banking and sovereign crises.

Finally, others have studied the effectiveness of bailout programs in ending or, at least, attenuating government debt crises, akin to our paper. Li, Sy, and McMurray (2015), for instance, study the history of IMF's bailouts. They assert that the design of such packages tends to overlook the recipient country's characteristics and, for the most part, focuses on short-term solutions that are often damaging in the long-term.

Discussion

In our model, a multilateral institution can provide conditional financial aid by transferring resources to the domestic government, and imposing certain fiscal conditions. In particular, it restricts the amount of resources that the government can borrow from foreign lenders. In turn, the government has the option to accept or decline financial aid. In short, there are both benefits and costs to such a decision. It captures them with the following features. A parameter accounts for the size of the financial aid transfer, modeled as the fraction of the country's outstanding debt. The other is a restriction on the debt's size that the country can take on the next period. We interpret this as a way of instilling some fiscal discipline. When a government is close to defaulting, it would be trying to reduce its leverage. The referred restriction should increase the pace at which a government would try to do so. Of course, *ex ante*, the restriction does not necessarily hold. It reduces the default probability and, consequently, lowers its borrowing costs, partially because investors know that fiscal

discipline is, commonly, part of a financial aid package. This underscores the relevance of aid by multilateral institutions.

We assume that the government can commit to fiscal discipline in the implementation of financial aid. This presupposes that there is no lack of commitment principle in this specific case. Such an assumption is not as strong as it may initially appear. This is the case as the implementation of the financial aid package and fiscal discipline take place in the same period. In contrast, the commitment issue in the general sovereign debt setup involves several periods. Thus, the latter case does not preclude the lack of commitment principle in the general sovereign debt setup.

We calibrate our model to Argentine data. The model is able to account for several business cycle's empirical regularities, such as, the countercyclical nature of interest rate spreads and the highly pro-cyclical character of domestic absorption. We then feed the model with a sequence of output shocks that resemble the dynamics of economic activity in Argentina from 1993 to 2002. The model is able to replicate the default episode of 2002. Then, we also use our model as a laboratory with which we assess whether a financial aid program would have been useful to prevent the default event. Our quantitative framework suggests that financial aid could have sufficiently reduced the incentives to default, for instance, as reflected in the lower interest rate premium.

More generally, one should consider that the difficulties faced by Argentina are multifaceted. They entail a plethora of historical, cultural, institutional, political, or even other economic aspects that we do not account for in the model. Of course, a single model attempting to account for all of them would swiftly become intractable. Nonetheless, we believe that our model captures key elements and essential economic incentives that policy makers face during a crisis. In particular, it emphasizes that under certain conditions, which we consider sensible, the decision to default on their sovereign debt might be the optimal one. It, moreover, highlights the role of financial aid provided by a multilateral institution in preventing a full-fledged sovereign default episode.

II. Model Economy

In our model, fiscal policy and the option of receiving financial aid are endogenous, in an otherwise standard sovereign default setup.[2] There are three types of agents: households, government, and foreign lenders. We consider fiscal policy to highlight two features. First, only the government can issue sovereign debt and engage in conditional financial support programs from multilateral institutions. Second, in a crisis in general, the government ultimately absorbs private debts.

The representative household has preferences over the expected value of the stream of its consumption in each period:

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{(C_t^{\alpha} G_t^{1-\alpha})^{1-\sigma}}{1-\sigma}, \tag{1}$$

where the subjective discount factor is $\beta \in (0,1)$ and σ is the coefficient of relative risk aversion. Households derive utility from private consumption C_t and public expenditure G_t. We consider a Cobb-Douglas aggregator, where the parameter α captures the private consumption weight. The households' output Y_t follows a Markov process, with conditional density function $Q(Y_{t+1}|Y_t)$ where Y_t has values belonging to a set $\mathcal{Y}$. Specifically, we estimate an AR(1) model $y_t = \mu + \rho y_{t-1} + \epsilon_t$ for the cyclical component of output. Then, we discretize it to obtain the Markov Chain, a procedure we explain in more detail. Private consumption plus public expenditure is equal to output. The government taxes output. Households take public expenditure and taxation as given. Thus, they consume according to the following equation.

$$C_t = (1 - T_t)Y_t. \tag{2}$$

The government maximizes households' utility by issuing one-period bonds, taxing households' output, and financing public expenditure.[3] Conditional on being in good credit

[2] Our model, without fiscal policy and financial aid, is similar to that of Arellano (2008).

[3] Hatchondo and Martínez (2009), and Chatterjee and Eyigungor (2012) propose sovereign default models with long-term debt bonds. The analytical challenge they solve is keeping the number of state variables small while introducing such bonds. Hatchondo and Martínez (2009) model accounts for a higher and more volatile interest rate, while Chatterjee and Eyigungor (2012) for a higher debt to output ratio and spreads.

standing, it decides between paying the outstanding debt, receiving financial aid, and defaulting on it. To take its decision, the government compares the costs and benefits associated with each of these options, as we later explain in detail. Receiving financial aid entails paying the outstanding debt in a given period. In short, in the model, the government cannot default and then opt for financial aid.

If the government decides on paying the outstanding debt, it maintains its access to external financing. Consequently, it would be able to issue new debt. The government then decides how much to borrow, the level of public expenditure and, in tandem, the tax rate. On the other hand, if the government opts to default on its debt, it would face exclusion from international financial markets. Such an exclusion would take place for a random number of periods.

When the country is in financial autarchy, the government only chooses the public expenditure and the tax rate. The benefits and costs of these two options are as follows. If it honors its debt, the government maintains access to international financial markets. However, it has to pay the debt service. Conversely, if it defaults, it can consume what it was going to allocate to pay its debt and its service. However, it would face financial autarchy and, in tandem, suffer an output loss, which we explain in detail below.

In our model, the government can opt for financial aid. It is as an option that the government has only when having access to financial markets. If it chooses to receive financial aid, it will incur in both benefits and costs, which we operationalize, respectively, as a direct transfer of resources (financial aid) and as a restriction on the government debt next period, which we broadly interpret as fiscal discipline. Importantly, financial aid lasts only for one period in the model, afterwards, the government has again to decide whether to pay its debt, receive financial aid once more or, perhaps, default. One can interpret such a financial aid as support provided by a multilateral institution, which enacts some restrictions to the indebted government.

This problem can be expressed as a dynamic program. The state variables are output Y and debt B. The government decides between honoring its debt, receiving financial aid, and defaulting on its debt, by comparing the value function associated with honoring its outstanding debt obligations $V^c(B,Y)$ with that of receiving financial aid $V^f(B,Y)$ and with

that of the value function $V^d(Y)$ which corresponds to defaulting.[4] The following value function will be relevant to describe the government's problem:

$$V_0(B,Y) = max\{V^c(B,Y), V^f(B,Y), V^d(Y)\}. \quad (3)$$

Accordingly, we define the decision function D as follows:

$$\begin{aligned} D(B,Y) &= 2 \text{ if } V^c(B,Y) > \max\{V^f(B,Y), V^d(Y)\}; \\ D(B,Y) &= 1 \text{ if } V^f(B,Y) > \max\{V^c(B,Y), V^d(Y)\}; \text{ and,} \\ D(B,Y) &= 0 \text{ otherwise.} \end{aligned}$$

This allows us to define the repayment (R), financial aid (F), and default (D) sets, which depend on the level of debt and debt level, as follows:

$$\begin{aligned} \mathrm{R}(B) &= \{Y \in \Upsilon : D(Y,B) = 2\}; \quad (4) \\ \mathrm{F}(B) &= \{Y \in \Upsilon : D(Y,B) = 1\}; \text{ and,} \\ \mathrm{D}(B) &= \{Y \in \Upsilon : D(Y,B) = 0\}. \end{aligned}$$

The government maximizes the representative household's utility subject to her budget constraint and its own budget constraint. When the government honors its debts and does not receive financial aid, its debt choices are not restricted. Thus, the problem, when the government has access to credit markets is as follows.

$$V^c(B,Y) = max_{T,G,B'} \left\{ \frac{(C^\alpha G^{1-\alpha})^{1-\sigma}}{1-\sigma} + \beta \sum_{Y'} V_0(B',Y')Q(Y'|Y) \right\} \quad (5)$$

$$s.t.$$

$$\begin{aligned} G + B &= TY + q(B',Y)B', \\ C &= (1-T)Y, \end{aligned}$$

where the price of the bond is $q(B',Y)$, B' is foreign debt next period, and Y is output in the current period.[5]

[4] As explained, the country's debt is set to zero if the agent decides to default.

[5] The timing convention is as follows. The government starts period t with debt B, observes output y, and decides debt for the next period B'. It does not observe output y' until the next period.

When the government opts for financial aid, it has to satisfy a borrowing restriction. We assume that the option of receiving financial aid is only available when output is low, specifically, when output is smaller than its mean value.[6] We motivate this by the fact that, absent financial aid, the probability of default is decreasing in the level of output. Hence, a default is less of a concern if the economy in question is performing relatively well economically. In this case the government´s problem is:

$$V^f(B,Y) = max_{T,G,B'}\left\{\frac{(C^\alpha G^{1-\alpha})^{1-\sigma}}{1-\sigma} + \beta\sum_{Y'} V_0(B',Y')Q(Y'|Y)\right\} \tag{6}$$

$$s.t.$$

$$G + B = TY + F(B) + q(B',Y)B',$$

$$B' \leq B,$$

$$C = (1-T)Y,$$

where

$$F(B) = \begin{cases} \delta B & \text{if} \quad Y < E(Y) \\ 0 & \text{if} \quad Y \geq E(Y) \end{cases}$$

and δB accounts for the transfer of resources (financial aid). In light of this transfer, the government has now, in principle, more resources to finance public expenditure. However, it also faces a restriction $B' \leq B$ on the future evolution of its debt. The government cannot borrow from foreign lenders more than what it has borrowed in the previous period. One can see such a restriction as some sort of fiscal discipline. In the period after receiving financial aid, once more, it faces international credit markets, possibly without the referred aid.

If the government defaults, financial markets exclude the country and it suffers an exogenous output loss. The output in autarky is equal to $h(Y)$, which accounts for the cost on output after defaulting. One can motivate its presence by costs not directly associated with the default, e.g., a loss in productivity (e.g., see Alonso-Ortiz, Colla, and Da-Rocha, 2017). We provide a precise definition of $h(Y)$ below. Accordingly, the output tax is the only source to finance public expenditure. In this case, the problem is:

[6] In the numerical exercises, we consider the cyclical component of output, so an endowment lower than the mean value of output corresponds to a negative output gap.

$$V^d(Y) = max_{T_d, G_d} \left\{ \frac{\left(C_d^{\alpha} G_d^{1-\alpha}\right)^{1-\sigma}}{1-\sigma} + \beta \sum_{Y'} [\mu V_0(0, Y') + (1-\mu) V^d(Y')] Q(Y'|Y) \right\} \quad (7)$$

$$s.t.$$

$$G_d = T_d h(Y),$$

$$C_d = (1 - T_d) h(Y).$$

Once excluded from the international credit markets, in the next period, it regains access to them with exogenous probability μ. If it regains access, it would do so with no debt, i.e., $B = 0$.[7]

In addition, foreign lenders are risk-averse and have access to an international risk-free bond with interest rate r_f. Assuming no-arbitrage, the equilibrium bond price is determined by:

$$q(B', Y) = \sum_{Y' \in R(B') \cup F(B')} \text{SDF}(Y', Y) Q(Y'|Y) \quad (8)$$

where the stochastic discount factor (SDF) is equal to $(1 + r_f)^{-1} - \lambda(\log Y' - \rho \log Y)$, akin to Arellano (2008). The coefficient λ measures the foreign lenders' degree of risk-aversion. Lenders are more risk-averse the larger λ is ($\lambda \geq 0$). In fact, one can broadly interpret it as a coefficient of relative risk aversion (CRRA).[8] The covariance between the output and the SDF is different from zero. The states of nature where there is no default depend directly on B' through the repayment sets.

Consider the cases in which the government would opt for financial aid. One can divide such cases in two. There are some cases in which, under the absence of financial aid the government would not default. There are other in which, under the absence of financial aid, the government would default. Of course, one can think of normative and, plausibly some political economy, arguments to minimize the former and maximize the latter.

[7] We assume there is a full-fledged default; i.e., the government does not pay any of its debt. In practice however, a credit event in which, for example, the government misses a payment, one might consider it a default. We do not consider credit events, only full-fledged defaults.

[8] Under a CRRA the SDF is given by $\beta \exp(-\lambda \Delta c_{t+1})$ where $\Delta c_{t+1} = \log(C_{t+1}/C_t)$, which is approximately $(1 + r_f)^{-1} - \lambda \Delta c_{t+1}$. The working assumption in Arellano (2008) is that the innovations in consumption growth coincide with those of (the log of) output.

The debt restriction $B' \leq B$ might or might not bind when the government decides to accept financial aid. In effect, if the government were close to default then it would be trying to reduce its debt level independently of such a restriction. It is thus in the multilateral institution's interest to procure a binding restriction, which would instill fiscal discipline and yet, in tandem, consider the trade-offs against the financial aid benefits.

Definition. For the model economy, a recursive equilibrium is:

1. A set of value functions V_0, V^c, V^d and V^f for the government.
2. A policy function for the household´s consumption.
3. Policy functions for government´s financial aid and default decision D, optimal asset holdings B', optimal government expenditures G and optimal tax rates T.
4. A bond price function $q(B', Y)$.

Such that:

1. Given the government and the bond price function, the household policies for consumption satisfy the household´s budget constraint; and,
2. Given the bond price function q and the optimal policies for households, the government´s value functions V_0, V^c, V^f and V^d, and the policy functions D, B', G and T solve (3), (5), (6) and (7).

III. Calibration and Numerical Exercise

We calibrate our model and then perform a numerical exercise in order to analyze the role of financial aid in preventing a sovereign default episode.[9] To that end, we first discuss the values of the parameters used in such an exercise. Second, we briefly analyze how changes in the parameters' values related to financial aid affect the incentives to pay, to opt for financial aid, or not to pay, in the model. Third, we simulate the model and present several of its key statistics. Finally, we choose a sequence of output shocks that match the evolution

[9] The model abstracts from some factors affecting the dynamics of sovereign debt, such as the role of banks.

of the Argentinian economic activity during the years prior to its sovereign default episode in 2002, and assess the model economy's dynamics. Below, we describe the precise way in which we do this. It is worth mentioning that we have used a yearly frequency.

Value of the Parameters

The values of the parameters used in the model´s solution are comparable to those utilized in the literature of sovereign default quantitative models (e.g., see Aguiar and Gopinath (2006), Arellano (2008), Cuadra et al. (2010)). We choose other parameters to emulate the most salient empirical features of the Argentinian economy. Table 1 presents their specific values and, if applicable, their sources. Our comments are as follows.

The parameter σ, the coefficient of relative risk aversion, is set equal to two, a standard value in the literature (e.g., see Aguiar and Gopinath (2006)). The subjective discount factor (β) is set equal to 0.81, in line with Arellano (2008).[10] During the period of study, the public expenditure to private consumption ratio in Argentina was, on average, around 0.20. A Cobb-Douglas utility functions and a private consumption weight α equal to 0.83 helps matching this fact.[11]

The parameter μ corresponds to the exogenous probability of reentering international financial markets after a default episode. It is set equal to 0.1. This value implies that a country expects to be able to regain access to financial markets ten years after defaulting.[12] There is output loss in autarky.[13] We capture with the function $h(Y)$, as in Arellano (2008). Specifically, we assume that a default episode entails an output cost of the following form: [14]

[10] Arellano (2008) explores the Argentinian case with quarterly data. She uses a value of 0.95, which is equivalent to 0.81 with annual data. For the subjective discount factor with annual data, values that are more standard fall within the 0.95-0.99 range. However, one drawback of sovereign default quantitative models is that they require a higher level of impatience (i.e., a smaller β) to generate default in equilibrium.

[11] The public expenditure to private consumption ratio in the model is $(1-\alpha)/\alpha$.

[12] According to Benjamin and Wright (2009), a process of sovereign debt restructuring takes, on average, eight years.

[13] Rose (2005) provides a rationale for the loss of output when countries face debt crises.

[14] This specification extends the range of debt values that carry positive default premiums, which allows the model to generate higher interest rate spreads, all else being equal (Arellano (2008)).

$$h(Y) = (1-\theta)E(Y) \quad \text{if} \quad Y > (1-\theta)E(Y), \tag{9}$$
$$h(Y) = Y \quad \text{if} \quad Y \leq (1-\theta)E(Y).$$

The default cost parameter θ is set equal to 0.03, as in Arellano (2008). Intuitively, if the economy is doing relatively well and the government opts for default, it will face an output cost of $Y - (1-\theta)E(Y)$. If it is not doing well and defaults, it will not face an output cost. A plausible interpretation is that in the former case, the government repudiates the debt and in the latter, it has to default.

We assume that the cyclical component of the GDP follows an AR(1) process:

$$z_t = \rho_z z_{t-1} + e_t \tag{10}$$

where e_t is i.i.d. $N(0, \sigma_z^2)$.

The parameters of the stochastic process for output ρ_z and σ_z are set to match the autocorrelation and standard deviation of the cyclical component of the Argentinian GDP.[15] To solve the dynamic program, the AR(1) process is approximated by a discrete Markov chain.[16] Then, we solve the model numerically using the value function iteration-algorithm.[17]

In terms of financial aid, we first consider as a benchmark scenario no financial aid, that is, the financial aid parameter δ is set equal to zero. Subsequently, we assess which value of this parameter would have (counterfactually) prevented the Argentinean sovereign default episode of 2002. As for the stochastic discount factor, following Lizarazo (2013), we have set $\lambda = 5$, which measures the foreign lenders' risk-aversion.[18] As explained, one can interpret the parameter λ as a coefficient of relative risk aversion. The risk-free rate r_f is equal to 0.04 (i.e., 4.00%), a standard value for annual data in the business cycles' literature.

[15] To estimate the cyclical component of GDP (z), we considered the log of the Argentinian GDP and the Hodrick-Prescott filter.
[16] The fact that we use an AR(1) process to approximate the cyclical component of output (z) allows us a direct conversion to a Markov Chain model in discrete time. This facilitates solving the model numerically. Once we have the discretized values of z, we consider $Y = \exp(z)$.
[17] To implement the value function algorithm, we discretize the state space with a grid. The choice of its dimension is such that the results are robust. In other words, they do not change when the dimension of the grid is increased. A finer grid allows for results that are more accurate. In the numerical solution of the model, we consider a grid of 1,000 values for debt and a grid of 25 values for output.
[18] We use Lizaraso's (2013) smaller coefficient as it is closer to an accepted CRRA value.

Financial aid entails benefits and costs, as explained. Specifically, increasing the financial aid parameter augments the incentives to opt for it. The government would be getting additional resources. Financial aid, everything else being equal, reduces the debt burden. That is, the aid increases the amount of resources available to the economy. Accordingly, defaulting on the outstanding debt becomes less attractive than opting for financial aid. On the other hand, a stricter fiscal discipline lessens the incentives to opt for financial aid.[19] This implies a tighter constraint on the debt that the government can take next period. The institution providing financial aid balance its costs and benefits to reduce the possibility of a default scenario and, in tandem, to mitigate moral hazard. Of course, in our model, we are not explicitly accounting for moral hazard.

Results

We simulate the model in order to assess how it matches the business cycle's features of Argentina. The first column of Table 2 shows a number of statistics for such an economy. We have considered annual data from 1980 to 2014. Output and absorption are in logs, and we express the fiscal balance as a fraction of GDP.[20] The interest rate spreads corresponds to the difference between the interest rate of Argentina and the U.S.[21] For the calibration, we filter all time series with the Hodrick-Prescott filter with the standard value for its parameter when using annual data. The second column of Table 2 shows the corresponding business cycle's statistics for the simulated model. These statistics depend on the average values of 100,000 simulations. Overall, the model is able to match several statistics. In the data and in the model, absorption is more volatile than output, while fiscal balance is less volatile than output. In addition, it is able to match the high pro-cyclical nature of absorption, as well as the counter-cyclical behavior of interest rate spreads.

[19] We could think of other schemes to introduce financial aid and the requirement of fiscal discipline, which would to different modeling implications.
[20] The national accounts data are from the World Bank and cover the period from 1980 to 2016. In turn, we obtained the fiscal balance data from the IMF. They cover the period from 1993 to 2016. The interest rate data are from the database of Neumayer and Perri (2005).
[21] The U.S. interest rate corresponds to the rate of 12 month T-Bills.

Having solved and calibrated the model, we choose a sequence of output shocks such that its path is as close as possible to the path of the cyclical component of the GDP estimated for Argentina.[22] Figure 1 shows such a path from 1993 to 2002 and the one matched with the model economy. As can be seen, economic activity increased from 1995 to 2000. Nonetheless, in 2001, it began contracting. Given this exogenous output path, we consider the policy functions to solve for the model economy's dynamics. In what follows, we discuss the economy's dynamics when there is no financial aid.

In our simulation, period 1 stands for 1993, 2 for 1994, …, and 10 for 2002. At the outset, the economy faces a negative shock in period 3 (which corresponds to the Tequila Crisis in 1995) and the interest rate spread increases at the time. Then, from period four to period six (1996-1998), the economy faces a sequence of positive shocks. The favorable economic performance leads to an interest rate spread equal to zero. Thereafter, in period seven, the economy starts to experience a sequence of negative output shocks and, thus, the expansion eventually comes to a halt. From that moment, foreign lenders demand a risk premium to keep on lending to the government, which increases the interest rate spread. Figure 2 depicts both the output level and the interest rate spread in the model economy. The negative correlation between these two variables is strong (Figure 2). Subsequently, the fall in economic activity in period 10 (2002) is of such a magnitude that it is optimal for the government to default on its financial obligations.

We next compare our results with the developments of the Argentinian economy in the period prior to 2002. Figure 3 exhibits the interest rate spreads of Argentina and those generated by the model. As the figure indicates, in the early years, an increase in the spread seems to be associated with the 1995 slowdown. In subsequent years, the spreads pointed to a very low sovereign risk. This could relate to the economic expansion observed until 1998 (Figure 1). However, after economic activity began faltering, interest rates spreads started to rise, a dynamic akin to that based on the model. In this way, the model is able to replicate the correlation between output and interest rate spreads as well as the sovereign default episode, which took place in Argentina in 2002.

[22] To be sure, when mentioning GDP or economic activity, we are referring to the cyclical component of GDP.

We also simulate the model with several values for the parameter δ to assess whether the government would have not defaulted. Based on this exercise, we determine that a value of 0.05 (five percent of the outstanding debt) would have been sufficient to prevent the default episode. As we know, the financial aid did not materialize at the time. Moreover, any value higher than 5% would had implied that the multilateral institution would be assigning to the financial aid program an amount of resources greater than the one necessary to avoid a default event.

Given the lackluster economic environment and the cost of paying the outstanding debt, in period nine (i.e., 2001) and afterwards the government opts to take on financial aid. In period nine, although taking financial aid is the optimal decision, the value of honoring the outstanding debt without receiving financial aid is higher than the value of defaulting. Figure 4 illustrates this point. In effect, it shows that the value functions for the output registered in period nine. We observe that for the level of debt that the government had to pay in that period (B^*), the value function of opting for financial aid is higher than the one corresponding to honoring the outstanding debt. In turn, the latter is higher than the one corresponding to default.

As a result, one can see financial aid as a direct subsidy for this specific period. We depict this situation in Region 3 in Figure 5. However, in period 10 (i.e., 2002), while opting for financial aid is the optimal decision, the value of defaulting is higher than the value of paying back the outstanding debt obligations without taking financial aid. That is, there are cases in which a full-fledged default episode would have taken place had it not been for the presence of financial aid. In other words, in our counterfactual experiment, financial aid is crucial to avert a sovereign default episode. We show this situation in Region 2 in Figure 5.

To illustrate this feature of the model, Figure 5 depicts the value functions associated with honoring the outstanding debt obligations, receiving financial aid and defaulting. These value functions assume a fixed value of output. One then can identify four regions. In Region 1 (R1), the value of defaulting is higher than the other two value functions (i.e., paying and financial aid), therefore defaulting on the outstanding debt is the optimal solution. In Region 2 (R2), the value of receiving financial aid is higher than the other two value functions (i.e., defaulting and paying). In addition, the value of defaulting is higher than the value of

honoring the outstanding debt. Accordingly, in such a region, financial aid prevents a default episode. In Region 3 (R3), the value of receiving financial aid is higher than the value of paying back debt and the latter is higher than the value of defaulting. Thus, financial aid works as a subsidy. In Region 4 (R4), the value of honoring the outstanding debt is higher than the other two value functions (i.e., defaulting and financial aid). This has as a repercussion that the agent will not default regardless of official aid.

Official aid, by changing the relative prices of the decision to default, reduces the incentives to do so. Given the non-contingent debt's size, official aid reduces the cost of its service in terms of the foregone consumption in the absence of such aid. Under such circumstances, the benefits of defaulting (preventing a fall in current consumption) decrease relative to the costs of doing so (entering autarky). In this context, a less generous transfer (financial aid), everything else constant, increases the incentives to default. As mentioned, if we consider a value of zero percent instead of, say, five percent for the financial aid parameter, the government defaults on its debt obligations in period 10 (i.e., 2002). In contrast, financial aid, in the form of a resource transfer and fiscal discipline, reduces the incentives to default. In turn, a lower probability of default leads to a lower interest rate spread and, consequently, borrowing costs fall. Figure 6 illustrates how financial aid reduces the interest rate spreads. Specifically, it shows the interest rate spreads for the model, with and without financial aid. In the latter case, the interest rate spreads are visibly higher.

To analyze the impact of financial aid on the level of sovereign debt, Figure 7 shows the dynamics of sovereign debt for the model without financial aid and with financial aid equal to five percent. With financial aid, everything else being constant, the government can maintain higher levels of sovereign debt. As mentioned, the presence of financial aid contributes reducing the incentives to default. As foreign lenders know this, they are willing to lend to the government at lower interest rates and, thus, one observes greater levels of sovereign debt.[23]

[23] Recall that, in our model, a one period non-contingent bond is considered.

In general, countries that find themselves in financial distress might reach a point where it is in their own interest to cease debt payments. Their decisions would account for the reputational costs and the lost access to international markets. More formally, if a country reaches a point in which the level of utility obtained under default is higher than the level of utility it gets from paying its debt´s service, it will then be a rational economic decision to default. Nonetheless, the possibility of receiving financial aid can certainly contribute to avoid a sovereign default episode.

Key Policy Implications

One of our main aims has been to explore the implications of financial aid. At the heart of its provision, there is the lack of an effective enforcement mechanism. Thus, a multilateral institution should provide financial aid along with some fiscal discipline, in lieu of an enforcement mechanism, which in the international scenario would be impossible to have. As mentioned, financial aid changes the relative prices that a sovereign faces when confronting the possibility of defaulting. In an ideal world, its provision should avoid a default, have no economic distortions, and contribute toward an economic recovery. However, the model underscores that the presence of financial aid does not assure that the sovereign will avoid a default. Moreover, under some contingencies, it may introduce economic distortions. Prominently, a multilateral institution could have provided financial aid when there was no real need.

In this context, it is illustrative to discuss some of the more specific policy implications. Although we could do this from one of various perspectives, but that of a multilateral institution providing the aid is the most useful. We then retake some of the results in our model to provide a richer context to our discussion. We have underscored the existence of four regions in the debt space as function of the sovereign actions, which we list next (Figure 5).

1. The sovereign will default regardless of the presence of financial aid.
2. It will opt for the financial aid and a default would have taken place under its absence.
3. It will take the financial aid, but no default would occur under its absence.
4. It will not default regardless of the existence of financial aid.

The most relevant regions are those denoted by two and three. One can then see region three as a distortion and region two where the financial aid makes a difference. The shape and size of these regions depend on the design of the implementation of financial aid. Policy wise, the key is to strike a balance between financial aid and fiscal discipline to minimize three and maximize two.

In our case, we have introduced financial aid as a discount on the sovereign debt. In turn, we introduce fiscal discipline as a constraint that limits government debt. While both are linear restrictions, the former is on a stock and the latter, on a flow, providing some flexibility to its implementation.[24] As a polar case, a burdensome discipline will make the sovereign shed any financial aid, making regions two and three empty sets. Accordingly, we focus on feasible levels of financial aid.

As Figure 5 shows, given a level of output, debt determines in which of the four regions the sovereign is. First, low levels of debt corresponds to Region 4 in which the sovereign honors its financial obligations and does not opt for financial aid. In this region, the debt burden is manageable and the risk premium is close or equal to zero. Under such circumstances, the sovereign has sufficient incentives to borrow, taking advantage of the low interest rates. Financial aid is not sufficiently attractive since it would impose a borrowing constraint. However, as the sovereign debt increases, the risk premium and, consequently, the interest rate tend also to rise. Thus, financing public expenditure and private consumption through borrowing becomes less appealing. Thus, the borrowing constraint associated with receiving financial aid from the multilateral institution would have more slackness. At the same time, the amount of debt is not sufficiently high to trigger a default episode. Under these circumstances, the sovereign would be located in Region 3 and the economic distortion would increase. If debt continues to grow, its burden would increase and defaulting would become more attractive. In this setting, financial aid would prevent a default episode (Region 2). Finally, for high enough levels of debt, the sovereign would default irrespective of the presence of financial aid (Region 1).

[24] Other ways of introducing financial aid can be non-linear, and have similar implications.

Once we have analyzed the sovereign's incentives in each region, it can be useful to ponder some policy measures that may help minimizing the size of Region 3. A key question then is how to change the relative prices to achieve this. For tractability reasons, we have modeled financial aid as a fraction of current debt. However, as our previous analyses illustrate, offering financial aid for low levels of debt will not make a significant difference (Region 4). In addition, relative prices must change in Region 3 in order to make financial aid less attractive. One can achieve this by reducing the amount of financial aid.

Based on the previous analysis, one way to do this is as follows. Instead of setting the amount of financial aid as a constant fraction of current debt, such a fraction could be an increasing function of debt. In this manner, for example, for relatively low levels of debt the resources that a country would receive as financial aid would be lower. Then, for relatively high levels of debt, financial aid would be proportionally greater. Of course, a multilateral institution would have to face the challenge of calibrating the amount of financial aid. To be sure, the model features a fixed amount of financial aid (relative to its debt). A possible extension of the model is to allow the agent to choose between several possible amounts of financial aid.

IV. Concluding Remarks

To have a better understanding of the key economic trade-offs and the dynamics of some macroeconomic variables in a (potentially) defaulting economy during the recent crisis, we have developed a dynamic stochastic quantitative model of sovereign debt with endogenous financial aid. In our model, the government can opt for aid or default on its debt obligations, thereby generating endogenous interest rate spreads.

In the quantitative analysis, we calibrate the model to match key features of the Argentinian economy. The model accounts for several business cycle's stylized facts, such as the countercyclical nature of interest rate spread and the highly pro-cyclical nature of domestic absorption. Moreover, we feed the model with a sequence of output shocks that resembles the dynamics of economic activity in Argentina before the default episode in early 2000's. The model predicts a default in 2002. Then, we have used the model as a laboratory to assess the amount of financial aid that would have prevented the default event.

Overall, the model illustrates how the introduction of financial aid changes the relative prices that the sovereign faces when dealing with the possibility of defaulting on its debt obligations. Ideally, the presence of financial aid should prevent a default event and minimize economic distortions. However, the model highlights that financial aid does not always ensure that the sovereign will pay its debts. What is more, in some cases, it may introduce economic distortions. In particular, there may be situations in which a multilateral institution grants financial aid to a country that would have repaid its debts in any case. By underscoring the presence of these distortions, the model highlights that a multilateral institution should consider them when designing financial aid programs. How to design the above goes beyond the scope of this paper. However, we have outlined some ideas that may be useful to address this and other related issues.

Finally, although we have not included the exchange rate as part of our model, we think that its consideration could be a useful line of research (e.g., see Da-Rocha, Gimenez and Lores, 2013). We next briefly discuss the possible role of financial aid when the government defaults and cannot commit to maintain the exchange rate. Accounting for the exchange rate regime leads to the modification of the trade-offs faced by the government if it knows a priori that it cannot commit to maintaining the exchange rate. On the one hand, it will more likely accept the financial aid. In effect, a depreciated exchange rate will imply that it will have fewer resources to pay its debt in the future. On the other hand, a depreciation could hasten the economy's recovery if a default does take place. Thus, the government might have fewer incentives to take the financial aid. Having rejected the financial aid, it defaults, but given the depreciation, net exports increase and the current account adjusts, which could prompt a swifter recovery.

Tables and Figures

Table 1: Parameter Values

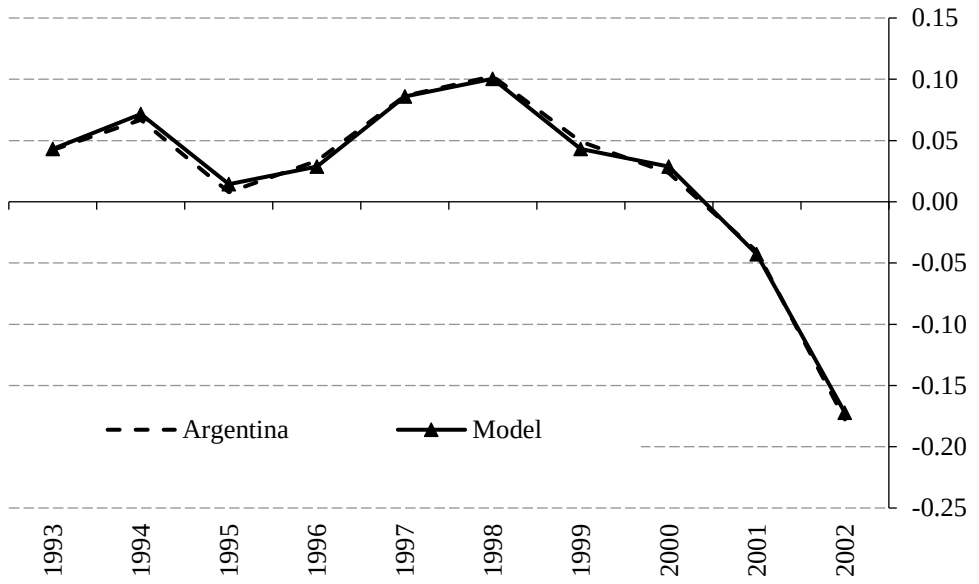
		Value	Source
Country Risk Aversion	σ	2.00	Standard in the literature
Discount Factor	β	0.81	Arellano (2008), annual frequency
Consumption Weight	α	0.83	Argentinian data
Re-entry Probability	μ	0.10	Benjamin and Wright (2008)
Output Loss in Autarky	θ	0.03	Arellano (2008)
Financial Aid	δ	0.00	
Investors Risk Aversion	λ	5.00	Lizarazo (2013)
Risk Free Interest Rate	r	0.04	Standard in the literature
Endowment Process´s Parameters	ρ_y	0.62	Argentinian data
	σ_y	0.054	

Table 2. Calibration Statistics

	Argentinian Data (1980-2016)*	Benchmark Model
σ(Absorption)/σ(GDP)	1.35	1.08
σ(Fiscal balance)/σ(GDP)	0.25	0.18
σ(Spread)/σ(GDP)	0.67	0.11
ρ(Absorption, GDP)	0.95	0.99
ρ(Fiscal balance, GDP)	-0.01	-0.40
ρ(Spread, GDP)	-0.78	-0.67

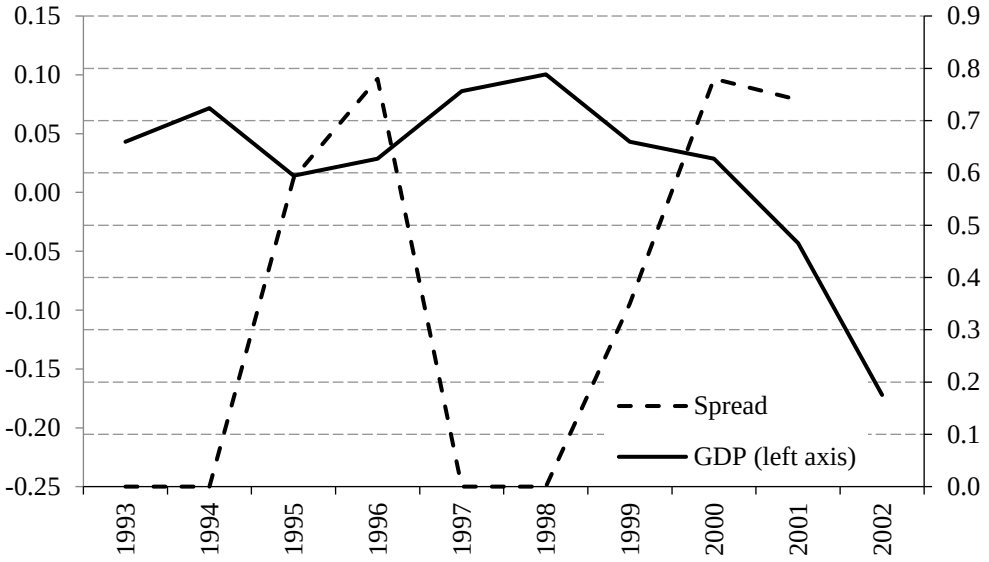
***Data of fiscal balance goes from 1993 to 2016. Data of spreads goes from 1983 to 2002**

Figure 1. Real GDP



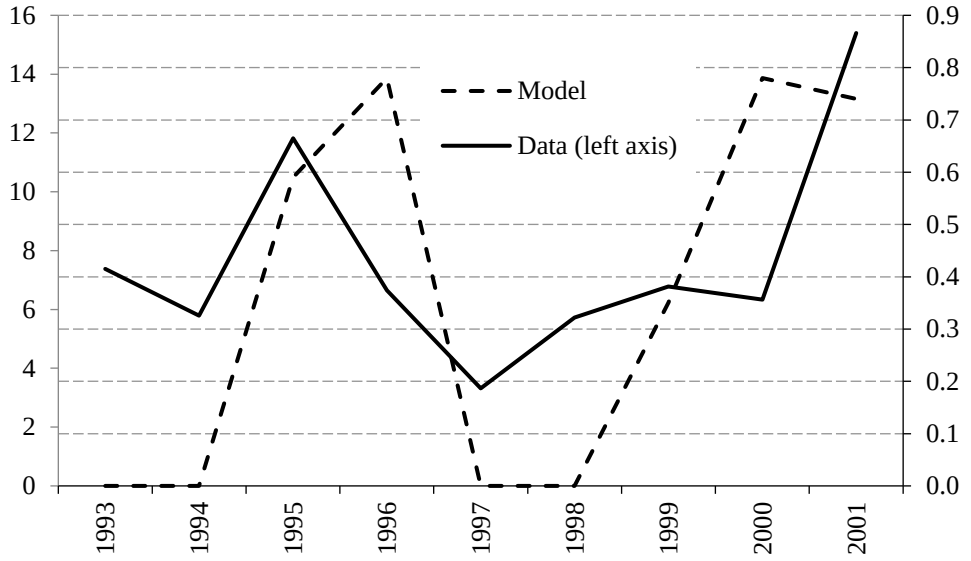
Source: World Bank.

Figure 2. GDP vs. Spread Model Economy (%)



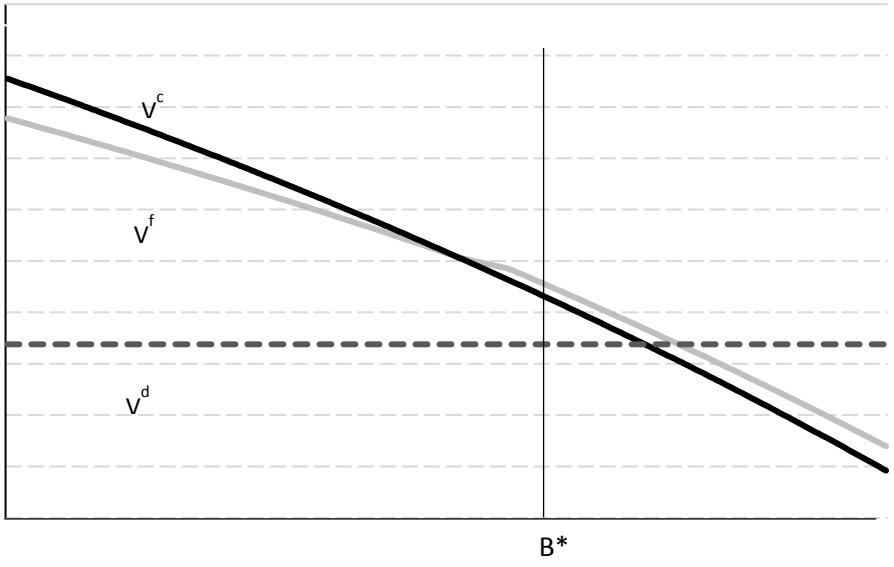
Source: Model Simulation

Figure 3. Interest Rate Spreads (%)



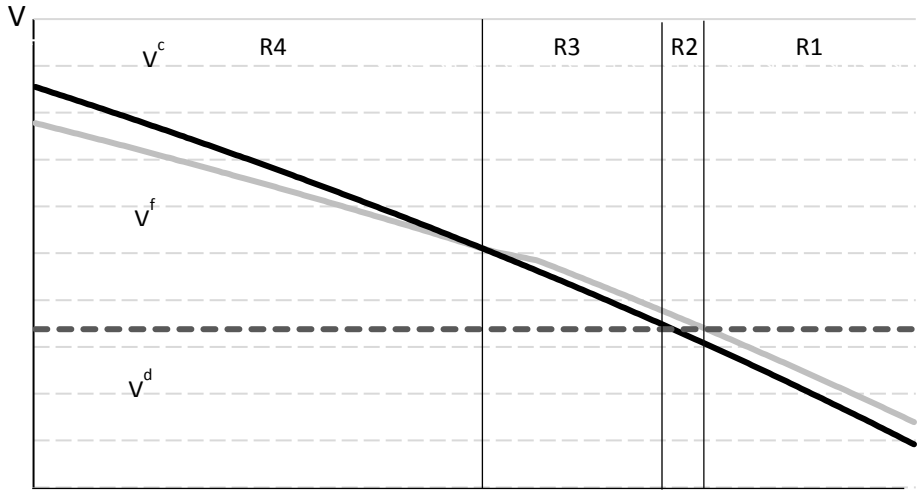
Source: Model and Database of Neumayer and Perri (2005)

Figure 4. Value Functions



Source: Model Simulation

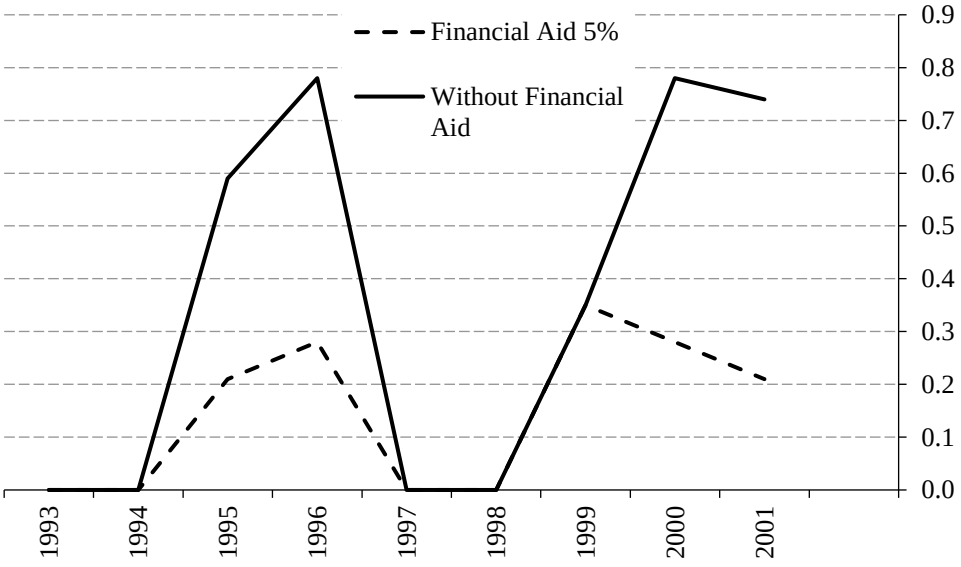
Figure 5. Value Functions



* The three value functions correspond to the same value of output

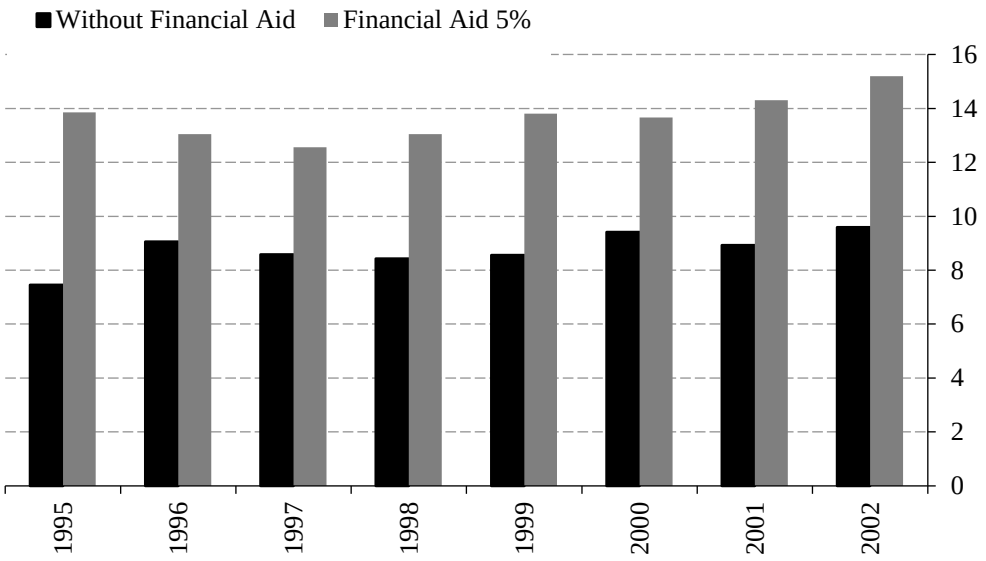
Source: Model Simulation

Figure 6. Interest Rate Spread Model Economy



Source: Model Simulation

Figure 7. Debt Model Economy (% of GDP)



Source: Model Simulation

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