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# Analysis of the Dynamics of Mexican Inflation Using Wavelets \*

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## Abstract

This paper studies the dynamics of Mexican inflation by using a wavelet multiresolution analysis on 16 indexes of the Mexican Consumer Price Index. This enables us to estimate the long-term trend, seasonality, and local shocks of the inflation series, even when the series are non-stationary. The energy distribution between the high frequency, seasonal, and trend components, as well as its evolution through time, are compared. In particular, headline and core inflations show a more stable behavior in all the scales since 2001. Also, an increase in the proportion of variance explained by short-term variations is detected in the inflation series. In relative terms, the short run is becoming as important for headline inflation as the medium and long run, and more important for non-core inflation. These results are in line with previous studies documenting the reduction in the Mexican inflation persistence.

**Keywords:** Inflation dynamics, wavelets, multiresolution analysis, energy decomposition.

**JEL Classification:** C19, C49, E31.

## Resumen

Este trabajo estudia la dinámica de la inflación en México usando el análisis de multiresolución con wavelets para 16 índices del Índice Nacional de Precios al Consumidor. Con ello es posible estimar la tendencia de largo plazo, la estacionalidad y los choques temporales de las series de inflación, aun cuando éstas no sean estacionarias. La distribución de la energía de los componentes de alta frecuencia, estacionalidad y tendencia, así como su evolución en el tiempo, son comparadas para las series. En particular la inflación general y subyacente muestran comportamientos estables en todas las escalas desde 2001. También se detecta un aumento en la proporción de varianza explicada por las variaciones de corto plazo en las series de inflación. En términos relativo, cabe destacar que, el corto plazo se ha vuelto tan importante como el mediano y largo plazo para la inflación general y más importante para la inflación no subyacente. Estos resultados son congruentes con estudios previos sobre la reducción de persistencia de la inflación en México.

**Palabras Clave:** Dinámica inflacionaria, wavelets, análisis de multiresolución, distribución de energía.

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# 1. Introduction

In the past decades, for different countries, low levels of inflation may have been related to changes in the inflation process, possibly due to monetary policies and stable expectations (Taylor (2000) and Sargent (1999)). Therefore, efforts have been made to characterize these dynamics in some countries.<sup>1</sup> In Mexico inflation dynamics have recently been studied in Ramos-Francia and Torres (2008), Capistrán and Ramos-Francia (2009), and Chiquiar et al. (2009). In this paper, even though we will also study the dynamics of inflation, we will take a completely different and complementary approach using wavelets.

Wavelets have not been extensively used in economics<sup>2</sup> although their application has given interesting results in other disciplines. Here we take advantage of the flexibility of wavelets to study the dynamics of Mexican inflation from 1969 to 2009<sup>3</sup> and to identify fundamental changes in its characteristics. By using wavelets, we will be able to identify the long-term trend and local shocks even when the series are non-stationary, to study the relative importance of the high frequency, seasonal, and trend components and how it has been changing through time. We will analyze the dynamics for the 16 indexes of the Mexican Consumer Price Index. Up to the knowledge of the author, this is the first time inflation dynamics have been studied using wavelets.

During the studied period, there have been episodes of high and low inflation in Mexico. Some shocks, due to devaluations, financial crisis, or increases in international commodity prices, have affected the domestic price level throughout the period. During the first half of the seventies inflation was relatively low; in contrast, the late seventies and most of the eighties, characterized by a fiscal dominance problem, exhibited very high levels of inflation. In the late eighties and at the beginning of the nineties, after restraining public expenditure and renegotiating the external

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<sup>1</sup>See for example Batini (2002), Castillo, et al. (2007), Cecchetti and Debelle (2006), Levin and Moessner (2005), Pincheira (2008), Pivetta and Reis (2006).

<sup>2</sup>There have been some economic applications using wavelets mainly related to non-stationarity, local phenomena, and time-scale decomposition. Some examples are: Aussem et al. (1998), Davison et al. (1998), Jensen (1999, 2000), Ramsey et al. (1995), Ramsey and Lampart (1998a, 1998b), Ramsey and Zhang (1996, 1997).

<sup>3</sup>All available monthly data from Banco de México website, [www.banxico.org.mx](http://www.banxico.org.mx), was used. Headline inflation starts on February 1969; food, other merchandise, housing, and other services indexes on January 1994, and core, non-core, and the rest of the indexes on January 1982.

public debt, inflation gradually decreased. Nevertheless, it went back to high levels during the financial crisis that took place in 1995. Afterwards, in a relatively short period, the economy was stabilized due to measures that contributed in restoring credibility in the financial system and monetary policy. Fiscal and monetary rules<sup>4</sup> brought important reductions to inflation levels from then onwards (for a comprehensive discussion of these measures see Ramos-Francia and Torres (2005)). In 2002 inflation targeting was fully implemented by the Mexican Central Bank, although some features of this regime were introduced since 1999 leading to a disinflation period. Chiquiar et al. (2009) documented that there was a change in persistence of the headline inflation in Mexico, which became stationary since the end of 2000, and of core inflation in April 2001.

All these different episodes will be clearly identified using wavelet analysis. More importantly, our main results show that since 2001 headline and core inflation series show a more stable behavior in all the scales. In relative terms, the short run is as important for headline inflation as the medium and long run, and more important for non-core inflation. Non-core inflation is really capturing the short-lived price variations while core inflation still reflects the medium and long-term pressures. These findings are in line with the reduction in persistence of headline and core inflation series previously stated.

Our analysis will be based on wavelet techniques, which generally have been disregarded by econometrics, where Fourier analysis is more prevalent when working in the frequency domain. The Fourier transform is a well-established technique for the analysis of stationary time series; it is based on sums of sines and cosines to represent a function on the frequency domain. Nevertheless, it assumes that the frequency content of the function is stationary through time; Fourier series have infinite energy that does not die out and finite power that cannot change over time.<sup>5</sup> With Fourier analysis a single disturbance affects all frequencies for the entire length of the series; changes in a few observations affect all the components of the expansion.

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<sup>4</sup>In 1995 the Mexican Central Bank defined a target on commercial banks' current accounts, which was used to signal a restrictive monetary policy since 1998. In 2000 quarterly inflation reports were published by the Central Bank to reinforce transparency and the concept of core inflation was introduced. The formal implementation of inflation targeting in 2002 was announced in 2001. In 2003 pre-established dates were settled for the monetary policy announcements. In 2004, in addition to the target on commercial banks' current accounts, Banco de México announced specific levels of interest rates (overnight interbank rate). Starting January 2008 the commercial banks' current accounts target substituted by a target for the overnight interbank rate (see Banco de México (2007)).

<sup>5</sup>Here energy can be understood as the area under the squared signal or function, and power as the energy per unit of time (see Section 3).

The basis functions in Fourier analysis are non-local, they stretch out to infinity. Also, given that the data is summarized as a function of frequency, it does not preserve information in the time domain.

On the contrary, wavelets are localized in time and scale. This feature makes them suitable for the representation of signals of finite length or those that show different behaviors through time. Wavelets are not homogenous over time (they have finite energy) and have compact support (they last for a short period of time) and therefore are indexed in the time domain. Wavelets permit us to observe the signal at a number of different scales; we can look initially from a distance at general features and then gradually zoom in to reveal finer details. As Schleicher (2002) states “wavelets can be compared to a wide-angle camera lens that allows one to take broad landscape portraits as well as zoom in on microscopic detail that is normally hidden to the human eye”; and, as Graps (1995) affirms, “wavelets enable us to see both the forest and the trees”. In contrast to Fourier series, when there are disturbances in a few observations, only the basis functions whose support includes the region of different behavior will be affected.

Wavelets can provide new insights into the analysis of economic data because they provide flexibility in handling irregular data series, precision to locate discontinuities and isolate shocks, and ability to deal with the non-stationarity of stochastic innovations.

In this paper, we shall first give a brief introduction about wavelets (Section 2). Data will be described in Section 3, followed by a wavelet analysis that will allow us to identify trends, seasonality, short-term variations, and shocks in the series. Also in this section, we will show how the relative importance of short, medium, and long-term variations has been changing, which is relevant to study under an inflation targeting framework. Finally conclusions and possible further work are presented in Section 4.

## 2. What are Wavelets? A basic Overview.

All square integrable functions in  $(-\infty, \infty)$  can be constructed using suitable bases. A basis function  $g(t)$  can be used to express data or functions in the form,

$$x(t) = \sum_{j,k} w_{j,k} g(t) \quad t = 1, 2, \dots, T$$

where  $w_{j,k}$  are weights or coefficients.

Wavelets are basis functions with specific characteristics. They can have indefinitely different shapes, but they all share the same basic construction. There are father wavelets,  $\phi$ , and mother wavelets,  $\psi$ . Father wavelets, also referred to as the scaling function, integrate to one and are used to represent the long scale, smooth baseline trend; mother wavelets integrate to zero and represent the deviations from the smooth component. Father wavelets may be viewed as low-pass filters capturing long-run phenomena, whereas the mother wavelets as high-pass filters capturing short-lived phenomena.

By selecting a father wavelet

$$\phi_{J,k}(t) = 2^{J/2}\phi(2^J t - k)$$

and a suitable mother wavelet function

$$\psi_{j,k}(t) = 2^{j/2}\psi(2^j t - k)$$

and considering its dilation (by changing  $j$ ) and translations (by changing  $k$ ), orthonormal bases can be constructed for all square integrable functions on  $(-\infty, \infty)$ . Here  $k$  is the time domain index and  $j$  is the scale. These bases break down a function at different scales. At fine scales (high frequency) short-term signal discontinuities can be isolated, while at coarse scales (low frequency) long-term phenomena can be captured. So we can look at a series from a long distance (low frequencies) and determine coarser details or discover finer characteristics when having a closer look (higher frequencies); this can be achieved by changing the scale parameter  $j$  and the location  $k$ .

So the wavelet transformation of a time series  $x(t)$  is given using both father and mother wavelets, by

$$x(t) = \sum_{k=1}^{N_J} v_{J,k}\phi_{J,k}(t) + \sum_{j=1}^J \sum_{k=1}^{N_j} w_{j,k}\psi_{j,k}(t)$$

where  $v_{J,k}$  and  $w_{j,k}$  are the coefficients and  $N_j$  are the number of coefficients in the  $j$ -th scale. This transformation, called the discrete wavelet transform (DWT), only allows a dyadic sample size, but all integer translations can be used, avoiding this restriction (called the Maximal Overlap Discrete Wavelet Transform (MODWT)). The MODWT is not sensitive to the starting point of the series, and the number of coefficients at each stage of the decomposition is equal to the sample size. The MODWT

is an energy-preserving transform, and it enables the alignment of the features of the original time series to those of the multiresolution analysis (multiresolution analysis will be briefly explained later in this section).

There exist different wavelet bases. Here we will use the Daubechies wavelets filters, which represent a collection of wavelets (including the Haar wavelet that can be seen as a length 2 Daubechies wavelet<sup>6</sup>) that are widely used (see Daubechies (1992)).

A multiresolution analysis (MRA) is the additive decomposition of a series (Mallat (1989)). A MRA can be thought of as estimating a signal from various distances, observing the signal at a range of different scales. So if  $d_j$  is the  $j - th$  level wavelet detail associated with changes in the series  $x(t)$  at a given scale  $j$ , a MRA can be defined as

$$x(t) = \sum_{j=1}^{J+1} d_{j,t}.$$

So the original time series will be decomposed in  $J+1$  new time series, each giving information about a given scale. The first  $J$  details are related to the mother wavelets and the last  $J + 1 - th$  detail, called wavelet smooth, to the father wavelet.

It is out of the scope of this paper to give a comprehensive explanation about wavelets. A brief and basic introduction was given here since there are already various complete reviews about the topic. For a mathematical approach consult Addison (2002), Daubechies (1992), Mallat (1998), Percival and Walden (2000), Walker(1999), Walnut(2002); for a review about wavelets applied to economics see Crowley (2005), Gencay et al. (2002), Ramsey (1999), Ramsey (2002), Schleicher (2002).

### 3. Inflation Dynamics

#### 3.1. Data

In this paper we will study sixteen indexes of the Mexican Consumer Price Index, *Índice Nacional de Precios al Consumidor (INPC)*; we look at the monthly percentage changes. We start our study with headline inflation, following with core and non-core inflation. The core component can be divided into two main indexes, merchandise and services, and the non-core component into agriculture and administered and regulated goods and services. Each of these indexes can be disaggregated into

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<sup>6</sup>Length 2 refers to the number of vanishing moments. If a wavelet basis has “ $m$  vanishing moments” means that a  $m$ -order polynomial will be passed through the mother wavelet, the projection integrates to zero and the polynomial component will be captured by the father wavelet.

subindexes: merchandises into food and other merchandises; services into housing, education, and rest of services; agriculture into fruits and vegetables and farm-related goods; and lastly administered and regulated goods and services into administered goods and regulated goods and services. Headline inflation goes from February 1969 to June 2009 (Figure 1.a, top row), while core and non-core inflation go from February 1982 to June 2009 (Figure 3.a and Figure 5.a, top row, respectively). All the other indexes will be studied from January 2000 to June 2009.<sup>7</sup>

The waveslim package in R from Brandom Whitcher was used in this section to obtain the wavelet decompositions (see Whitcher (2007)).

### 3.2. Trends, Seasonality, Variance and Shocks

Here we will decompose the original time series in new series incorporating the short, medium, or long-term information. The overall aim is to decompose the time series as follows:

$$\text{Series} = \text{High Frequency Variance} + \text{Seasonality} + \text{Trend}.$$

Wavelets provide a unique decomposition of time series that reveal properties of the data by deconstructing it. By using a multiresolution analysis of a maximum overlapping discrete wavelet transform, the information can be extracted from the series at different scales (called “details”) which are additive in their effect (the series can be reconstructed by adding up the details). So if the original time series has too much noise and strong seasonality effects, the trend will not be clear at first sight. Successive details contain progressively less high-frequency information, so with the higher frequencies removed, the overall trend is left in the wavelet smooth.

In this paper we use five scales,<sup>8</sup> or wavelet details, corresponding to 2 to 4 month cycles, 4 to 8 month cycles, 8 to 16 month cycles, 16 to 32 month cycles, and 32 to 64 months cycles. So the lower scales, including the highest frequencies, represent the short-term variations of the series. Seasonality is mainly captured in the third scale which corresponds to cycles of 8 to 16 months. Once the information of the five details has been removed from the series, we are left with an approximation of the trend (wavelet smooth), which only reveals information of longer-term cycles, above 64 months.

A wavelet basis needs to be selected to perform the decomposition. Here the

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<sup>7</sup>This, as explained later, is to focus our analysis around and after the time when headline inflation changed persistence, following Chiquiar et al. (2009).

<sup>8</sup>Different number of scales were used for the decomposition obtaining similar results. Already, the fifth scale is not giving substantial information, therefore there is no need to increase the number of scales.



results will only be presented for the level 8 Daubechies least asymmetric (LA(8)), although they were robust to other wavelet bases used here (Haar, Daubechies (4), Daubechies (8), Minimum Bandwidth (8)).

Figure 1.a presents the original headline inflation time series (top row) and its corresponding five wavelet details and wavelet smooth. The smoothed periodograms in Figure 1.b indicate the frequencies incorporated in each wavelet detail and wavelet smooth of Figure 1.a. It can be noticed that each detail captures certain periodicities. The lower scales capture the higher frequencies and the higher scales the lower frequencies. The third scale is capturing most of the seasonality component.

From the original series of Figure 1.a, we can appreciate how inflation was low at the beginning of the period of study but progressively started increasing in the second half of the seventies to reach very high levels in the eighties. At the end of the eighties the levels of inflation fell, but the financial crisis of 1995 triggered high levels of inflation for some months. In 1996 inflation started decreasing until reaching historically low and stable levels around the end of 2000. Nevertheless, some inflationary pressures can be detected around 1998 due to the impacts of the international financial crises (East Asia, Russia, and Brazil) and around 2007 and 2008 due to high international commodity prices, firstly, and then due to the Mexican Peso devaluation.

From the wavelet details and the wavelet smooth, the first thing to be noticed is the absence of regular periodicities and of constant amplitudes. Each detail changes continuously across time. Only after 2000, series seem to be more stable in all the frequencies. Also the presence of shocks can be detected across all the scales; they are well located temporally and stand out across the frequency range. Observe that the high levels of inflation during the eighties affect all the frequencies. Also the 1995 shock increased the short-term variability, the amplitude of the seasonality cycles, and the trend. The down-sloped trend of the wavelet smooth was interrupted temporarily between mid-1997 and early 1999 because of the financial crises of East Asia, Russia, and Brazil.

Notice as well, how the seasonal component changes drastically through time; this is mainly revealed in the third detail. This feature can complicate the seasonal adjustment of the series. Finally, the variance and level of the inflation seem to be positively correlated; in periods of low inflation variability seems to be low, in periods of high inflation variability seems to be high.

In Figure 2 the original headline inflation series, its wavelet details and wavelet smooth are shown from January 2000 to June 2009. This will enable us to look

closely at the recent dynamics of inflation, specifically after headline inflation had a change of persistence, becoming stationary (Chiquiar et al. 2009). Notice how at the beginning, the trend declines sharply reaching the lowest levels at the beginning of 2006 reaching monthly values of around 0.284%. Nevertheless, from the second half of 2006 onwards, the trend starts increasing. The wavelet smooth incorporates the inflation pressures of the hike in international commodity prices from mid 2006 until mid 2008 and of the Mexican Peso depreciation at the end of 2008 and beginning of 2009. Nevertheless, the levels are still historically low, and it seems the trend has been decreasing since the end of 2008.<sup>9</sup>

Also from Figure 2, it can be seen that the fourth scale (16 to 32 month cycle) is relatively stable from 2001 onwards. The third detail (representing seasonality) changes amplitude during the period, although the periodicity and amplitude seem to be relatively constant from 2003 until the end of the series. However, there seem to be some problems during 2005; we will look closer at this when analyzing non-core inflation (Figure 6). Notice a sudden burst of activity in the lowest scale at the beginning of 2002, this is also due to non-core inflation and will be addressed later in this paper.

These results confirm the fact that a reduction of mean and variance of headline inflation has been taking place since 1996, and that since 2001 its behavior has been stable. This price stability was ensured by sound monetary policies (see Ramos-Francia and Torres (2005)).

Figure 3 shows the LA(8) multiresolution analysis of the maximum overlapping discrete wavelet transform for the core inflation (a) and its corresponding smoothed periodogram (b). This series starts on February 1982. The MODWT MRA of core inflation from January 2000 to June 2009 is plotted in Figure 4.

The medium and long-term dynamics of core inflation are similar to those presented by headline inflation, reaching historically low levels at the end of 2005 (around 0.283% monthly inflation). The trend started increasing in 2006 reflecting the soaring international commodity prices at the time and the later depreciation of the Mexican Peso. A change of slope seemed to have occurred in January 2009, although the reduction has been much slower than in headline and non-core inflation. The amplitude of the seasonal component is low in core inflation, especially after 2001. Also the levels of high frequency variance are rather small, compare to headline and

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<sup>9</sup>The MODWT does not subsample the filtered output to relax the dyadic sample size requirement of the DWT. When the end of the vector is reached, we use the reflection method for computing the remaining wavelet coefficients. This will bias the estimations at the boundaries (after phase shifting); so the actual levels may be lower.

non-core inflation. By definition, it is desirable that core inflation mainly incorporates the medium and long-term changes of the price index and not the short-term relative price changes. Core inflation seems to present these characteristics even before inflation targeting was implemented in Mexico, and with inflation targeting it has been reinforced. This will be analyzed in more depth when studying the energy of the series in the last part of this section.

In Figure 5.a the original non-core inflation series (from February 1982 to June 2009), its wavelet details and wavelet smooth from the LA(8) MODWT MRA are plotted, while in 5.b the corresponding smoothed periodograms are shown. The seasonal component incorporated in the third detail (8 to 16 month cycles) seems to be the dominant frequency of the series. This fact can also be seen in Figure 6 where the series are plotted from January 2000 onwards to reveal specific features during these last years.

The high frequency variance, captured in the first and second details, is very strong. This feature contrasts with the one of core inflation (Figure 4). Non-core inflation is mainly driven by high frequency variance and seasonality, while core inflation reflects the medium and long-term trend. Notice that here again all the scales are contaminated during the high inflation periods of the eighties and the crisis of 1995.

When examining closely scale by scale the non-core inflation in Figure 6, at the lowest scale (2 to 4 month cycles) a burst of energy can be spotted starting at the beginning of 2002. This is due to substantial changes in the prices of fruit and vegetables during that year (Figure 12.a). Also during 2002, some problems in the seasonal component (third detail) can be detected as a result of significant increases in public prices (Figure 13.a), mainly in residential electricity and gas (see Banco de México (2002) for more information). High levels of volatility on the prices of fruit and vegetables at the end of 2004 and beginnings of 2005 are affecting the first three details. The wavelet smooth reaches its lowest level at mid-2006 (around 0.277% monthly inflation). Since mid-2006, non-core inflation starts showing an increasing trend (with a sudden temporary decrease in mid-2007 due to fruits and vegetables (Figure 12.a)); this is caused by the effect on local prices of rising international commodity prices. Food, energy and public prices soared, pushing inflation up. Also the pass-through of the depreciation of the Mexican Peso influenced this increase from the second half of 2008. Nevertheless, an important decline has been taking place since the end of 2008.

Although our main goal is studying the dynamics of headline, core, and non-

core inflation, the other 13 indexes of the INPC will be considered here to examine what is going on in a more disaggregate level (which explains the aggregate one). These series will be analyzed from January 2000 until June 2009, as the last period comprises the years of our main interest, the inflation targeting regime.

Starting with the indexes of the core component, merchandise and services inflation series and its MODWT MRA are shown in Figure 7, followed by its subindexes in Figure 8 (food and rest of merchandise), Figure 9 (housing and education), and Figure 10 (rest of services). Merchandise presents a very important and stable seasonal component at the third scale (revealing yearly seasonality). Nevertheless, its trend has been largely affected by both high international commodity prices and the Mexican Peso depreciation, and it seems to have started to decrease slightly only since April/May 2009. Yearly seasonality is mainly explained by the food subindex, but the trend behavior is due to increases in both the food and rest of merchandise.

Services inflations also has an important seasonal component but mainly explained by the second scale (4 to 8 months cycles); this is due to the fact that it is highly influenced by the changes in education prices (that mainly happen in January, August, and September). Notice that the changes in its trend and the trend of its components have been affected in much less degree by the recent inflationary pressures.

In Figure 11 the two indexes of non-core inflation, agriculture and administered and regulated goods and services, and its MODWT MRA are graphed. Agriculture presents very high levels of the higher frequencies variance (compare the axis scales), intensified from mid 2004 until the end of 2006. This was mainly due to fruits and vegetables (Figure 12.a), though farm-related goods (Figure 12.b) also registered a burst of activity in its two first scales during 2004. Seasonality is also very important for these series (mainly in fruit and vegetables), and they were substantially affected by international commodity prices and the depreciation of the Mexican peso. Nevertheless, since the end of 2008, there seems to be a change of trend, now downward sloping.

When focusing in administered and regulated goods and services (Figure 11.b), notice how seasonality dominates the dynamics of this series (mainly explained by electricity prices included in the administered goods (Figure 13.a)). The increases of gasoline prices during the second half of 2008, were mainly absorbed by the fourth and fifth scale, so the long-term trend was unaffected. An upward sloping of the trend of regulated goods (Figure 13.b) can be spotted since mid-2006 until almost the end of the sample, nevertheless it was relatively small, practically not affecting

the long-term trend of the aggregate index.

More could be said about these disaggregated series, nevertheless our main objective is to analyze the dynamics of headline, core, and non-core inflation. For a detailed analysis of inflation series in an even more disaggregate level, using multivariate analysis, see Ysusi (2009).

### 3.3. Energy Decomposition

Once the series have been decomposed, it is relevant to study the relative importance of the short, medium, and long-term dynamics and how they have changed in the recent disinflation process. As the monetary policy became a successful mechanism to reduce inflation, the degree of persistence of the headline and core inflation series have declined (see Chiquiar et al. (2009)). Therefore, a reduction in the importance of the long run in this series will be expected; the shocks will die out in the short or medium term.

Here we will use the energy of each level of the inflation's wavelet decomposition, i.e. the energy of each scale, to measure the relative importance of the short, medium, and long run. The energy is analogous to the variance of each detail level, and it will be given as the percentage of the overall energy. Hence, we will examine the percentage of variance that each scale is explaining.

As described in Percival and Walden (2000), the discrete wavelet transform has the ability to decompose the energy in a time series across scales. Percival and Mofjeld (1997) proved that the MODWT is also an energy-preserving transform (the variance of the time series is preserved in the variance of the coefficients from the MODWT). So a time series  $x(t)$ , with wavelet coefficients for scale  $j$ ,  $\tilde{w}_{j,t}$  and scaling coefficient  $\tilde{v}_{J,t}$  from a MODWT, has the following energy decomposition:

$$\sum_{t=1}^N x^2(t) = \sum_{j=1}^J \sum_{t=1}^N \tilde{w}_{j,t}^2 + \sum_{t=1}^N \tilde{v}_{J,t}^2$$

where  $N$  is the number of observations used in the calculation.<sup>10</sup> This allows us to separate the contribution of energy in the time series due to changes at a given scale.

Table 1 presents the energy of each scale (as percentage of the overall energy) for three different periods for headline, core, and non-core inflation.<sup>11</sup> For this table the

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<sup>10</sup>An unbiased estimator of the energy will be calculated with the coefficients unaffected by the boundary, so  $N$  is not always equal to  $T$ .  $N$  will depend on the basis and the number of scales used.

<sup>11</sup>Table 2 presents it for the merchandise, services, agricultural, and administered and regulated

coefficients affected by the boundaries were not accounted (sixteen coefficients were disregarded out of 485 for headline inflation and out of 329 for core and non-core inflation), in order to get an unbiased estimator. Notice that here only four scales were used (the fifth scale is included in the smooth); this is done to disregard as few of the boundary observations as possible, not to lose too much information. The Haar basis was used for this table given that it is less affected by the boundaries than the other Daubechies bases. Nevertheless, results are quite robust to different wavelet bases used but not reported.

The energy is firstly calculated for headline, core, and non-core inflation using the entire sample. For all of the series, the long run dominates all the other frequencies, explaining most of the variance (83% for headline inflation, 87% for core inflation, and 57% for non-core inflation); the high frequency variations and seasonality components are relatively unimportant except for non-core inflation. We now divide the series into two parts: before and after the headline inflation became stationary according to Chiquiar et al. (2009). So the second column of the table gives the results for the period before January 2001 and the third column presents them for the stationary period of headline inflation, from January 2001 onwards.

There seem to be substantial differences between the two periods. Notice that for headline inflation there is a considerable decline in the amount of variance explained by the lower frequencies. Instead, high frequency variations and seasonality went from explaining just around 12% of the energy to more than 35% of it. The distribution of energy of core inflation seems to be a bit more constant throughout the different periods, although after 2001 the energy explained by the higher frequencies grew from around 4% to 11%. Noticeably, 62% of the variance of non-core inflation was included in the cycles above 16 months when studying the series before 2001, but it went down drastically to almost 20% over the most recent period.

The previous results are consistent with studies about inflation persistence in Mexico. Given that the degree of persistence of the Mexican inflation has been falling during the last decade, it was to be expected that the short-term component of the headline inflation would explain relatively more variance compared to the previous decades.

The differences between the energy distribution of core and non-core inflation come from the fact that core inflation excludes more volatile items, it tries to isolate what is happening to general prices without distraction from spikes in more volatile

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goods and services inflation series. The results are similar to the ones explained here for the main indexes.

prices. Core inflation is a more reliable indicator of the underlying inflation trend, so it should mainly reflect the medium and long-term pressures. Therefore the energy distribution of core inflation is likely to suffer less changes, with most of the weight on the lower frequencies.<sup>12</sup> Meanwhile non-core inflation is subject to higher variability and reflects the short-term shocks. As this shocks tend to affect the lower scales and die out, the higher frequencies will explain more of the variance of the non-core inflation.

The results in Table 1 give a first insight into how the energy distribution has been changing for the Mexican inflation. Nevertheless, each sub-sample has a different length and characterizes a specific period. Now, we will estimate the relative energy of the scales at each point in time. With this, it will be possible to detect changes through time of the relative importance of the different frequencies.

Figure 14 shows the energy decomposition through time for headline inflation. The first detail represents the highest frequency variability, the two following details are added together to incorporate the information between 4 and 16 month cycles representing seasonality, and the fourth detail and the wavelet smooth are added up to show the cycles above 16 months, representing the longer term. It illustrates how the higher frequencies energy-contribution exhibit a U-shaped pattern over the complete period of study. Short-term variations and seasonality explain a high proportion of the total variance at the beginning of the seventies and after 2001, and few of it during the eighties and nineties. Conversely, the lower frequencies explain a lower share of the variance since 2001. In 2007 the first three details were explaining in average more than 45% of the energy of the headline inflation; in contrast, they were explaining in average less than 10% during 1997 (the cycles above 16 months were explaining around 90%). During most part of the eighties, the variance was mainly explained by the wavelet smooth, reaching yearly average levels above 95%; less than 5% was explained by the three lower scales. This sharp decline on the percentage of energy explained by the medium and long run coincides with the most recent part of the disinflation period, where inflation became stationary.

The energy decomposition through time for the core inflation (Figure 15) also exhibits some changes in 2001. Nevertheless, notice an increase in importance of the higher frequencies around 1988 and 1995; this is due to the fact that big shocks are incorporated in all the scales. So the higher frequencies during this years are

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<sup>12</sup>Core inflation suffered a change in persistence during 2001 as documented in Chiquiar et al. (2009). This fact is reflected in the core inflation series as a drastic decrease in variance of the lower frequency (and not in the percentage that it explains). The long term trend has an estimated sample variance equal to 11.8 before January 2001 and of 0.13 after that same date.

contaminated by the shocks and do not really indicate a change in the inflation process.

The increase of explained-variance of the first three scales of core inflation is relatively small compared to the increase of importance of these same frequencies in non-core inflation (Figure 16). For example in 2002, for core inflation 16% in average is explained by these lower scales, but above 50% in average for non-core. Given that core inflation is constructed as an indicator of the trend and it mainly reflects the medium and long-term pressures, this result was somewhat anticipated.

The non-core inflation has experienced major changes in its energy distribution (Figure 16). Since 2001, the first three wavelet details, i.e. the short run, explain most of the variance of the non-core inflation. The lower frequencies (cycles above 16 months) have gone from yearly average levels of explained-energy of around 70% before 2000, down to 16% during 2007. Excluding a short period at the beginning of the nineties, the recent stationary period is the first episode when the non-core inflation variance is mainly explained by the high-frequency variations and seasonality. Notice how before 2001, the series representing the higher frequencies was lower than the one representing the low-frequencies, but since the end of 2000 this fact reversed. Before 2001, it seems that the non-core inflation was driven by the medium and long run instead of the short run as expected. Nevertheless, in the inflation-targeting period, non-core inflation is incorporating the more volatile items and is mainly reflecting the short-term shocks that die out quickly.

## 4. Conclusions

In this paper we apply the wavelet methodology to study inflation dynamics in Mexico. Particular focus is put on the recent disinflation period, during which important efforts have been made by the monetary authorities to stabilize prices. The purpose of this study is to contribute to the debate on the Mexican inflation dynamics under the inflation targeting framework. Our approach is purely statistical, complementing previous structural and econometric research.

As explained in the paper, the Mexican experience is characterized by high and low inflation periods. Characteristics of the inflation dynamics have changed in the last years due to actions taken by Banco de México during the period, which are consistent with a monetary policy seeking long-term price stability. Using the wavelet decomposition this long-term convergence is evidenced statistically.<sup>13</sup>

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<sup>13</sup>From mid-2006 until 2008 international commodity prices, and later the depreciation of the



Since 2001 headline and core inflation series show a more stable behavior in all the scales. An increase in the proportion of variance explained by short-term variations is detected in all headline, core, and non-core inflation series. In relative terms, the short run is becoming as important for headline inflation as the medium and long run, and more important for non-core inflation. Non-core inflation is really capturing the short-lived price variations while core inflation still reflects the medium and long-term pressures. These results are in line with the reduction in inflation persistence documented in Chiquiar et al. (2009).

Wavelets have not been extensively used in economics, but the potential benefits are substantial. Further analysis could be performed using wavelets, which may provide new insights into economic phenomena. Future possible research related to Mexican inflation include the estimation of the persistence of fractionally integrated processes, an alternative measure of core inflation, and the improvement of forecasts by extracting information from different scales that are hidden at the aggregate level. These are just few of the many opportunities of research that are offered by the use of wavelets in economics.

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Mexican Peso, have caused upward pressure on inflation, nevertheless the monetary authorities have adopted various measures consistent with the inflation target framework and the objective of price stability. A declining trend in all the series can be observed since the end of 2008 and beginning of 2009.

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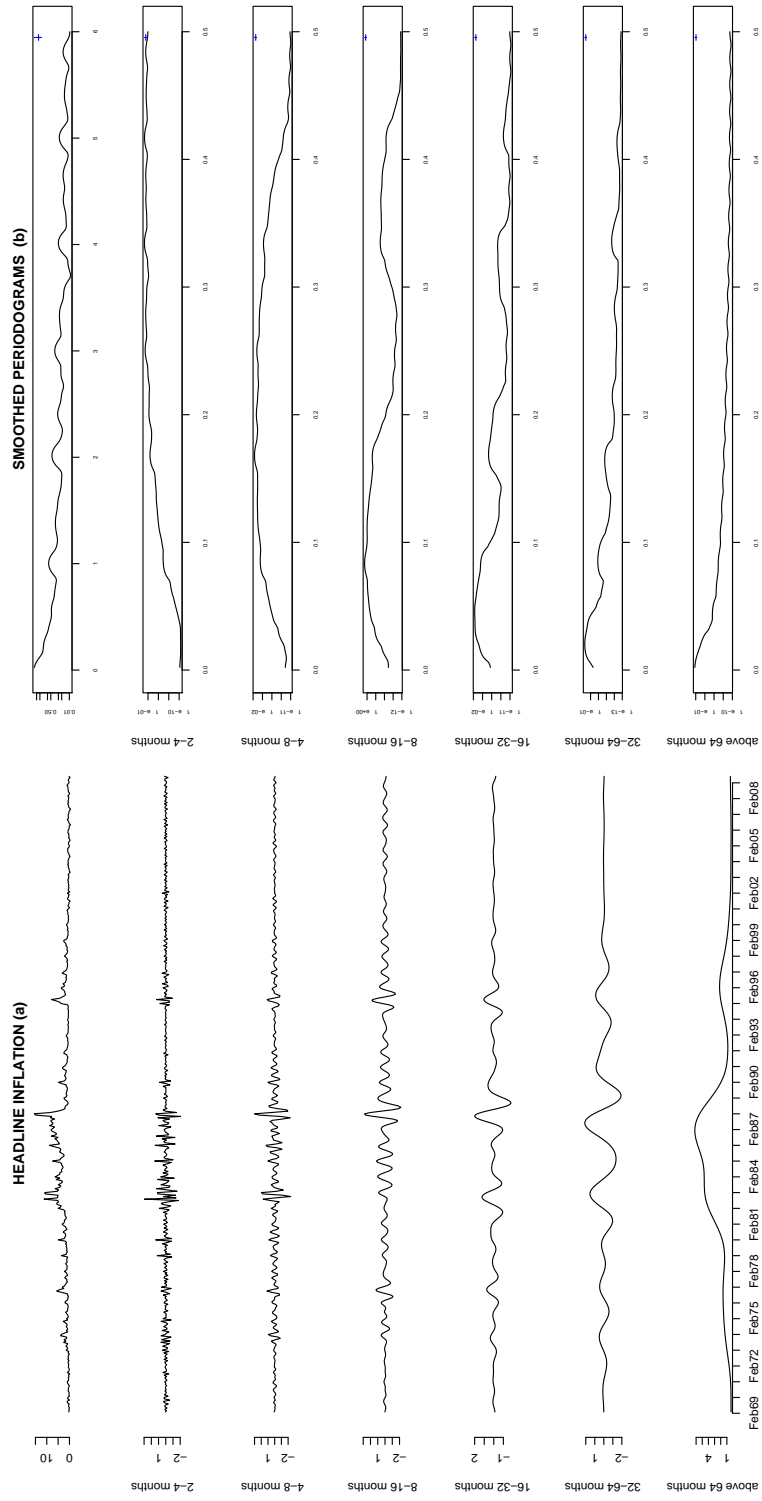


Figure 1. a)  $LA(8)$  MODWT MRA of the headline inflation series. The original time series is in the upper row. Below it, the wavelet details and the wavelet smooth are plotted. b) The corresponding smoothed periodograms; they indicate the periodic components included in each scale. The x-axis is given in cycles.

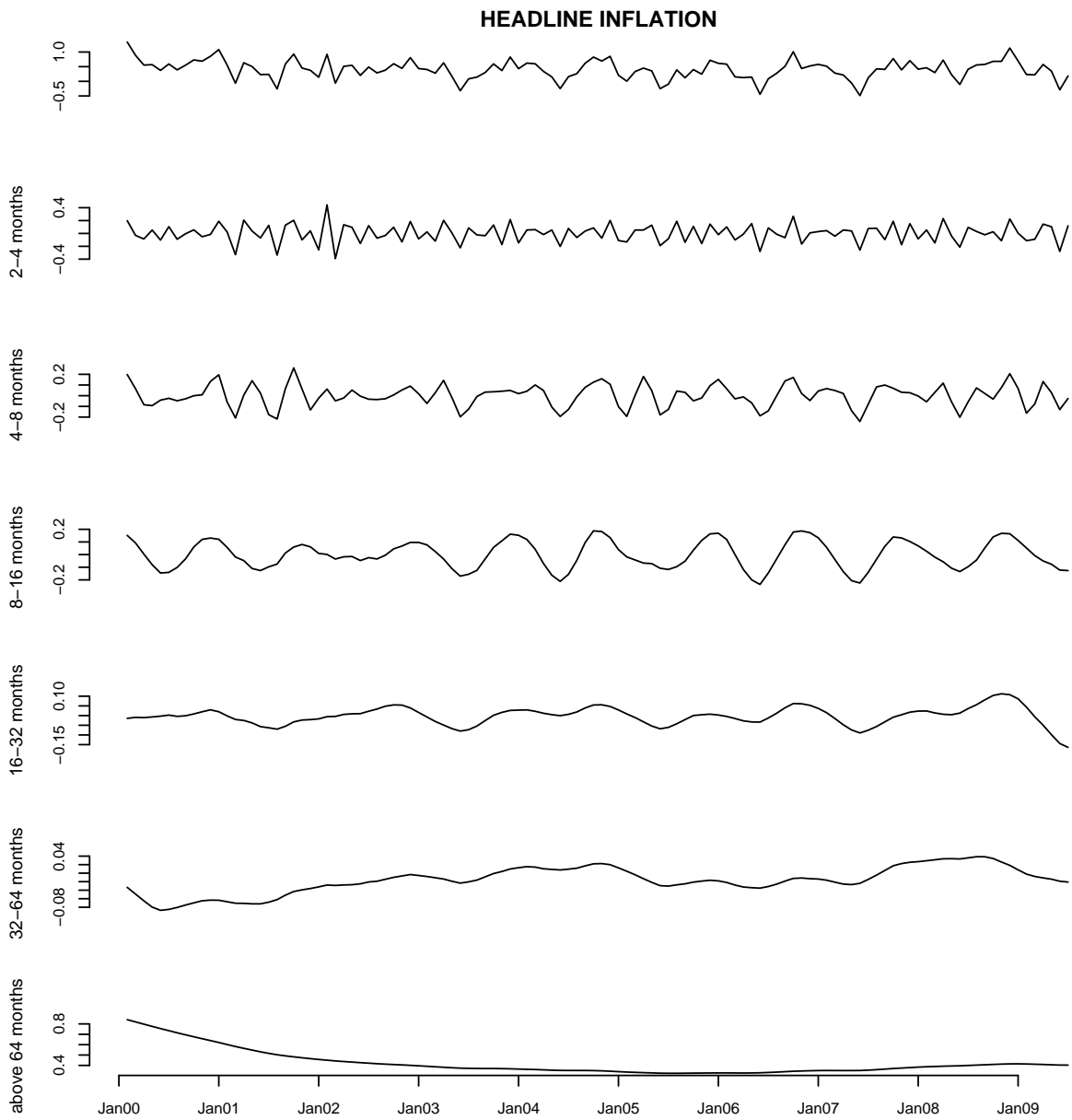


Figure 2. *MODWT MRA of the headline inflation series from January 2000 to June 2009. The original time series is in the upper row, followed by the wavelet details and smooth.*

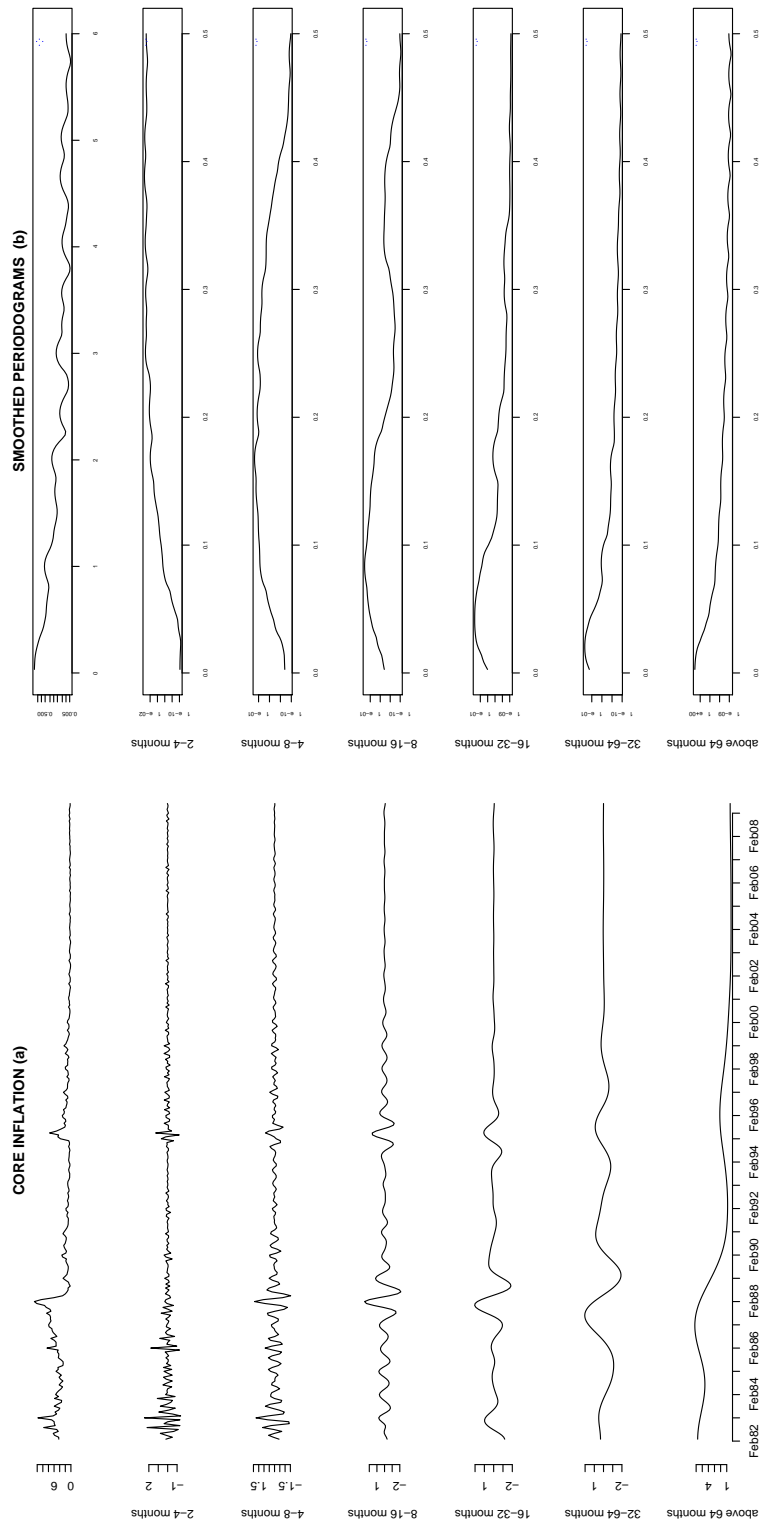


Figure 3. a)  $LA(8)$  MODWT MRA of the core inflation series. The original time series is in the upper row. Below it, the wavelet details and the wavelet smooth are plotted. b) The corresponding smoothed periodograms; they indicate the periodic components included in each scale. The x-axis is given in cycles.

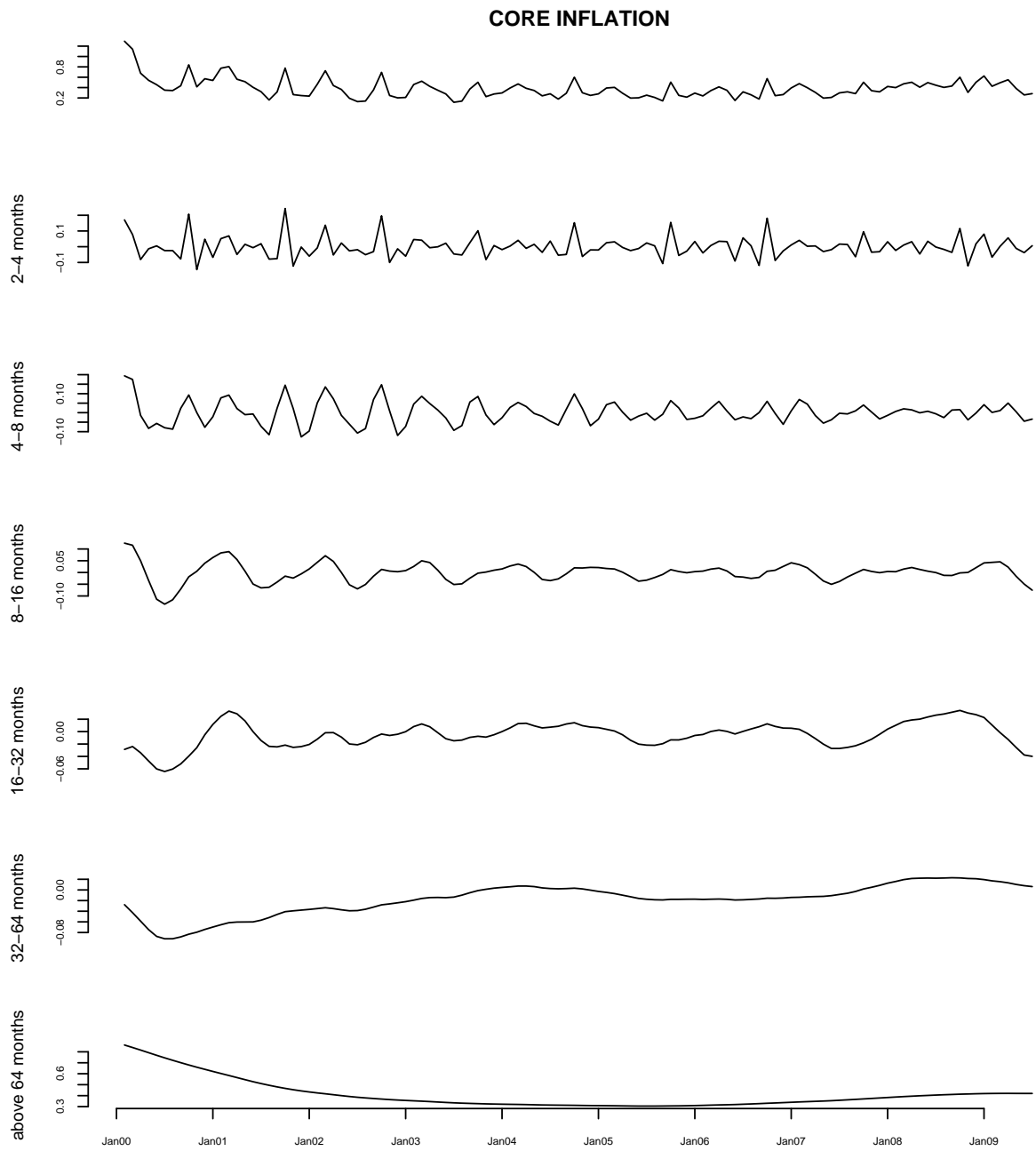


Figure 4. *MODWT MRA of the core inflation series from January 2000 to June 2009. The original time series is in the upper row, followed by the wavelet details and smooth.*



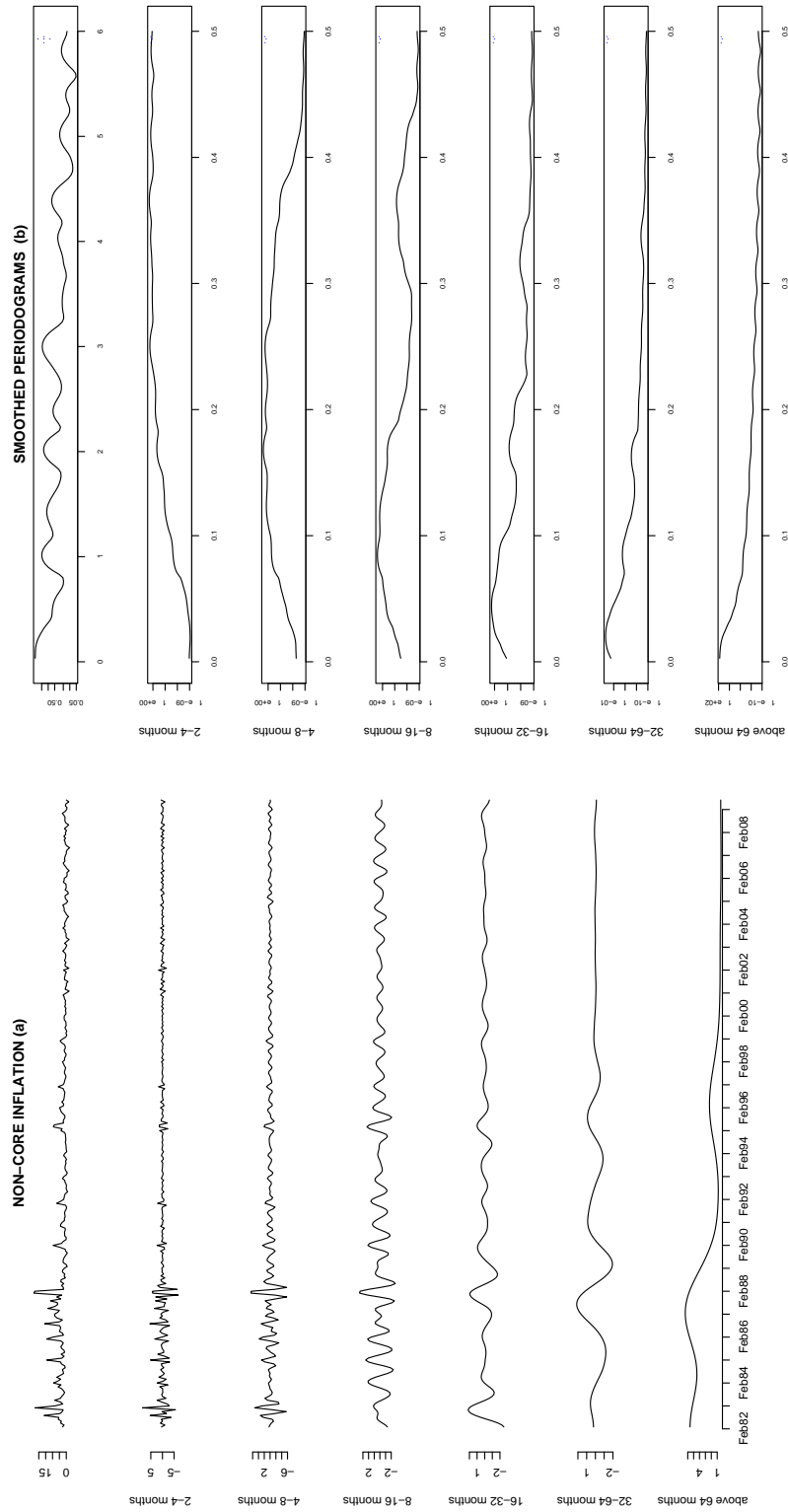


Figure 5. a)  $LA(8)$  MODWT MRA of the non-core inflation series. The original time series is in the upper row. Below it, the wavelet details and the wavelet smooth are plotted. b) The corresponding smoothed periodograms; they indicate the periodic components included in each scale.

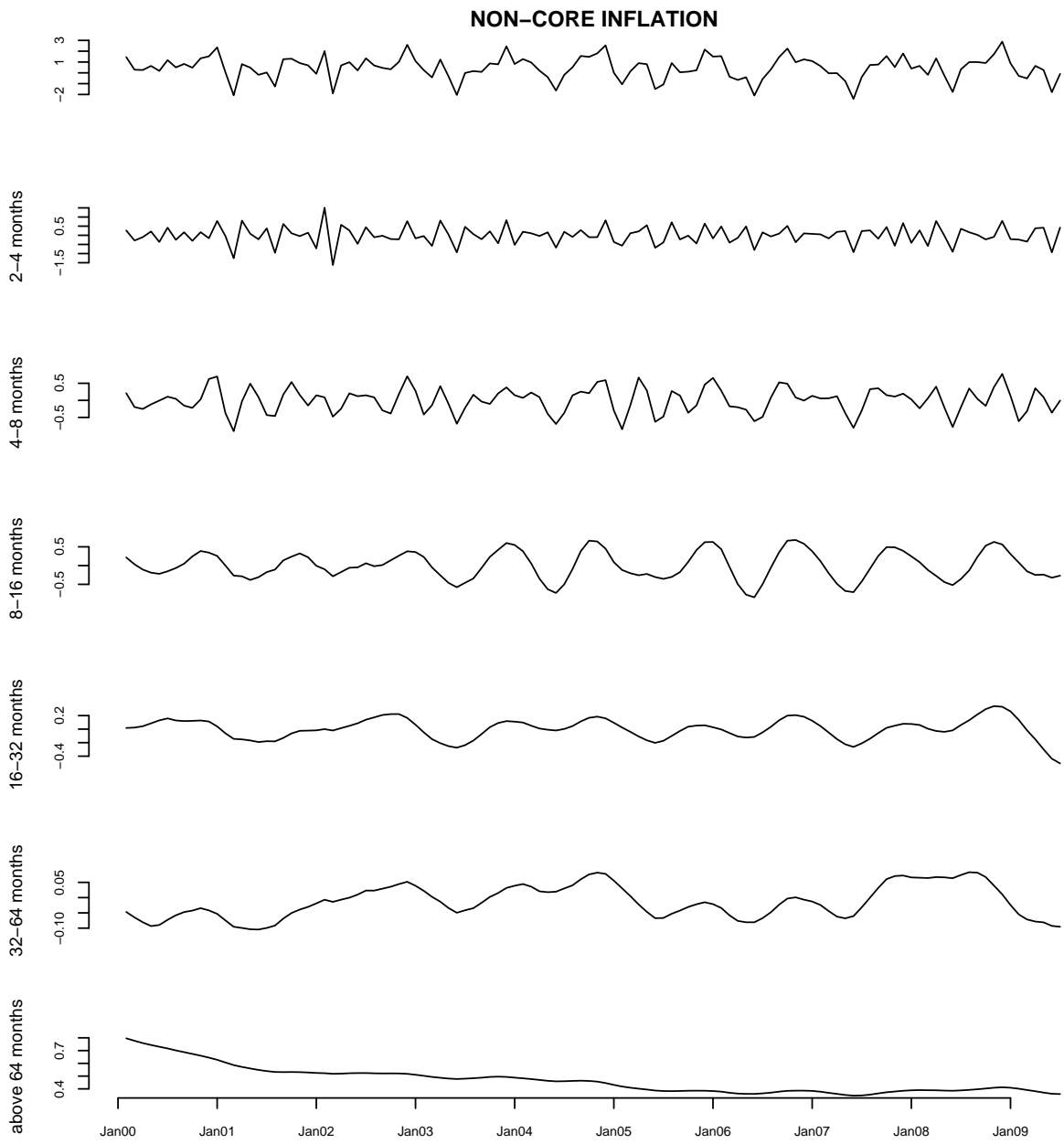


Figure 6. *MODWT MRA of the non-core inflation series from January 2000 to June 2009. The original time series is in the upper row, followed by the wavelet details and smooth.*

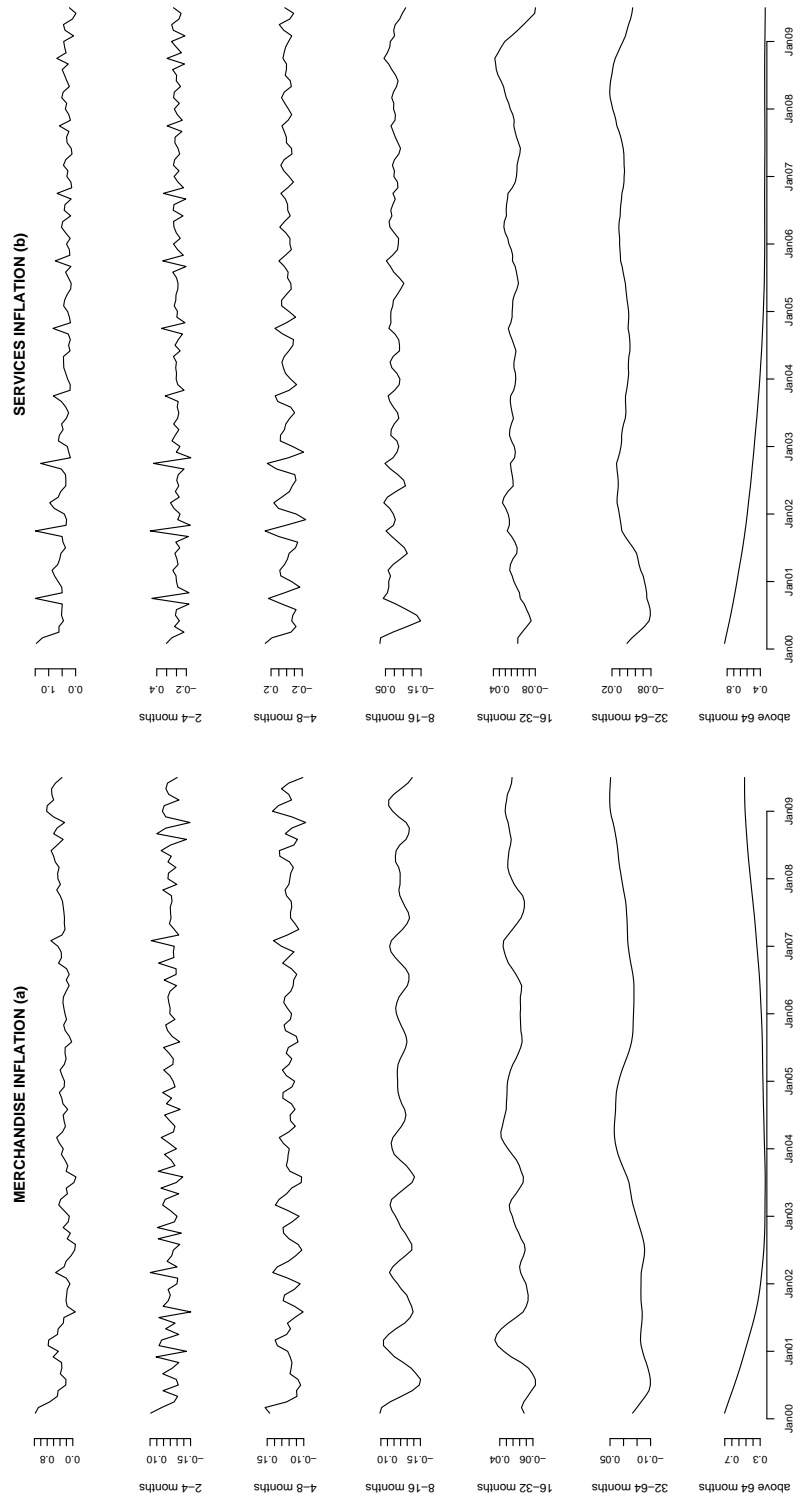


Figure 7.  $LA(8)$  MODWT MRA of inflation of the merchandise (a) and services (b) indices from January 2000 to June 2009. The original time series is in the upper row, followed by the wavelet details and smooth.

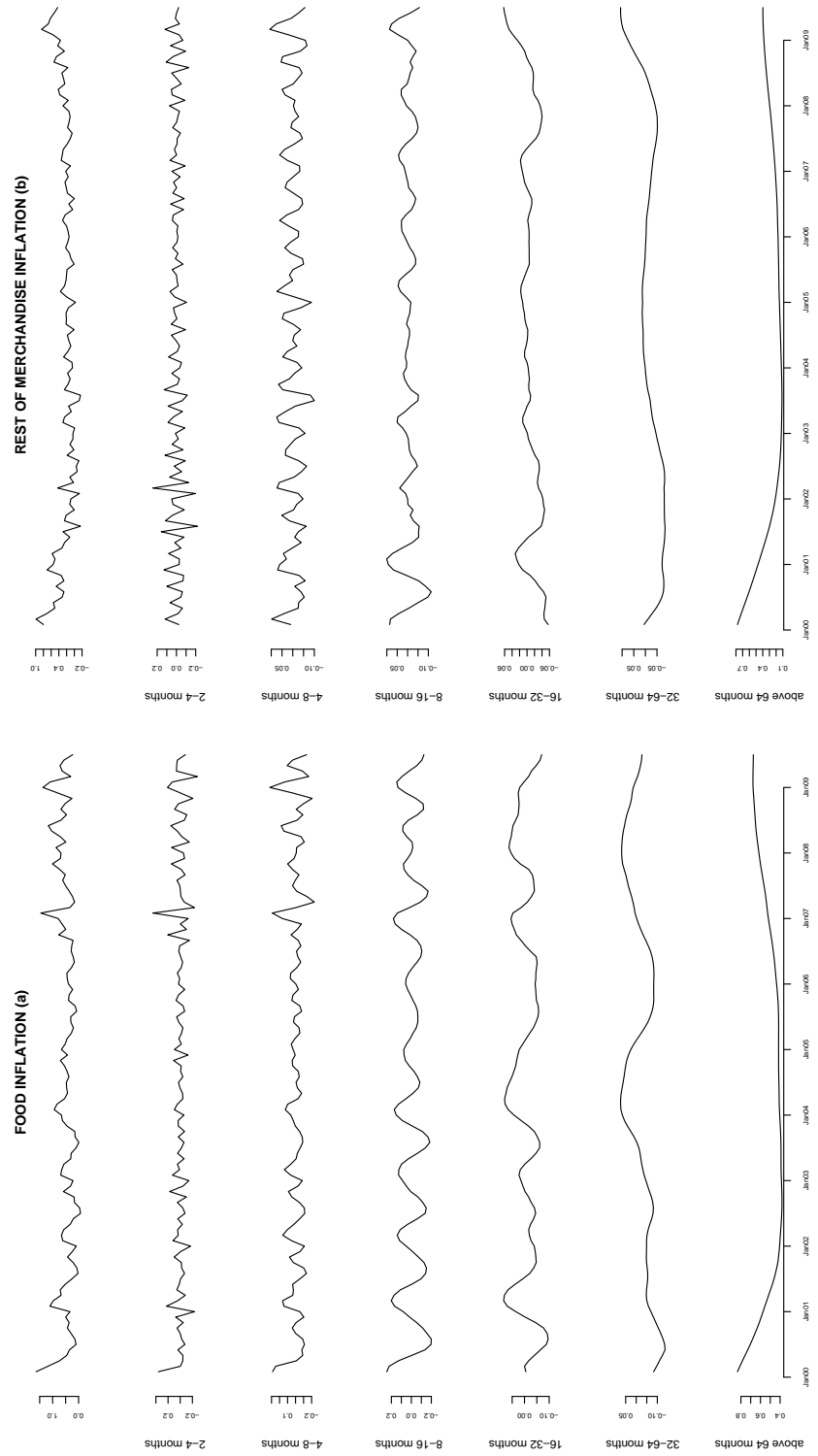


Figure 8. *MODWT MRA of inflation of the food (a) and other merchandise (b) indices from January 2000 to June 2009. The original time series is in the upper row, followed by the wavelet details and smooth.*

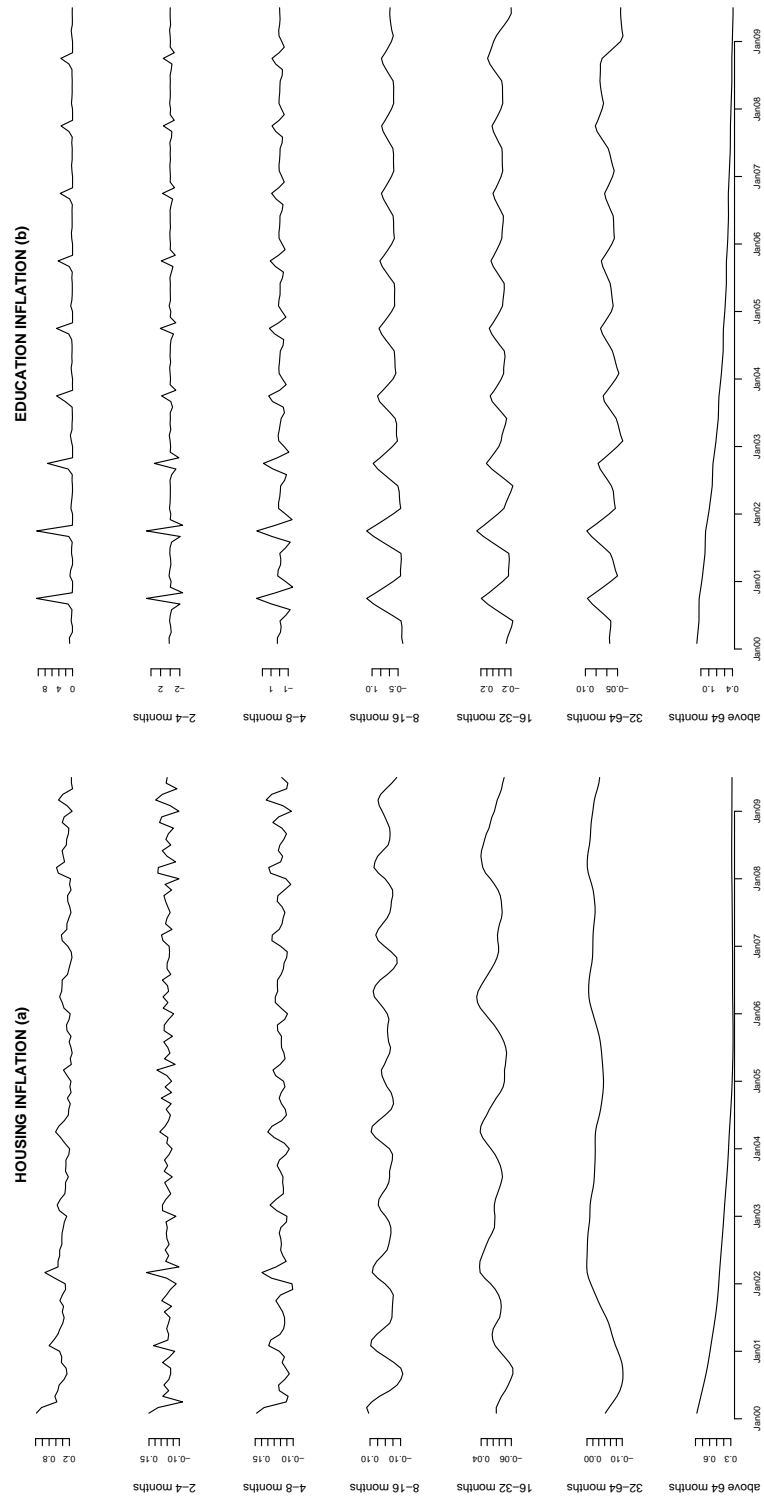


Figure 9. *MODWT MRA of inflation of the housing (a) and education (b) indices from January 2000 to June 2009. The original time series is in the upper row, followed by the wavelet details and smooth.*

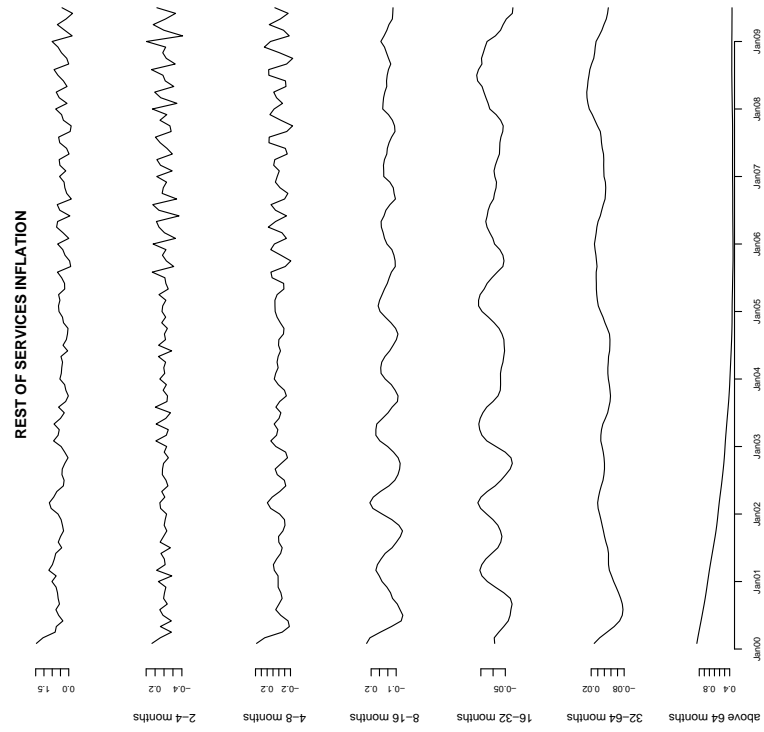


Figure 10. *MODWT MRA of inflation of the other services indices from January 2000 to June 2009. The original time series is in the upper row, followed by the wavelet details and smooth.*

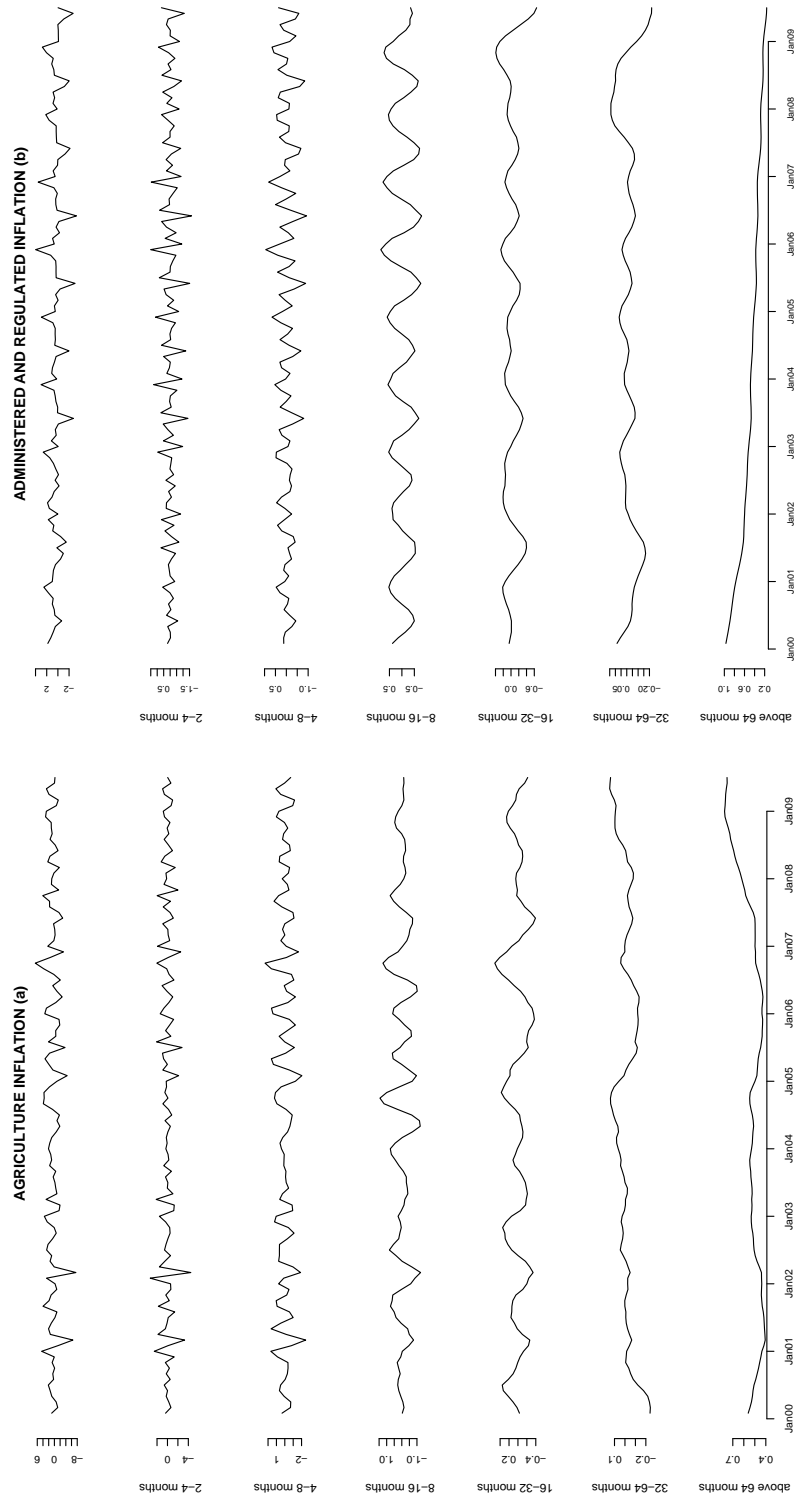


Figure 11. *MODWT MRA of inflation of the agriculture (a) and administered and regulated (b) indices from January 2000 to June 2009. The original time series is in the upper row, followed by the wavelet details and smooth.*

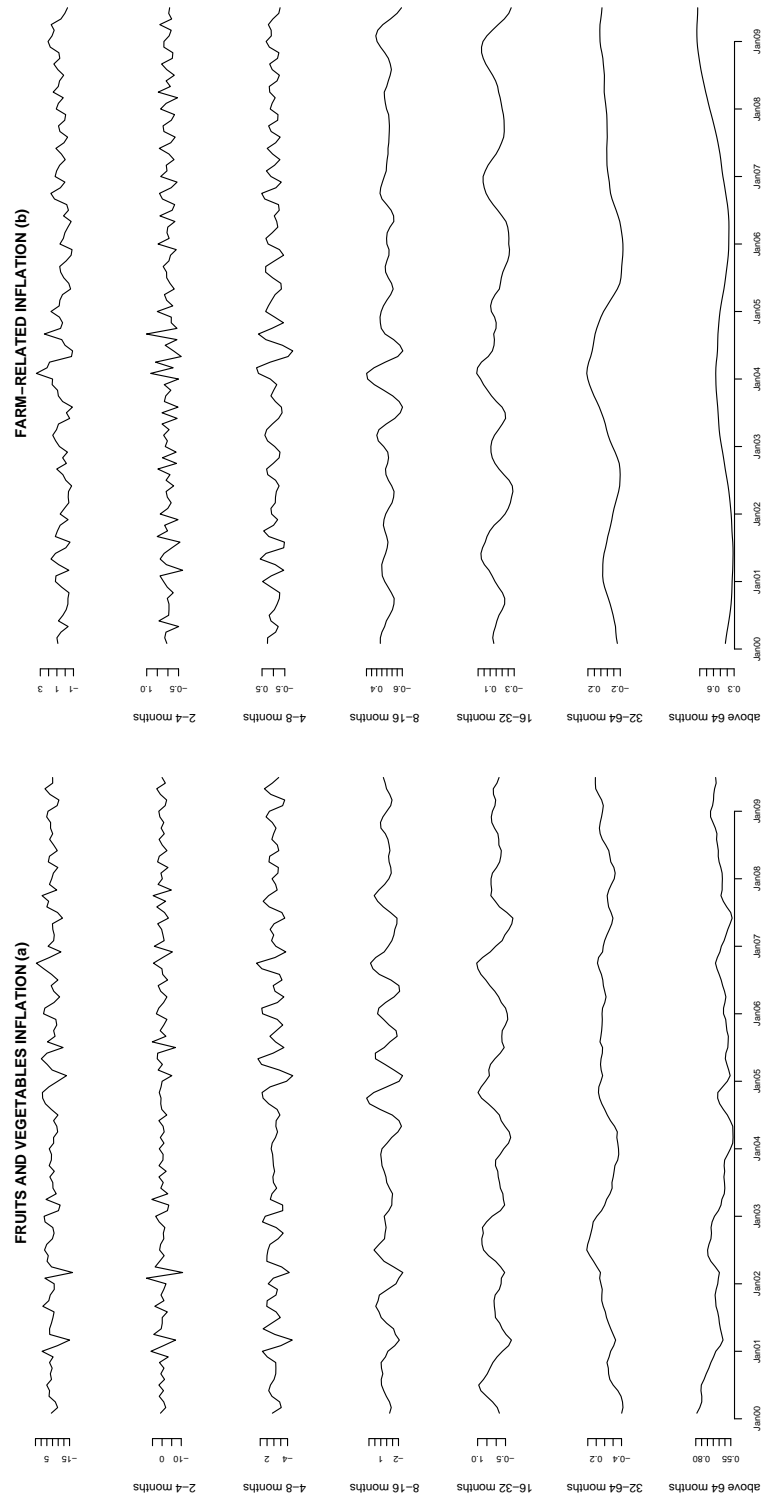


Figure 12. *MODWT MRA of inflation of the fruits and vegetables (a) and farm-related (b) indices from January 2000 to June 2009. The original time series is in the upper row, followed by the wavelet details and smooth.*



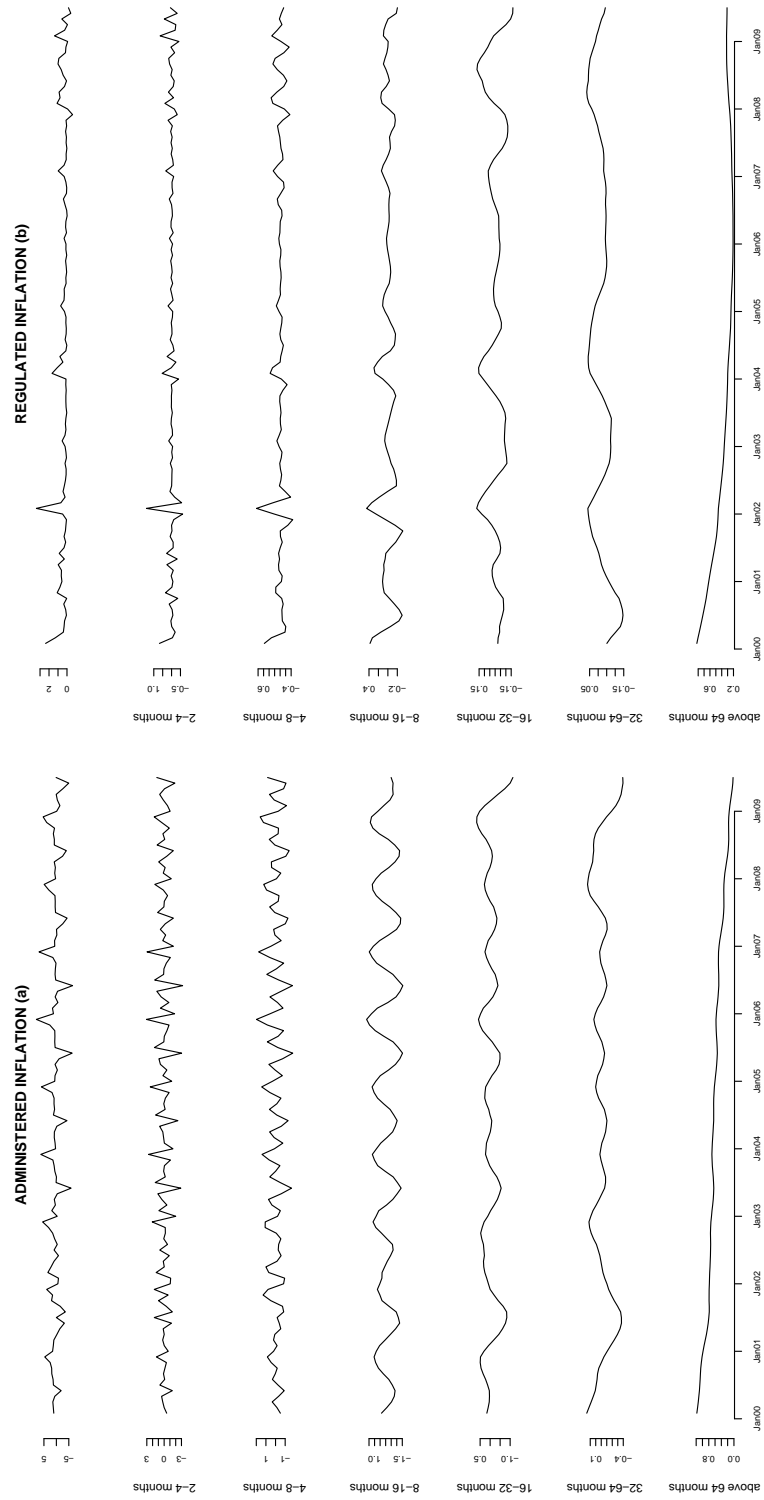


Figure 13. *MODWT MRA of inflation of the administered (a) and regulated (b) indices from January 2000 to June 2009. The original time series is in the upper row, followed by the wavelet details and smooth.*

### HEADLINE INFLATION: PERCENT OF ENERGY

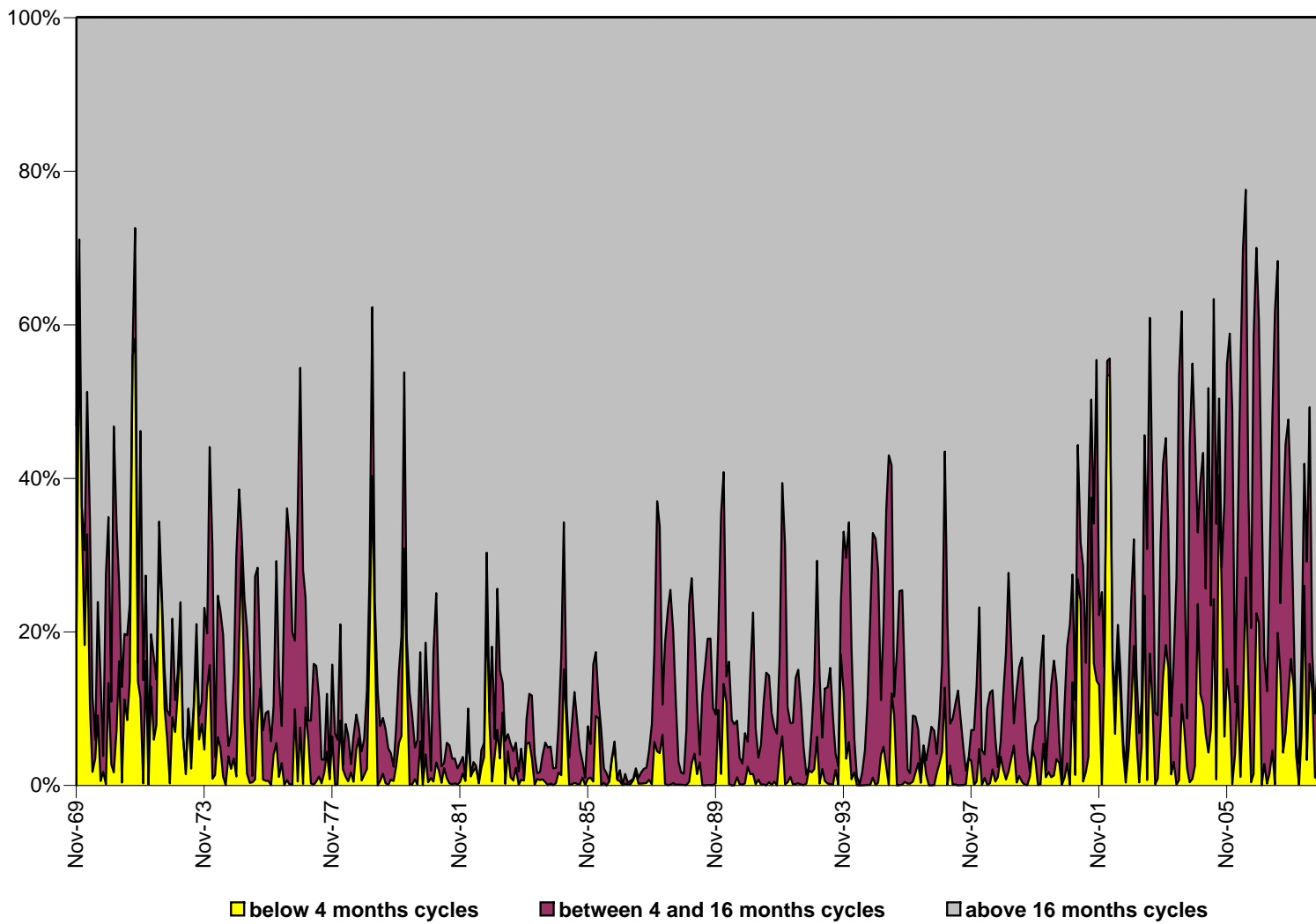


Figure 14. *Energy decomposition for headline inflation through time. The energy of each scale is expressed as fraction of the overall energy.*

### CORE INFLATION: PERCENT OF ENERGY

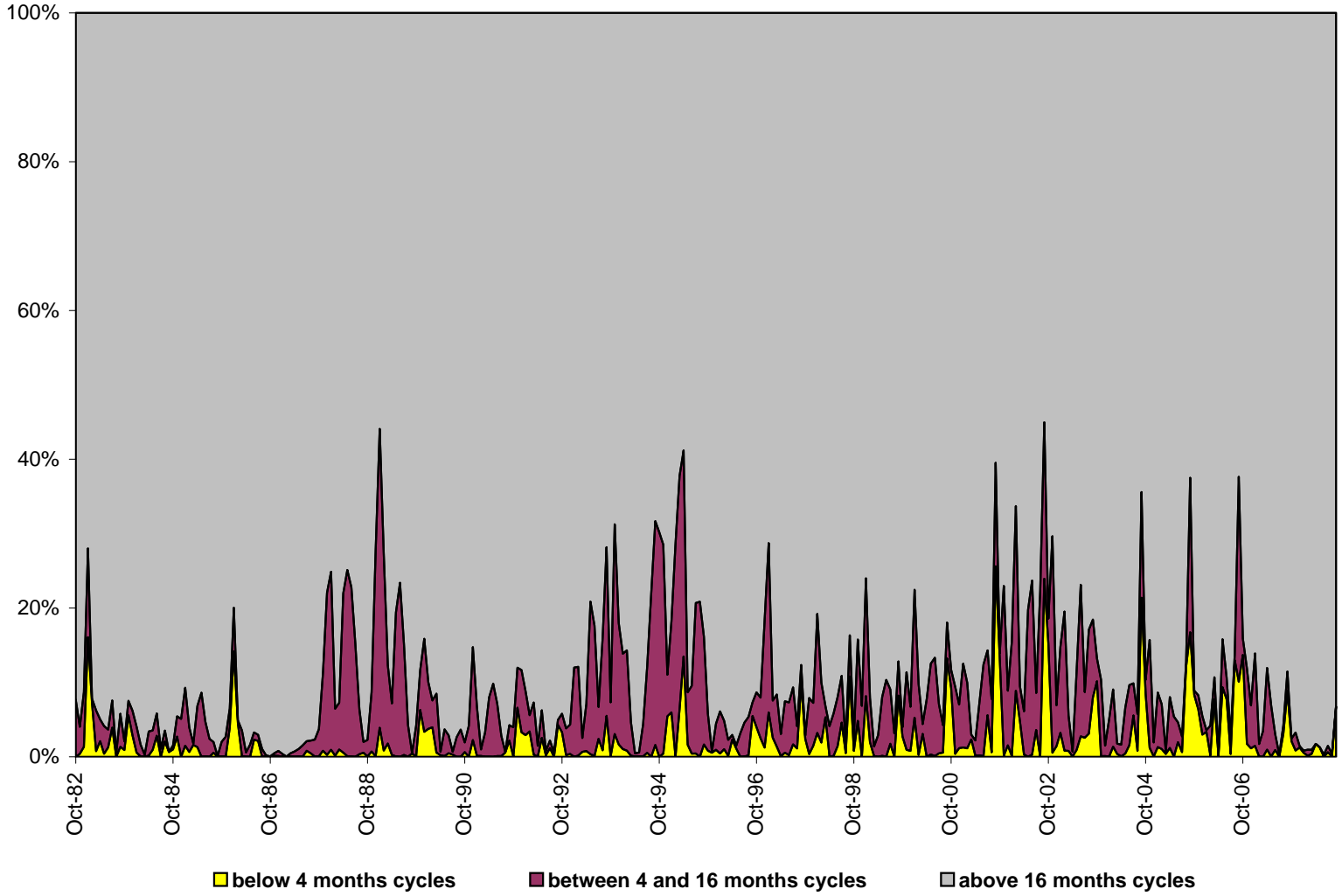


Figure 15. *Energy decomposition for core inflation through time. The energy of each scale is expressed as fraction of the overall energy.*

### NON-CORE INFLATION: PERCENT OF ENERGY

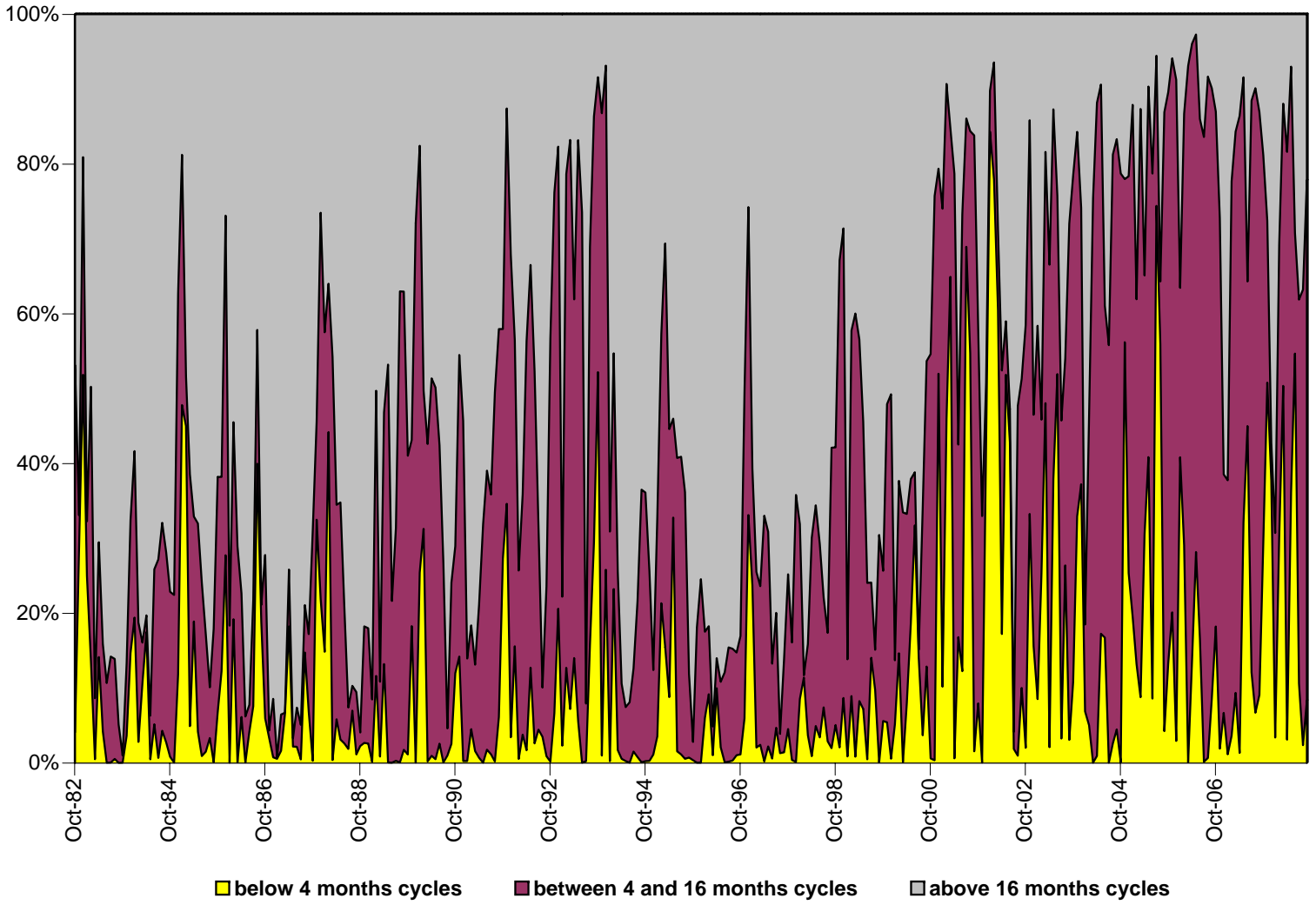


Figure 16. *Energy decomposition for non-core inflation through time. The energy of each scale is expressed as fraction of the overall energy.*

HEADLINE INFLATION	1969-2008	1969-2000	2001-2008
2-4 month cycles	0.04	0.04	0.14
4-8 month cycles	0.04	0.04	0.11
8-16 month cycles	0.04	0.04	0.11
16-32 month cycles	0.05	0.05	0.03
above 32 month cycles	0.83	0.83	0.61
CORE INFLATION	1982-2008	1982-2000	2001-2008
2-4 month cycles	0.02	0.01	0.05
4-8 month cycles	0.03	0.03	0.06
8-16 month cycles	0.04	0.04	0.02
16-32 month cycles	0.04	0.05	0.01
above 32 month cycles	0.87	0.87	0.86
NON-CORE INFLATION	1982-2008	1982-2000	2001-2008
2-4 month cycles	0.16	0.16	0.29
4-8 month cycles	0.15	0.14	0.24
8-16 month cycles	0.08	0.08	0.24
16-32 month cycles	0.04	0.04	0.06
above 32 month cycles	0.57	0.58	0.17

Table 1. *Energy decomposition for headline, core and non-core inflation for different periods. The energy of each scale is expressed as fraction of the overall energy.*

MERCHANDISE	1982-2008	1982-2000	2001-2008
2-4 month cycles	0.02	0.02	0.04
4-8 month cycles	0.03	0.03	0.05
8-16 month cycles	0.04	0.04	0.05
16-32 month cycles	0.06	0.06	0.03
above 32 month cycles	0.85	0.85	0.83
SERVICES	1982-2008	1982-2000	2001-2008
2-4 month cycles	0.04	0.04	0.10
4-8 month cycles	0.03	0.03	0.06
8-16 month cycles	0.03	0.03	0.02
16-32 month cycles	0.03	0.03	0.01
above 32 month cycles	0.87	0.87	0.81
AGRICULTURE	1982-2008	1982-2000	2001-2008
2-4 month cycles	0.13	0.09	0.42
4-8 month cycles	0.12	0.09	0.32
8-16 month cycles	0.07	0.06	0.16
16-32 month cycles	0.03	0.04	0.04
above 32 month cycles	0.65	0.72	0.06
ADMINISTERED AND REGULATED	1982-2008	1982-2000	2001-2008
2-4 month cycles	0.29	0.29	0.32
4-8 month cycles	0.23	0.23	0.23
8-16 month cycles	0.11	0.11	0.24
16-32 month cycles	0.05	0.05	0.07
above 32 month cycles	0.32	0.32	0.14

Table 2. *Energy decomposition of inflation of the merchandise, services, agriculture, and administered and regulates indices for different periods. The energy of each scale is expressed as fraction of the overall energy.*