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Should monetary policy lean against the wind in a small-open economy? Revisiting the Tinbergen rule*

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Abstract: It has been debated whether monetary policy should lean against the wind, i.e., if central banks should also respond to the build-up of financial imbalances. I contribute to the debate by showing that targeting the two policy objectives with a single instrument is more costly for a small-open economy than for a closed one. To this end, I develop a small-open economy DSGE model with the Bernanke-Gertler-Gilchrist financial accelerator that features financial frictions and monopolistic competition in goods markets. I then estimate this model for Mexico to explore the policy regimes yielding the lowest welfare cost. My main finding is that the Tinbergen rule is alive and well. In addition, my model is useful to gauge macroprudential measures effectiveness when discriminating against foreign liabilities.

Keywords: Monetary policy, Macroprudential policies, Leaning against the wind, Tinbergen rule, Capital controls.

JEL Classification: C51, E32, E44, E52, E58, E61, F41, G21, G28.

Resumen: Se ha debatido si la política monetaria debería suavizar el ciclo crediticio de la economía, es decir, si los bancos centrales deberían también responder a desequilibrios financieros. Este artículo contribuye a este debate mostrando que es más costoso para una economía pequeña y abierta, respecto a una economía cerrada, perseguir dos objetivos con un solo instrumento de política. Para esto se desarrolla un modelo de Equilibrio General Dinámico Estocástico para una economía pequeña y abierta que incorpora el acelerador financiero de Bernanke-Gertler-Gilchrist donde se presentan fallas de mercado: fricciones financieras y poder monopolístico en los mercados de bienes. Después, estima el modelo para México y explora los regímenes de política que generan el menor costo en bienestar para la economía. El principal hallazgo es que la regla de Tinbergen se cumple. Adicionalmente, el modelo es útil para medir la efectividad de las medidas macroprudenciales que discrimina contra deuda externa.

Palabras Clave: Política monetaria, Políticas macroprudenciales, "Leaning against the wind", Regla de Tinbergen, Controles de capital.

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1 Introduction

The debate on whether to use monetary policy to address risks to financial instability, colloquially known as \textit{leaning against the wind}, is a relevant topic of discussion in policy and academic circles, and it was exacerbated in the global financial crisis (GFC) aftermath (Kockerols and Kok, 2019). In particular, it has been argued that some emerging market and developing economies’ (EMDEs) central banks appear to lean against financial imbalances and often pursue financial stability goals in addition to inflation stabilization. So, to achieve their objectives, the policy toolkit of these central banks has complemented interest rate policy with additional instruments such as foreign exchange intervention, capital flow measures, and macroprudential policies.\footnote{Following Smets (2014), macroprudential policies are defined as regulatory policies that aim to prevent or at least contain the build-up of financial imbalances and ensure the stability of the financial system against shocks.} According to Tobias et al., (2020), domestic considerations dominate in small-open advanced economies, and adverse external shocks are usually met with monetary policy easing. On the other hand, many EMDEs’ central banks with inflation-targeting frameworks have continued to rely on macroprudential and capital flow management tools in their monetary policy operations during episodes of volatile capital flows. In this line, while conventional monetary policy maintains its role in counteracting inflation, there are doubts that it is sufficient to guard against the risks of financial instability, especially in EMDEs and small-open economies.\footnote{Some economists point out that near-term price stability is not a sufficient condition for financial stability. For instance, Bailliu, Meh and Zhang (2012) argue that most economies experienced severe recessions in 2008–09 even though they had all been pursuing monetary policies focused on price stability for many years.} As a result, there have been increased calls for the development of macroprudential financial regulation and the incorporation of financial stability considerations into monetary policy analysis.

This paper asks whether, in addition to pursuing the objective of price stability, central banks should also respond to the build-up of financial imbalances, such as those associated with unsustainable credit expansion. To shed light on the issue, I present a dynamic stochastic general equilibrium (DSGE) model with the Bernanke, Gertler, and Gilchrist (1999)—also known as the BGG model—financial accelerator in an open-economy fashion and estimate the...
main structural parameters of the model through Bayesian estimation using Mexican data. Based on these estimates, (i) I conduct simulations under different policy regimes—centered on price stability to another set in which policy-makers also respond to financial imbalances—and analyze the trade-offs between monetary and macroprudential policy rules in mitigating the impact of financial and productivity shocks that trigger capital flows.\textsuperscript{3} Besides, (ii) I search numerically for the optimal policy regime that delivers the lowest welfare cost to the economy. The results indicate that the optimal policy mix is when monetary and macroprudential policies focus on price and financial stability, respectively. In other words, an additional objective for the monetary authority, like the one of financial stability, should not rely on the central bank policy rate as a target. This joins Tinbergen’s rule that from a welfare standpoint, monetary and macroprudential policies work in a complementary way towards the achievement of apparently mutually exclusive objectives and thus reinforce each other.\textsuperscript{4} Also, I show that macroprudential policies help monetary policy stabilize the economy in the face of financial shocks. Nevertheless, macroprudential measures may not help economic stability under various types of shock, particularly a technology shock. Hence, there is a trade-off between financial and macroeconomic stability objectives in the face of a productivity shock.

The main contribution of the present paper is related to the following two experiments. First, considering that in the past, capital flow reversals intensified and caused significant local currency depreciation and consequent inflation in many EMDEs, these economies adopted a variety of policy tools aimed at curbing credit growth (Aoki et al., 2018). Therefore, I extend the framework to see how effective macroprudential measures are when discriminate against foreign liabilities. Specifically, I assess macroprudential measures’ stabilization performances that discriminating against foreign liabilities (capital controls) \textit{versus} broad macroprudential measures. Second, much of the existing literature considers the potential

\textsuperscript{3}These regimes consider: (1) the standard Taylor rule, (2) the augmented Taylor rule, (3) the standard Taylor rule plus macroprudential tool, and (4) the augmented Taylor rule plus macroprudential tool.

\textsuperscript{4}As noted by Tinbergen (1952), the policy-maker cannot intend to hit targets for more objective variables than the number of instruments available. The Tinbergen rule states that achieving the desired targets requires an equal number of independent instruments.
gains from complementing monetary policy rules with macroprudential rules. However, most of these studies analyze closed economies in their frameworks. In this regard, I study the relevance of the Tinbergen rule in a closed economy, and I examine the welfare cost differences between an open economy and a closed economy. Experiment results suggest that broad macroprudential measures are more effective than capital controls, as the latter only bring a shift from foreign debt to domestic debt and hence affect the composition of economic agents’ debt, rather than the total debt volume. On the other hand, Tinbergen’s rule violations entail large welfare costs, but these costs are higher in an open economy than in a closed economy. This happens because, in an economy in which economic agents can access international financial markets, domestic monetary policy is likely to have less impact on financial imbalances since firms can borrow at the foreign interest rate. As a result, financial vulnerabilities can trigger capital outflows and amplify negative shocks on economic activity.

This paper is in line with literature that supports the idea that financial frictions and information problems can make monetary policy-making more complicated than setting a policy rate and allowing the exchange rate to adjust flexibly, as in the Mundell-Fleming framework that abstracts from many real-world imperfections. The Modigliani-Miller (1958) theorem implies that the financial structure is indeterminate and irrelevant to real economic outcomes. Nonetheless, when credit markets are incomplete, characterized by asymmetric information and financial imperfections, the Modigliani-Miller irrelevance theorem no longer applies. In the model developed, I relax the Modigliani-Miller assumptions—there is no complete market’s assumption—through the BGG financial accelerator in which the lender has to pay an auditing cost due to information problems, and the optimal debt contract gives rise to a risk premium. These inefficiencies imply allocations that are not Pareto-efficient, therefore, a constrained social planner or policy-maker who takes the financial market imperfections as given can coordinate private agents’ behavior in a way that generates

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5See, for instance, Basu et al., 2020.
6According to Faia and Monacelli (2007), agency costs have a twofold effect. In the long run, they produce an inefficiently low level of capital and investment, and hence output, since the economy suffers a dead-weight loss associated with the monitoring activity of the lender. In the short run, the presence of an endogenous risk premium (in the sense of BGG) distorts the dynamic allocation of capital and investment. In practice, agency costs act as an implicit, and time-varying, investment tax.
a Pareto improvement. So, why is monetary policy ineffective in achieving price and financial stability simultaneously as a unique policy tool? Why does Tinbergen’s rule hold on in the presented framework? In short, the Tinbergen rule applies because two instruments are needed to tackle two inefficiencies: sticky prices and agency costs. Price rigidities induce a role for monetary policy to affect the real interest rate and correct the associated distortion. Similarly, the presence of financial frictions associated with the cost of monitoring defaulted borrowers suggests the potential role of a macroprudential instrument to reduce this other distortion.

I do not explicitly model systemic risks from first principles due to their complex nature. Instead, following the work of Bailliu, Meh and Zhang (2012), I use deviations in credit growth from its steady state value as a proxy for financial imbalances and propose policy regimes that are simple enough for a monetary authority to implement. Specifically, the use of macroprudential tools is assumed to influence the funding costs of firms directly. A period of excessive credit expansion would trigger the use of macroprudential tools, changes in risk premiums, an increase in firms’ funding costs, and a dampening of investment. So, by design, macroprudential measures could address the procyclicality of financial markets by making it harder to borrow during booms, making the subsequent reversal less dramatic and thus reducing the amplitude of the boom-bust cycle. Hence, macroprudential policies are able to affect aggregate demand and supply as well as financial conditions through cushioning or amplifying the economic cycle by directly affecting the provision of credit. This mechanism is intended to capture the effects of macroprudential tools such as the countercyclical capital buffer, a key measure in the Basel III package. Notwithstanding, it is important to mention that the way I modeled this instrument is relatively simple for a regulatory policy of this kind. Some instruments can be used to reach the objectives of the macroprudential policy in better practical design. Following Lim et al. (2011), the first type is credit-related instruments, including caps on the loan-to-value (LTV) and debt-to-income (DTI) ratios, limits on foreign currency lending, and ceilings on credit growth. The second type is liquidity-related instruments, which involve reserve requirements, limits
to net open currency positions, and controls on maturity mismatches. Finally, the third type includes countercyclical capital requirements and restrictions on profit distribution. Literature addressing this limit that provides more comprehensive tools can be found, for instance, in Gruss and Sgherri (2009), Glocker and Towbin (2012) and Bianchi and Mendoza (2011).\footnote{In an international model with financial frictions, Gruss and Sgherri (2009) study the role of loan-to-value (LTV) limits in reducing credit cycle volatility in a small open economy. The main aim of Glocker and Towbin (2012) is to analyze under which circumstances reserve requirements (as taxes on the banking system) are effective as an additional monetary policy tool to achieve price stability or as a macroprudential tool to achieve financial stability. For their part, Bianchi and Mendoza (2011) analyze the effectiveness of macroprudential taxes to avoid the externalities associated with overborrowing.}

In some economies, the central bank is at the center of the macroprudential arrangement, while, in others, it has no explicit financial stability mandate. Even so, most central banks devote considerable resources to promoting financial stability. In the case of Mexico, macroprudential policy is currently not part of the monetary policy toolkit. However, Mexico had been an early adopter of macroprudential tools (adopted the Basel III capital rules in 2013) and has been widely recognized for its prompt reaction to the 2007-2009 financial turmoil.\footnote{Furthermore, in 2010, Mexican authorities created the Consejo de Estabilidad del Sistema Financiero (CESF), following the recommendations of the G20, which is the institution in charge of coordinating action for the implementation of macroprudential policies, along with cooperation for a permanent assessment of risks.} Cardenas (2015) points out that the resilience of the Mexican banking system and its good financial regulation relative to other countries during the GFC was a product of the lesson learned in the 1994-1995 financial crisis. After the so-called Tequila Crisis in 1995, the Bank of Mexico started to implement prudential regulation, which follows the traditional microprudential approach.\footnote{Among the prudential regulation measures that Mexico implemented in the nineties are regulation of banks’ foreign currency operations, a cap on exposure to related counterparties, caps on interbank exposures, and tighter limits on value at risk for pension-fund portfolios at times of high volatility (Calafell, 2013).}

**Related literature.** Although there is a growing amount of literature that explores how monetary and macroprudential policies might be coordinated,\footnote{Paoletti and Paustian (2017) study how monetary and macroprudential policy should be coordinated to minimize the social costs of macroeconomic fluctuations. They find that when policy-makers act independently, there is a costly tug-of-war between authorities with different objectives and similar instruments. For instance, Angelini et al. (2011) find that in all cases, cooperation between the central bank and the macroprudential authority yields superior outcomes.} there is no consensus
yet on whether monetary policy should take financial stability considerations into account. Several papers suggest that there may be benefits to including financial and credit conditions in monetary policy rules. As discussed in Smets (2014), while the macroprudential policy framework should be the primary tool for maintaining financial stability, monetary policy authorities should also keep an eye on financial stability.

Bailliu, Meh and Zhang (2012), Kannan et al. (2012), Christensen et al. (2011), Quint and Rabanal (2014) and Sámano (2011) consider the potential gains from complementing monetary policy rules with macroprudential rules. In a framework for a closed economy, Bailliu, Meh and Zhang (2012) find that welfare is higher in regimes where policy-makers respond to financial imbalances using the policy rate and/or a macroprudential tool. In Kannan et al. (2012), the authors compare the behavior of their model economy under different policy regimes, assuming that policy-makers have two instruments at their disposal: A nominal short-term interest rate and a macroprudential instrument. They find that including a credit term in the monetary policy reaction function and a macroprudential rule can improve macroeconomic stability in the face of a financial shock but not in the presence of a productivity shock. On the other hand, Christensen et al. (2011) focus mainly on the interaction between monetary policy and countercyclical capital buffers. They find that countercyclical bank leverage regulation can have desirable stabilization properties, particularly when financial shocks are an important economic fluctuation source. In related work, Quint and Rabanal (2014) find that the introduction of a macroprudential rule would help in reducing macroeconomic volatility, improve welfare and partially substitute for the lack of domestic monetary policies. To study the relevance of Tinbergen’s rule, Carrillo et al. (2020) compare the effectiveness of the simple Taylor rule (STR) and augmented Taylor rule (ATR) versus a dual rules regime (DRR) with a Taylor rule and a separate financial policy rule. One of the key results of this paper is that the welfare costs of violating Tinbergen’s rule are large.

For their part, Glocker and Towbin (2012) consider a small open-economy model with sticky prices, financial frictions, and a banking sector that is subject to legal reserve require-
ments as macroprudential instruments. Despite that work and the present paper overlapping in interpreting the Tinbergen rule in their respective results, differences between them ultimately result in different views on policy tools' adequacy for macroeconomic-stabilization. Glocker and Towbin (2012) indicate that reserve requirements can support the price stability objective only if financial frictions are important (in the sense of the BGG financial accelerator) and if the central bank has an explicit financial stability objective that aims to contain fluctuations in credit in addition to output and prices. On the other hand, my results indicate that macroprudential policies help monetary policy stabilize the economy in the face of financial shocks. However, an additional objective for the monetary authority, like financial stability, should not rely on the central bank’s policy rate as a target. Not surprisingly, these differences originated from the objective of the central bank in each study. Glocker and Towbin (2012) assume that the central bank receives an exogenous mandate from the government in the form of a loss function it has to minimize: (i) In the first setting, the central bank aims to minimize a weighted average of output and inflation variability. (ii) In the second setting, the variability of loans enters additionally. Nevertheless, an alternative objective for the central bank would be a welfare criterion, implied by the utility function, instead of an ad hoc loss function. This paper follows that approach for the optimal policy regime that delivers the lowest welfare cost to the economy. In particular, I conduct simulations under the various regimes and rank them using standard compensating, as originally proposed by Lucas (1987). An advantage of such an approach is that the objective function is derived endogenously from the model and does not require additional judgment on what variables to enter. However, it is important to mention that some economists argue that is not obvious whether a central bank should or does try to maximize a household’s welfare (Svensson, 2008; Blanchard, 2009; Glocker and Towbin, 2012).

Why is the analysis of macroprudential policy relevant to public policy? This is because the presence of financial vulnerabilities can amplify negative shocks to economic activity. Consequently, financial crises are more prolonged and more painful when macroprudential tools are not part of the policy toolkit. Nonetheless, research about macroeconomic stability
and the welfare implications of macroprudential tools is limited in the context of small-open economies. Likewise, there are few studies on the macroprudential implications for the Mexican economy; most of them focus on a partial equilibrium analysis that does not consider the effects of monetary, financial, and credit conditions on the real economy. Therefore, a well-designed regulatory framework that relies on a general equilibrium model is necessary to prevent financial imbalances and inflation pressures.

Road map. In Section 2, I present the model. In Section 3, I discuss the data and estimation strategy employed. Using the estimated model, Section 4 discusses the model economy’s performance under the policy regimes regarding key shocks. In Section 5, I compare the performance of the policy regimes using a welfare criterion. Section 6 examines the dynamics of the economy when policy-makers establish macroprudential capital controls, and I explore the welfare cost differences between an open economy and a closed economy. Section 7 contains some concluding remarks.

2 The Model

2.1 A Sticky-Price DSGE Model with Agency Costs

The model presented here is an extension of the dynamic New Keynesian framework modified to allow for financial accelerator effects on investment ‘a la Bernanke to an open-economy context. In this model, financial and credit conditions play a central role in the propagation of cyclical fluctuations due to a financial accelerator effect. The model includes the following types of agents: There are households, entrepreneurs, producers (final goods producers and intermediate goods producers or retailers), capital producers and financial intermediaries. Also, there is a central bank that conducts monetary and macroprudential policies. I describe their respective problems below. In what follows, variables without superscripts refer to the domestic economy, while variables with asterisks indicate the rest-of-the-world unless indicated otherwise.
2.2 Households

There is a continuum of households of length unity. Households live forever and are assumed to be identical, i.e., they have the same preferences and endowments. Each household supplies labor, $0 \leq h_t \leq 1$, consumes a homogeneous consumption good, $c_t$, and participates in domestic and foreign financial markets. In particular, the intertemporal preferences of the households are characterized by:

$$U_0 = E_t \left[ \sum_{t=0}^{\infty} \beta^t \left( \log(c_t) - \theta \frac{h_t^{1+\eta}}{1 + \eta} \right) \right],$$

where $E_t$ is the expectation operator conditional on information available at time $t$, $\beta \in (0, 1)$ is the discount factor, $\eta > 0$ is the inverse elasticity of labor supply and $\theta > 0$ is the weight of leisure in the utility function.\(^{12}\) On the other hand, the period $- t$ household budget constraint equals consumption plus savings with the income:

$$P_t c_t + B_t - ner_t B^*_t = W_t h_t + R_{t-1} B_{t-1} - R^*_{t-1} \Theta_{t-1} ner_t B^*_{t-1} + \Upsilon_t.$$

The households’ income in period $- t$ derives from labor (where $W_t$ is the nominal wage), dividends and transfers received from their ownership of retail firms and from the financial intermediary, $\Upsilon_t$, and returns from previous periods’ holding of financial assets. Household savings can be invested in two types of financial assets: deposits, $B_t$ with a return of $R_t$ in $t$ (I assume the deposit contract is nominal, short-term and non contingent); foreign bonds, $B^*_t$, with a foreign currency return $R^*_t \Theta_t$ in $t$.\(^{13}\) Let $P_t$ and $P^*_t$ be the consumption price index (CPI) and the aggregate price index for foreign country’s consumption goods, respectively.\(^{14}\) Foreign bonds are expressed in foreign currency and, $ner_t$ is the nominal exchange rate

\(^{11}\)The leisure is defined as: $1 - h_t$.

\(^{12}\)This utility function has a long tradition in literature on business cycles; for instance, Unsal (2011), Gruss and Sgherri (2009) and Rubio and Carrasco-Gallego (2014).

\(^{13}\)In other words, households hold their financial wealth in the form of bank deposits, $B_t$, in a financial intermediary that pays the risk-free gross rate $R_t$. On the other hand, as Chari, Kehoe and McGrattan (2002), I assume complete international markets for state-contingent claims. This means that households also have access to state-contingent bonds, $B^*_t$, paying the stochastic gross return $R^*_t \Theta_t$.

\(^{14}\)Foreign inflation and foreign interest rates are exogenously given and follow a stochastic AR(1) process.
The real exchange rate is defined as \( rer_t = ner_t P_t^* / P_t \). On the other hand, \( \Theta_t \) is a risk premium for foreign bonds (liabilities), which is taken as given by the households but is a function of the total indebtedness of the economy, \( \Theta_t = \Theta(B_t^*) \). Similar to the work of Medina and Roldós (2018), I model this function as \( \Theta(B_t^*) = (B_t^*/B^*)^{\rho} \).

The first-order conditions for the representative consumer-maximization problem imply that the following efficiency conditions must hold:

\[
1 = \beta R_t E_t \left[ \frac{c_t}{c_{t+1}} \frac{1}{1 + \pi_{t+1}} \right], \tag{1}
\]

\[
1 = \beta R_t^* \Theta_t E_t \left[ \frac{c_t}{c_{t+1}} \frac{rer_{t+1}}{rer_t} \frac{1}{1 + \pi_{t+1}} \right], \tag{2}
\]

\[
\theta h_t c_t = \frac{W_t}{P_t}. \tag{3}
\]

Equation (1) is the stochastic Euler equation describing the optimal intertemporal consumption pattern, equation (2) is the stochastic Euler equation for foreign bonds, and equation (3) is the intratemporal consumption-leisure efficiency condition. Finally, to rule out the possibility of Ponzi schemes, the present value of financial wealth converges to zero:

\[
\lim_{k \to \infty} E_t \left[ F_{t+k}(B_{t+k} + ner_{t+k} B_{t+k}^*) \right] = 0. \tag{4}
\]

### 2.3 Capital Producers

Competitive capital-producing firms purchase raw output as a material input (i.e., investment goods purchased from final producers) to produce new capital goods. These capital goods are then sold at price \( Q_t \) to entrepreneurs. Capital is managed and rented to firms by a continuum of entrepreneurs, who use their net worth and a bank loan to finance the capital expenditures. In this way, capital producers differentiate their goods according to the needs of entrepreneurs. Let \( i_t \) denote aggregate investment expenditures. The aggregate
capital stock evolves according to:

\[ k_{t+1} = \chi_t i_t + (1 - \delta)k_t. \] (5)

where \( k_t \) is the capital stock, \( \delta \in (0, 1) \) is the depreciation rate, \( \chi_t i_t \) are efficient investment goods and \( \chi_t \) is an investment shock that follows a first-order autoregressive process.\(^{15}\) There are increasing marginal adjustment costs in the production of capital, \( \Phi(i_t/k_t) \), which capture the fact that aggregate investment expenditures of \( i_t \) yield a gross output of new capital goods.\(^{16}\) Following Bailliu, Meh and Zhang (2012), I assume that capital producers are subject to a quadratic capital adjustment cost of the form:

\[ \Phi \left( \frac{i_t}{k_t} \right) = \xi \left( \frac{i_t}{k_t} - \delta \right)^2 k_t. \] (6)

Let \( Q_t \) be the price of a unit of capital in terms of the consumption good. Since the capital-producing technology assumes constant returns to scale, these capital-producing firms earn zero profits in equilibrium. Capital producers maximize their profit:

\[ \max_{\{i_t\}_{t=0}^{\infty}} E_t \left\{ \chi_t Q_t i_t - i_t - \Phi \left( \frac{i_t}{k_t} \right) \right\}, \]

yielding the following first-order condition:

\[ E_t \left\{ \chi_t Q_t - 1 - \xi \left( \frac{i_t}{k_t} - \delta \right) \right\} = 0. \] (7)

### 2.4 Entrepreneurs

To induce the financial accelerator effect, entrepreneurs play a key role in the model. There is a continuum of entrepreneurs indexed by \( j \) in the interval \([0, 1]\). Following BGG model, these individuals are assumed to be risk-neutral and each entrepreneur has a constant probability \(^{15}\)\(^{16}\)

\[ \log(\chi_t) = \rho \log(\chi_{t-1}) + \varepsilon_{\chi,t}, \quad \varepsilon_{\chi,t} \sim i.i.d. N(0, \sigma^2_{\chi}). \]

\[ \Phi(\cdot) \] is increasing and concave and \( \Phi(0) = 0 \). As in Kiyotaki and Moore (1997), the idea is to have asset price variability contribute to volatility in entrepreneurial net worth.
ζ of surviving to the next period.\textsuperscript{17} I assume that the birth rate of entrepreneurs is such that the fraction of agents who are entrepreneurs is constant. Entrepreneurs transform unfinished capital goods into an intermediate good $y_{j,t}$ and sell it to retailers at the price $P_{j,t}$.\textsuperscript{18}

2.4.1 Entrepreneurs’ Optimization Problem

In each period $t$, entrepreneur $j$ acquires physical capital and entrepreneurs who “die” in period $t$ and are not allowed to purchase capital but instead simply consume their accumulated resources and depart the scene. Physical capital acquired in period $t$ is used in combination with hired labor to produce an intermediate good $y_{j,t}$ by a constant-returns to scale Cobb–Douglas production function:

$$y_{j,t} = f(k_{j,t}, h_{j,t}) = z_t k_{j,t}^{\alpha} h_{j,t}^{1-\alpha}, \quad (8)$$

where $k_{j,t}$ is the capital purchased by entrepreneur $j$ in period $t-1$ and $h_{j,t}$ is the labor hired by entrepreneur $j$. $z_t$ is an exogenous technology shock common to all entrepreneurs which follows:

$$\log(z_t) = \rho z \log(z_{t-1}) + \varepsilon_{z,t}, \quad \varepsilon_{z,t} \sim i.i.d. N(0, \sigma^2_z). \quad (9)$$

Entrepreneurs’ optimization problem consists of maximizing profits (revenues from the production process minus production costs). First order conditions imply that production factors are paid according to their marginal productivity and an equation for the expected return on capital. The demand curve for household labor and the demand for capital is given by:

$$\frac{(1 - \alpha) y_{j,t}}{h_{j,t}} = \frac{w_t}{P_{j,t}}, \quad (10)$$

$$\frac{\alpha y_{j,t}}{k_{j,t}} = \frac{r r_t}{P_{j,t}}, \quad (11)$$

\textsuperscript{17}Implying an expected lifetime of $\frac{1}{1 - \frac{1}{\zeta}}$.

\textsuperscript{18}Equivalently, $\frac{1}{P_{j,t}}$ is the gross markup of retail goods over wholesale goods.
where $P_{j,t}$ is the nominal price of the intermediate good and $rr_t$ is the rental rate of capital. For each unit of capital, a successful entrepreneur obtains a nominal payoff equal to the marginal productivity of capital and the price of the undepreciated capital:

$$R_{k,t} = rr_t + (1 - \delta)Q_t.$$  \hspace{1cm} (12)

The expected gross return to holding for a unit of capital from $t$ to $t + 1$ can be written as:

$$E_t [R_{k,t+1}] = E_t \left\{ \frac{P_{j,t+1} \alpha y_{j,t+1} + (1 - \delta)Q_{t+1}}{Q_t} \right\}. \hspace{1cm} (13)$$

### 2.4.2 Capital Funding

Capital purchased at the end of period $t$, $k_{j,t+1}$, is partly financed from the entrepreneur’s net worth, $n_{j,t+1}$, and partly from issuing nominal debt, $L_{j,t}$, through perfectly competitive financial intermediaries:

$$Q_t k_{j,t+1} = n_{j,t} + L_{j,t+1}. \hspace{1cm} (14)$$

### 2.4.3 Financial Market Imperfection

The productivity of each entrepreneur is subject to an idiosyncratic shock not observed by the bank. This creates agency problems, so interest charged by the banking sector is subject to an external premium over the risk-free rate paid by banks on households’ deposits $R_t$. The financial market imperfection arises due to asymmetric information between the borrower and the lender.\(^{20}\)

The lender has to pay an auditing cost to observe the output. With costly monitoring, the optimal debt contract that gives rise to a risk premium associated with external funds is

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\(^{19}\) Also, you can see this equation as the average rate of return of capital.

\(^{20}\) To endogenously motivate the existence of an external finance premium, Bernanke, Gertler and Gilchrist (1999) postulate a simple agency problem that introduces a conflict of interest between a borrower and his respective lenders in this model.
one in which monitoring only takes place in the case of default. Let $S_t$ be the risk premium, also called the default premium on domestic borrowing. The risk premium is defined as the ratio of the entrepreneur’s cost of external funds to the cost of internal funds:

$$S_t = E_t \left[ \frac{R_{k,t+1}}{R_t \frac{P_t}{P_{t+1}}} \right],$$

where $E_t [R_{k,t+1}]$ is the expected rate of return of capital, which is equal to the expected cost of external funds in equilibrium, and $E_t \left[ R_t \frac{P_t}{P_{t+1}} \right]$ is the cost of internal funds (or the same as the real interest rate). The magnitude of this premium varies with the leverage of the entrepreneurs, linking the terms of credit to balance sheet conditions. In this way, the optimal contract implies that the external finance premium, $s(\cdot)$, increases with leverage (i.e., depends on the entrepreneur’s balance sheet position), and thus can be characterized at the aggregate level by the following reduced-form equation:

$$S_t = s \left( \frac{Q_t k_{t+1}}{n_{t+1}} \right),$$

where $s'(\cdot) > 0$, $s(1) = 1$ and $\kappa_t = Q_t k_{t+1}/n_{t+1}$ is the leverage. For the numerical analysis, I suppose that $s(\cdot) = (\cdot)^\psi$, where $\psi > 0$. With credit-market frictions present, this standard model of lending with asymmetric information implies that the external finance premium depends inversely on borrowers’ net worth. This inverse relationship arises because, when borrowers have little wealth to contribute to project financing, the potential divergence of interests between the borrower and the suppliers of external funds is greater, implying increased agency costs; in equilibrium, lenders must be compensated for higher agency costs by a larger premium. To the extent that borrowers’ net worth is procyclical, the external finance premium will be countercyclical, enhancing the swings in borrowing and thus in investment, spending and production.
2.4.4 The Role of Net Worth

Entrepreneurs’ net worth comes from two sources: Profits accumulated from previous capital investment and income from supplying labor. As stressed in the literature, entrepreneurs’ net worth plays a critical role in the dynamics of the model. Given such, fluctuations in borrowers’ net worth can amplify and propagate exogenous shocks. Net worth matters because a borrower’s financial position is a key determinant of their cost of external finance. Higher levels of net worth allow for increased self-financing, mitigating the agency problems associated with external finance and reducing the external finance premium faced by the entrepreneur in equilibrium.

Let \( V_t \) be entrepreneurial equity (wealth accumulated by entrepreneurs), and \( W_{e,t} \) be the entrepreneurial wage. Then, aggregate entrepreneurial net worth at the end of period \( t \), \( n_{t+1} \), is given by:

\[
\begin{align*}
\text{(17)} \quad n_{t+1} &= \zeta V_t + W_{e,t},
\end{align*}
\]

where \( \zeta V_t \) is the equity held by entrepreneurs at \( t - 1 \) who are still in business at \( t \). Entrepreneurial equity equals gross earnings on holdings of equity from \( t - 1 \) to \( t \) less repayment of borrowings. Furthermore, this equity may be highly sensitive to unexpected shifts in asset prices, especially if firms are leveraged. In the general equilibrium, there is a multiplier effect: An unanticipated rise in asset prices raises net worth more than proportionately, which stimulates investment demand and, in turn, raises asset prices even further, and so on. This phenomenon will become evident in the model simulations ahead. Suppose that these agents do not supply labor, thus, \( W_{e,t} = 0 \). Thus, the aggregate net worth of entrepreneurs at the end of period \( t \), \( n_{t+1} \), is the sum of equity held by entrepreneurs surviving from period \( t - 1 \):

\[\text{Bernanke, Gertler and Gilchrist (1999) prove that an unexpected one percent change in the ex-post return to capital leads to a percentage change in entrepreneurial equity equal to the ratio of gross holdings of capital to equity. To the extent that entrepreneurs are leveraged, this ratio exceeds unity, implying a magnification effect of unexpected asset returns on entrepreneurial equity. The authors also argue that the key point here is that unexpected movements in asset prices, which are likely the largest source of unexpected movements in gross returns, can have a substantial effect on firms’ financial positions.}\]
\[ n_{t+1} = \zeta \left\{ R_{k,t} Q_{t-1} k_t - E_{t-1} [R_{k,t}] (Q_{t-1} k_t - n_t) \right\}. \quad (18) \]

The last equation suggests that the difference between the realized rate of return on capital in period \( t \), \( R_{k,t} \), and the expected rate of return on capital in the previous period, \( E_{t-1} R_{k,t} \), is the main source of change in entrepreneurial net worth. Using the risk premium relationship, we obtain

\[ n_{t+1} = \zeta \left\{ R_{k,t} Q_{t-1} k_t - S_{t-1} R_{t-1} E_{t-1} \left[ \frac{P_{t-1}}{P_t} (Q_{t-1} k_t - n_t) \right] \right\}. \quad (19) \]

The latter equation describes the link between the external finance premium and the net worth of potential borrowers. Finally, entrepreneurs going out of business will consume their residual equity:

\[ c_{e,t} = (1 - \zeta) n_{t+1}, \quad (20) \]

where \( c_{e,t} \) is the aggregate consumption of the entrepreneurs who exit in period \( t \).

### 2.5 Financial Intermediaries

Savers cannot lend to borrowers directly. Instead, financial intermediaries take deposits from savers, \( B_t \), and lend them to borrowers, charging a spread that depends on the borrowers’ net worth. In this framework, macroprudential measures entail an increase in financial intermediaries’ lending costs, which are then passed on to borrowers in the form of higher interest rates. Similar to Unsal (2011), I refer to the increased lending rates brought about by macroprudential measures as the regulation premium \( \tau_t \), which is a function of nominal credit growth. In the presence of macroprudential regulations, the spread between the lending rate and the policy rate is affected by both the default premium and the regulation premium. Hence, the lending costs for borrowing become
\[ S_t = f_t s(\cdot) \tau_t = f_t \left( \frac{Q_t k_{t+1}}{n_{t+1}} \right)^\psi \tau_t, \]  

where:

1. \( f_t \) is the financial shock. As the authors argue, changes in \( f_t \) can be thought of as a reduction in the margin banks charge over funding costs, caused by an increase in competition and a quest for market share, or by a reduction in perceived lending risk. These financial shocks follow a stochastic AR(1) process:

\[ \log(f_t) = \rho_f \log(f_{t-1}) + \varepsilon_{f,t}, \varepsilon_{f,t} \sim i.i.d.N(0, \sigma_f^2). \]  

2. \( s(\cdot) \) is an increasing function of the leverage of borrowers.

3. \( \tau_t \) is a macroprudential instrument that allows the policy-makers to affect market rates by imposing additional capital requirements or loan provisions whenever credit growth is above its steady state value. Thus, I modeled the macroprudential tool as an exogenous component of the external finance premium.

### 2.6 Goods Producers

#### 2.6.1 Intermediate Goods Producers: Retailers

The introduction of retailers allows for the presence of price rigidities. I assume that monopolistic competition occurs at the retail level.\(^{22}\) There is a continuum of monopolistically competitive retailers of measure 1 indexed by \( i \) who buy domestic and foreign intermediate goods. Entrepreneurs sell their output \( y_{j,t} \) to retailers at price \( P_{j,t} \). Also, retailers buy goods from foreign economy \( y^*_i,t \), at \( P^*_i ner_t \). Then, they produce an intermediate composite good, \( da_{i,t} \), and resell it to final producers at price \( P_{i,t} \). I assume that profits from retail activity

\(^{22}\)The monopoly power of retailers provides the source of nominal stickiness in the economy; otherwise, retailers play no role.
are rebated in a lump-sum to households. Each intermediate composite producer has the same technology:

\[ da_{i,t} = \left[ (1 - \alpha_d)^{1/\theta_d} (y_{j,t})^{1-1/\theta_d} + \alpha_d \left( (y_{i,t}^*)^{1-1/\theta_d} \right) \right]^{\theta_d-1} , \] (23)

where \( \alpha_d \) is the share of foreign goods and \( \theta_d \) is the elasticity of substitution between domestic and foreign goods in the composite good; \( y_{j,t} \) and \( y_{i,t}^* \) are the amount of domestic and foreign goods, respectively, used by the retailer \( i \). The cost minimization implies the following:

\[ \frac{y_{j,t}}{y_{i,t}^*} = \left[ \frac{1 - \alpha_d}{\alpha_d} \right] \left[ \frac{rer_t}{P_{j,t}/P_t} \right]^{\theta_d} . \] (24)

Following Medina and Roldós (2018), in an open economy, the real marginal cost is defined as

\[ mc_t = \left[ (1 - \alpha_d) \left( \frac{P_{j,t}}{P_t} \right)^{1-\theta_d} + \alpha_d \left( \frac{ner_t P_t^*}{P_t} \right)^{1-\theta_d} \right]^{1/\theta_d}, \forall i, \forall t, \] (25)

which, is the same for all intermediate goods producers, because they face the same prices of domestic and foreign goods and their technology is constant return to scale. For that reason, we can obtain:

\[ \int_0^1 y_{j,t} d\alpha_t = \frac{y_t - x_t}{y_t^*} = \left[ \frac{1 - \alpha_d}{\alpha_d} \right] \left[ \frac{rer_t}{P_{j,t}/P_t} \right]^{\theta_d} , \] (26)

which means that the total demand for domestic goods is composed by the demand of intermediate composite producers and exports of domestically produced goods, \( x_t \). Note that \( y_t^* \) represents the imports of foreign goods.

Each retailer \( i \) sells retail good \( da_{i,t} \) at price \( P_{i,t} \). To introduce price inertia, the retailer is free to change its price in a given period only with probability \( 1 - \nu \), following Calvo (1983). Thus, in each period, a fraction \( 1 - \nu \) of retailers reset their prices, while the remaining retailers keep their prices fixed. Each retailer selects its price to maximize its expected real
total profit over the periods during which its price remains fixed subject to future demand:

\[
\max_{\{P_i,t\}_{t=0}^\infty} \mathbb{E}_t \left\{ \sum_{S=0}^{\infty} \nu^S \Delta S_{t+1} \left[ \frac{P_{i,t+1} - mc_{t+S} d_{i,t+1}}{P_{t+1}} \right] \right\}
\]

\[s.t. \quad d_{i,t+1} = \left( \frac{P_{i,t+1}}{P_{t+1}} \right)^{-\varepsilon} d_{i,t+1}.
\]

where \(\Delta_{t,S} \equiv \beta^S \frac{c_t}{c_{t+1}}\) is the stochastic discount factor and \(mc_t\) is the real marginal cost. Let \(\hat{P}_t\) be the optimal price chosen by all firms adjusting at time \(t\). The first-order condition is as follows:\(^{23}\)

\[
\hat{P}_{i,t} = \frac{\varepsilon}{\varepsilon - 1} \left\{ \frac{\mathbb{E}_t \sum_{S=0}^{\infty} \nu^S \Delta S_{t+S} mc_{t+S} d_{t+S} \left[ \frac{1}{P_{t+S}} \right]^{-\varepsilon}}{\mathbb{E}_t \sum_{S=0}^{\infty} \nu^S \Delta S_{t+S} d_{t+S} \left[ \frac{1}{P_{t+S}} \right]^{1-\varepsilon}} \right\}.
\]

Roughly speaking, the retailer sets its price so that the expected discounted marginal revenue equals the discounted marginal cost. Given that a fraction \(\nu\) of retailers do not change their price in period \(t\), the aggregate price evolves according to

\[
P_t = \left( \nu P_{t-1}^{1-\varepsilon} + (1 - \nu)(\hat{P}_{i,t})^{1-\varepsilon} \right)^{1/\varepsilon}.
\]

### 2.6.2 Final Goods Producers

There is a continuum of final goods producers that operate under perfect competition and flexible prices. Total domestic demand, \(da_t\), is the following composite of individual retail goods:

\[
da_t = \left[ \int_0^1 (da_{i,t})^{\varepsilon-1} di \right]^{\frac{1}{\varepsilon-1}}.
\]

where \(d_{i,t}\) is the quantity of output sold by retailer \(i\), and \(\varepsilon > 1\) is the intermediate-good elasticity of substitution. Final goods producers purchase intermediate goods and aggregate

\(^{23}\)See Aoki, Proudman and Vlieghe (2004) and Rubio and Carrasco-Gallego (2014) for more details.
them according to the last equation; hence, the profit maximization is

$$\max_{\{da_{i,t}\}_{t=0}^{\infty}} P_t da_t - \int_0^1 P_{i,t} da_{i,t} d_i$$

s.t. $$da_t = \left[ \int_0^1 da_{i,t} \right]^{\frac{1}{1-\varepsilon}}$$,

where $$P_{i,t}$$ is the price of the intermediate composite good $$i$$. This problem delivers the following demand for individual intermediate goods:

$$da_{i,t} = \left( \frac{P_{i,t}}{P_t} \right)^{-\varepsilon} da_t, \quad \forall i,$$  \quad (30)

where the aggregate price level of domestic demand is given by imposing the usual zero-profit condition:

$$P_t = \left[ \int_0^1 P_{i,t}^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}}.$$  \quad (31)

### 2.7 Market Clearing

In equilibrium, household deposits in financial intermediaries are equal to the total debt held by the entrepreneurs:

$$B_t = L_t, \quad \forall t.$$  \quad (32)

Total demand for final goods is given by

$$da_t = c_t + c_{e,t} + i_t + \Phi \left( \frac{i_t}{k_t} \right), \quad \forall t.$$  \quad (33)

In this model, price stickiness induces price dispersion across final goods, and this price dispersion is inefficient and causes output loss. Thus, when aggregating, some adjustment needs to be made to take this inefficiency into account. Consider the equilibrium condition at the firm level:

$$F(k_{j,t}, h_{j,t}) = \left[ c_t + c_{e,t} + i_t + \Phi \left( \frac{i_t}{k_t} \right) \right] \left[ \frac{P_{i,t}}{P_t} \right]^{-\varepsilon}.$$  \quad (34)
Integrating over all firms yields the following:

$$F(k_t, h_t) = \left[ c_t + c_{e,t} + i_t + \frac{\xi}{2} \left( \frac{i_t}{k_t} - \delta \right)^2 k_t \right] \int_0^1 \left[ \frac{P_{i,t}}{P_t} \right]^{-\varepsilon}.$$  \hfill (35)

Let $$\Gamma_t = \int_0^1 \frac{P_{i,t}}{P_t} dj > 1$$ be the inefficiency attributed to price dispersion. Using the properties of Calvo’s pricing mechanism, Schmitt-Grohe and Uribe (2006) show that

$$\Gamma_t = (1 - \nu) \left[ \frac{\hat{P}_t}{P_t} \right]^{-\varepsilon} + \nu \Pi_t^{\varepsilon} \Gamma_{t-1}. \quad \hfill (36)$$

Thus, the resource constraint for final goods is

$$da_t = \left[ c_t + c_{e,t} + i_t + \frac{\xi}{2} \left( \frac{i_t}{k_t} - \delta \right)^2 k_t \right] \Gamma_t. \quad \hfill (37)$$

The relationship between total domestic demand and total supply of final goods is given by

$$da_t \Gamma_t = \omega a_t, \ \forall t, \quad \hfill (38)$$

where $$\omega a_t$$ is the aggregate supply of the composite goods, defined as

$$\omega a_t = \left[ (1 - \alpha_d)^{1/\theta_d} (y_t - x_t)^{1-1/\theta_d} + (\alpha_d)^{1/\theta_d} (y_t^*)^{1-1/\theta_d} \right] \frac{\theta_d}{\theta_d - 1}.$$ \hfill (39)

Finally, the balance-of-payments identity implies the following:\footnote{Current Account Balance + Financial Account Balance + Capital Account Balance = 0.}

$$rer_t B_t^* = R_t^* \Theta_{t-1} rer_t \frac{B_{t-1}^*}{(1 + \pi_t^*)} - \frac{P_{i,t}}{P_t} x_t + rer_t y_t^*,$$ \hfill (40)

where $$B_t^*$$ is the stock of foreign debt of the economy, $$R_t^*$$ is the gross foreign interest rate and $$(1 + \pi_t^*)$$ is the foreign inflation rate.\footnote{Both $$R_t^*$$ and $$(1 + \pi_t^*)$$ follow an AR(1) stochastic process.} The foreign demand for exports is modeled as follows:
\[ x_t = \left[ \frac{r_{rt}^t}{P_{jt}/P_t} \right]^{\theta^*}, \]  

(41)

where \( \theta^* \) is the price elasticity of the foreign demand for domestic goods.

2.8 Policy Regimes: Monetary Policy and Macropurudential Rules

2.8.1 Regime 1 (R1): Standard Taylor Rule

The baseline policy regime is a standard Taylor rule with interest rate smoothing, a standard way to characterize monetary policy under an inflation-targeting regime. According to the Taylor rule, the central bank adjusts the nominal interest rate, \( R_t \), in response to deviations in inflation, \( \Pi_t \) and \( y_t \), from their steady state values, \( \Pi \) and \( Y \). It is assumed that the central bank smooths interest rates, adjusting them gradually to the desired value. The baseline policy rule thus takes the following form:

\[
\frac{R_t}{R} = \left[ \left( \frac{\Pi_t}{\Pi} \right)^{\phi_\Pi} \left( \frac{y_t}{Y} \right)^{\phi_y} \right]^{1-\phi_r} \left[ \frac{R_{t-1}}{R} \right]^{\phi_r} e_t, \tag{42}
\]

where \( \{\phi_\Pi\} \in (1, \infty) \), \( \{\phi_y\} \in (0,\infty) \), \( \{\phi_r\} \in [0,1] \) and \( e_t \) is an exogenous monetary shock that follows \( \log(e_t) = \rho_e \log(e_{t-1}) + \varepsilon_{e,t} \), \( \varepsilon_{e,t} \sim i.i.d. N(0,\sigma_e^2) \).

2.8.2 Regime 2 (R2): Augmented Taylor Rule

The second policy regime is an augmented Taylor rule in which the baseline policy rule is augmented to allow the policy interest rate to also react to changes in nominal credit. In other words, the central bank can also use the policy rate to lean against the build-up of emerging financial imbalances. Thus, the baseline policy rule is augmented to allow the policy interest rate to also react to deviations in credit growth from its steady state value as follows:

\[
\frac{R_t}{R} = \left[ \left( \frac{\Pi_t}{\Pi} \right)^{\phi_\Pi} \left( \frac{y_t}{Y} \right)^{\phi_y} \left( \frac{c_{gt}}{cg} \right)^{\phi_c} \right]^{1-\phi_r} \left[ \frac{R_{t-1}}{R} \right]^{\phi_r} e_t, \tag{43}
\]
where \( \{\phi_c\} \in (-\infty, \infty) \), and
\[
cg_t = \frac{B_t + B^*_t}{B_{t-1} + B^*_{t-1}}
\]
is the growth rate of nominal credit; \( \phi_c \) is the policy coefficient chosen by the central bank that captures the extent to which it responds to deviations in credit growth \( cgt \). Hence, the second regime is implemented as a Taylor-type rule in which monetary policy reacts to the growth rate of nominal credit (I use deviations of credit growth from its steady state as the variable that triggers an interest rate response from the central bank). In this definition of broad macroprudential policy, it is implicit that the policy objective is defined in terms of aggregate credit activity.

### 2.8.3 Regime 3 (R3): Macroprudential Regime with a Standard Taylor Rule

The third regime combines a macroprudential rule with a standard Taylor rule. In this regime, it is assumed that policy-makers have both interest rates and the macroprudential instrument at their disposal to stabilize the macroeconomy. The macroprudential rule specifies the reaction of a macroprudential instrument to lagged nominal credit changes. Modeling the macroprudential tool as an exogenous component of the external finance premium results in the following:

\[
S_t = f_t S \left( \frac{Q_t K_{t+1}}{N_{t+1}} \right) \tau_t = f_t \left( \frac{Q_t K_{t+1}}{N_{t+1}} \right)^{\psi} \left( \frac{cgt}{cg} \right)^{\rho_\tau}, \tag{45}
\]

where \( \tau_t = \left( \frac{cgt}{cg} \right)^{\rho_\tau} \) and \( \rho_\tau \) is the policy coefficient chosen by the policy-makers, which may or may not be the central bank. Thus, in this model, the use of the macroprudential tool is triggered by signs of emerging financial imbalances (as proxied by deviations in credit growth from its steady state value when \( \tau_t > 1 \)) and is assumed to have a direct influence on the funding costs of firms (via the external finance premium). Thus, \( \tau_t \) is a macroprudential instrument that allows the policy-makers to affect market rates by imposing additional capital requirements or loan provisions whenever credit growth is above its steady state value (I modeled the macroprudential tool as an exogenous component of the external finance
2.8.4 Regime 4 (R4): Macroprudential Regime with the Augmented Taylor Rule

The fourth regime combines a macroprudential instrument with the augmented Taylor rule. In this regime, it is assumed that policy-makers have both interest rates and the macroprudential instrument at their disposal to stabilize the macroeconomy. To help the reader follow the analytical derivations, Table 1 defines all regimes with their respective policy.

<table>
<thead>
<tr>
<th>Regime</th>
<th>Monetary Policy Tool</th>
<th>Macroprudential Policy Tool</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>( \frac{R_t}{R_t} = \left( \frac{\Pi_t}{\Pi_t} \right)^{\phi_\pi} \left( \frac{y_t}{Y_t} \right)^{\phi_y} \left( \frac{cg_t}{cg} \right)^{\phi_c} ) ( 1-\phi_r \left( \frac{R_t}{R} \right)^{\phi_r} e_t, \text{if } \phi_c = 0 )</td>
<td>( S_t = f_t \left( \frac{Q_t K_{t+1}}{N_t + 1} \right) \left( \frac{cg_t}{cg} \right)^{\rho_r}, \text{if } \rho_r = 0 )</td>
</tr>
<tr>
<td>R2</td>
<td>( \frac{R_t}{R_t} = \left( \frac{\Pi_t}{\Pi_t} \right)^{\phi_\pi} \left( \frac{y_t}{Y_t} \right)^{\phi_y} \left( \frac{cg_t}{cg} \right)^{\phi_c} ) ( 1-\phi_r \left( \frac{R_t}{R} \right)^{\phi_r} e_t, \text{if } \phi_c &gt; 0 )</td>
<td>( S_t = f_t \left( \frac{Q_t K_{t+1}}{N_t + 1} \right) \left( \frac{cg_t}{cg} \right)^{\rho_r}, \text{if } \rho_r = 0 )</td>
</tr>
<tr>
<td>R3</td>
<td>( \frac{R_t}{R_t} = \left( \frac{\Pi_t}{\Pi_t} \right)^{\phi_\pi} \left( \frac{y_t}{Y_t} \right)^{\phi_y} \left( \frac{cg_t}{cg} \right)^{\phi_c} ) ( 1-\phi_r \left( \frac{R_t}{R} \right)^{\phi_r} e_t, \text{if } \phi_c = 0 )</td>
<td>( S_t = f_t \left( \frac{Q_t K_{t+1}}{N_t + 1} \right) \left( \frac{cg_t}{cg} \right)^{\rho_r}, \text{if } \rho_r &gt; 0 )</td>
</tr>
<tr>
<td>R4</td>
<td>( \frac{R_t}{R_t} = \left( \frac{\Pi_t}{\Pi_t} \right)^{\phi_\pi} \left( \frac{y_t}{Y_t} \right)^{\phi_y} \left( \frac{cg_t}{cg} \right)^{\phi_c} ) ( 1-\phi_r \left( \frac{R_t}{R} \right)^{\phi_r} e_t, \text{if } \phi_c &gt; 0 )</td>
<td>( S_t = f_t \left( \frac{Q_t K_{t+1}}{N_t + 1} \right) \left( \frac{cg_t}{cg} \right)^{\rho_r}, \text{if } \rho_r &gt; 0 )</td>
</tr>
</tbody>
</table>

Where: \( \{ \phi_\pi \} \in (1, \infty), \{ \phi_y \} \in (0, \infty), \{ \phi_r \} \in [0, 1] \) and \( \{ \phi_c \} \in (-\infty, \infty) \), R1: Standard Taylor Rule; R2: Augmented Taylor Rule; R3: Macroprudential Regime with a Standard Taylor Rule; R4: Macroprudential Regime with the Augmented Taylor Rule.

### 3 Quantitative Analysis

In this section, I use Bayesian methods to estimate the model’s parameters without macroprudential policy, which means under the R1 regime. I only add macroprudential policy to the model after the estimation is complete. The emphasis on financial factors in this paper leads me to consider the estimation of several quantities that are important to identify various shocks given the data. I estimate the model using Mexican quarterly data from 2000-Q1 to 2019-Q2. The model allows for six shocks. Following usual practice, I use as many shocks as observable variables. The observables are consumption, federal funds rate,
The interbank interest rate of equilibrium (TIIE, by its Spanish initials) and effective exchange rate were important to consumer wealth and confidence (web link: https://fred.stlouisfed.org/series/FEDFUNDS). which is the rate banks charge their customers with higher credit ratings. Additionally, the federal funds rate is the central interest rate in the U.S. financial market. It influences other interest rates such as the prime rate, which is the rate banks charge their customers with higher credit ratings. Additionally, the federal funds rate indirectly influences longer-term interest rates such as mortgages, loans, and savings, all of which are very important to consumer wealth and confidence (web link: https://fred.stlouisfed.org/series/FEDFUNDS). The interbank interest rate of equilibrium (TIIE, by its Spanish initials) and effective exchange rate were obtained from the Bank of Mexico.

Because there is not much literature regarding the parameters driving the macro-financial dynamics in Mexico, I focus my estimation on these parameters, and I calibrate the others using values established in the literature (Table 3 reports the choice of parameter values from

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26Gross domestic product (GDP), consumption and inflation were obtained from the Instituto Nacional de Estadística y Geografía (INEGI). The GDP was obtained by removing the public expenditure component of the national accounts because in the model there is no government. Effective federal funds rate was obtained from the Federal Reserve System. The federal funds rate is the interest rate at which depository institutions trade federal funds (balances held at Federal Reserve Banks) with each other overnight. The federal funds rate is the central interest rate in the U.S. financial market. It influences other interest rates such as the prime rate, which is the rate banks charge their customers with higher credit ratings. Additionally, the federal funds rate indirectly influences longer-term interest rates such as mortgages, loans, and savings, all of which are very important to consumer wealth and confidence (web link: https://fred.stlouisfed.org/series/FEDFUNDS). The interbank interest rate of equilibrium (TIIE, by its Spanish initials) and effective exchange rate were obtained from the Bank of Mexico.
Table 2: Summary statistics of the data set (period: 2000Q1-2019Q2).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Median</th>
<th>Maximum</th>
<th>Minimum</th>
<th>Std. Dev.</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption, percentage</td>
<td>0.021</td>
<td>0.026</td>
<td>0.055</td>
<td>-0.105</td>
<td>0.025</td>
<td>78</td>
</tr>
<tr>
<td>Federal funds rate</td>
<td>1.780</td>
<td>1.085</td>
<td>6.520</td>
<td>0.073</td>
<td>1.945</td>
<td>78</td>
</tr>
<tr>
<td>Effective nominal exchange rate</td>
<td>13.051</td>
<td>12.433</td>
<td>20.327</td>
<td>9.113</td>
<td>3.227</td>
<td>78</td>
</tr>
<tr>
<td>GDP, percentage</td>
<td>0.019</td>
<td>0.025</td>
<td>0.070</td>
<td>-0.089</td>
<td>0.024</td>
<td>78</td>
</tr>
<tr>
<td>Inflation, percentage</td>
<td>0.043</td>
<td>0.041</td>
<td>0.074</td>
<td>0.022</td>
<td>0.010</td>
<td>78</td>
</tr>
<tr>
<td>Interbank interest rate</td>
<td>7.199</td>
<td>7.311</td>
<td>18.105</td>
<td>3.293</td>
<td>3.379</td>
<td>78</td>
</tr>
</tbody>
</table>

Source: Banco de Mexico, INEGI and Board of Governors of the Federal Reserve System (US).
Consumption and GDP are in annual percentage change. N refers to the number of observations.

I begin with the conventional parameters. Following the work of Fernández and Meza (2011) and Aguiar and Gopinath (2007), the discount factor $\beta$ is set at 0.987, the inverse elasticity of labor supply $\eta$ at 1, and the weight on leisure in the utility function $\theta$ at 1.4. The quarterly depreciation rate $\delta$ and the capital share in production $\alpha$ are set at 0.03 and 1/3, respectively, consistent with Aguiar and Gopinath (2007). Following Bailliu, Meh and Zhang (2012), the intermediate-goods elasticity of substitution is taken to be 6. The survival rate of entrepreneurs is set at 0.9728 as in the work of Bernanke, Gertler and Gilchrist (1999) which implies that the average working life for entrepreneurs is 36 quarters. According to Medina and Roldós (2018), the share of foreign goods in the final goods composite is 30% ($\alpha_d = 0.30$), while the elasticity of substitution between home and foreign goods is less than one ($\theta_d = 0.5$) and the elasticity of foreign supply of funds $\varrho = 0.001$. These parameters are calibrated to resemble a prototypical emerging market economy such as Mexico.

Policy-makers have two instruments at their disposal to offset the effects of financial imbalances: the policy rate and the macroprudential tool. I set $\phi_c = 0.5$, where $\phi_c$ is the policy coefficient chosen by the central bank that captures the extent to which it responds to deviations in credit growth in the Taylor rule (i.e., the augmented Taylor rule). On the other hand, the policy coefficient that weighs the regulation premium $\rho_r$, which also responds to deviations in credit growth, is set at 1.25. These values are set for illustrative purposes. In Section 4 (simulation results), I set the coefficient values for $\phi_c$ and $\rho_r$ exogenously, as they are meant to highlight the differences across rules. However, in Section 5, I set these coefficient values endogenously to find the coefficient values for the optimal rules that maximize the
Table 3: Parameter calibration.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Definition</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>Discount factor</td>
<td>0.987</td>
</tr>
<tr>
<td>( \eta )</td>
<td>Inverse elasticity of labor supply</td>
<td>1</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Weight on leisure in the utility function</td>
<td>1.4</td>
</tr>
<tr>
<td>( \varrho )</td>
<td>Elasticity of foreign supply of funds</td>
<td>0.001</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Capital depreciation rate</td>
<td>0.03</td>
</tr>
<tr>
<td>( \zeta )</td>
<td>Survival rate of entrepreneurs</td>
<td>0.9728</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Capital share in production function</td>
<td>1/3</td>
</tr>
<tr>
<td>( \Pi )</td>
<td>Gross steady state inflation rate</td>
<td>3</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>Intermediate-goods elasticity of substitution</td>
<td>6</td>
</tr>
<tr>
<td>( \rho_r )</td>
<td>Policy coefficient in the macroprudential tools</td>
<td>1.25</td>
</tr>
<tr>
<td>( \phi_c )</td>
<td>Policy coefficient in the augmented Taylor rule</td>
<td>0.5</td>
</tr>
<tr>
<td>( \alpha_d )</td>
<td>Share of foreign goods</td>
<td>0.30</td>
</tr>
<tr>
<td>( \theta^* )</td>
<td>Price elasticity of the foreign demand for domestic goods</td>
<td>0.5</td>
</tr>
<tr>
<td>( \theta_d )</td>
<td>Elasticity of substitution between domestic and foreign goods in the composite good</td>
<td>0.5</td>
</tr>
</tbody>
</table>

\( \rho_r \) and \( \phi_c \) are chosen by the policy-makers.

welfare of the economy.

The remaining parameters are estimated using a Bayesian approach.\(^{27}\) Figure 2 and Table 4 show the prior and posterior distributions for the model’s parameters. Additionally, Table 4 reports the mode, the mean and the 5th and 90th percentiles of the posterior distribution of the parameters. I assume that all parameters are independent \textit{a priori}. For the monetary policy rule, I set the prior of the reaction on inflation \( \phi_r \) to have a gamma distribution with a mean of 1.60 and a standard deviation of 0.10. The estimated value of this coefficient which captures the response of monetary policy to a deviation of inflation from its steady state, is 1.6192. The coefficient of the reaction on the output gap is assumed to have a normal distribution of mean 0.1 and a standard deviation of 0.05. The estimated value for \( \phi_y \) is close to zero, implying that the monetary policy does not react very strongly to the output gap. The interest rate smoothing parameter \( \phi_r \) is assumed to have a beta distribution of mean 0.9 and a standard deviation of 0.05. The estimated interest rate smoothing parameter is 0.7667. In sum, the coefficients on the Taylor rule suggest a strong response to inflation.

\(^{27}\)I use Dynare to estimate the model through the Metropolis–Hastings algorithm to perform the simulations.
fluctuations in Mexico, a null response to real GDP growth and a high degree of interest rate inertia.

Table 4: Estimation results.

<table>
<thead>
<tr>
<th>Coef.</th>
<th>Density</th>
<th>Prior Mean</th>
<th>Str. Dev.</th>
<th>Posterior Mean</th>
<th>Mode</th>
<th>5%</th>
<th>90%</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\psi)</td>
<td>N</td>
<td>0.05</td>
<td>0.0020</td>
<td>0.0481</td>
<td>0.0476</td>
<td>0.0451</td>
<td>0.0508</td>
</tr>
<tr>
<td>(\xi)</td>
<td>N</td>
<td>0.250</td>
<td>0.0020</td>
<td>0.2605</td>
<td>0.2608</td>
<td>0.2586</td>
<td>0.2643</td>
</tr>
<tr>
<td>(\phi_r)</td>
<td>B</td>
<td>0.9</td>
<td>0.05</td>
<td>0.7667</td>
<td>0.7537</td>
<td>0.7052</td>
<td>0.7956</td>
</tr>
<tr>
<td>(\phi_\pi)</td>
<td>G</td>
<td>1.60</td>
<td>0.10</td>
<td>1.6192</td>
<td>1.6075</td>
<td>1.5060</td>
<td>1.7539</td>
</tr>
<tr>
<td>(\phi_y)</td>
<td>N</td>
<td>0.1</td>
<td>0.05</td>
<td>0.0883</td>
<td>0.0876</td>
<td>0.0434</td>
<td>0.1307</td>
</tr>
<tr>
<td>(\nu)</td>
<td>B</td>
<td>0.480</td>
<td>0.0020</td>
<td>0.4792</td>
<td>0.4824</td>
<td>0.4756</td>
<td>0.4818</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>Exogenous process: AR(1) Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\rho_z)</td>
<td>B</td>
<td>0.90</td>
</tr>
<tr>
<td>(\rho_e)</td>
<td>B</td>
<td>0.90</td>
</tr>
<tr>
<td>(\rho_\chi)</td>
<td>B</td>
<td>0.90</td>
</tr>
<tr>
<td>(\rho_f)</td>
<td>B</td>
<td>0.90</td>
</tr>
<tr>
<td>(\rho_{r^*})</td>
<td>B</td>
<td>0.90</td>
</tr>
<tr>
<td>(\rho_{p^*})</td>
<td>B</td>
<td>0.90</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>Exogenous process: Standard deviations</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma_z)</td>
<td>IG</td>
<td>0.01</td>
</tr>
<tr>
<td>(\sigma_e)</td>
<td>IG</td>
<td>0.01</td>
</tr>
<tr>
<td>(\sigma_\chi)</td>
<td>IG</td>
<td>0.01</td>
</tr>
<tr>
<td>(\sigma_f)</td>
<td>IG</td>
<td>0.01</td>
</tr>
<tr>
<td>(\sigma_{r^*})</td>
<td>IG</td>
<td>0.01</td>
</tr>
<tr>
<td>(\sigma_{p^*})</td>
<td>IG</td>
<td>0.01</td>
</tr>
</tbody>
</table>

For the priors of the shocks affecting the economy, I set the autoregressive coefficients to have a beta distribution with a mean of 0.90 and a standard deviation of 0.005. The standard deviations of the innovations are assumed to follow an inverse-gamma distribution with a mean of 0.01. Among the shocks, productivity shock is the most persistent and foreign price shock is the most volatile. The estimate of the sticky price parameter (the Calvo probability), \(\nu\), is 0.4792, suggesting that the average duration of price contracts is about two quarters. The estimate of the capital adjustment cost, \(\xi\), is 0.2605, and the weighted parameter in the external finance premium \(\psi\) is 0.0481.
4 Simulation Results

In what follows, I take the posterior mode of each of the parameters reported in Table 4 and the value of those reported in Table 3.

4.1 A Negative Total Factor Productivity (TFP) Shock

Figure 3 depicts the response of the model following a negative TFP shock. As shown, output, capital and consumption all decline, and inflation rises, consistent with the responses in a standard New Keynesian model. To close the gap between aggregate demand and aggregate supply, the exchange rate appreciates and makes exports less competitive. In the baseline model, the decline in capital implies a contraction in credit and asset prices: The first occurs because as the income of the economy declines, households and entrepreneurs now have fewer resources in financial markets, so capital inflows decline; the second occurs because as the demand for capital declines, the price of this asset declines. There is also a contraction in the risk premium, which is a product of the reduction in the demand for capital. Eventually, as the cost of borrowing declines, because of the rise of the interest rate following the Taylor rule, entrepreneurs increase their use of external financing, so the nominal credit growth eventually expands. Then, the lower policy rates partially offset the impact of the low default premium.

Figure 2: Prior and posterior distributions.
on lending rates and stabilize output and consumption. The stabilization of demand helps to
decrease inflation, and the economy returns to normalcy. I now compare how the economy
would respond to a negative technology shock under different policy regimes. The responses
of inflation and output are relatively similar across the four regimes. Thus, all four regimes
yield similar results in terms of macroeconomic stabilization. However, the monetary policy
differs across the regimes. In the R1 regime (blue line), the central bank tightens monetary
policy in response to the rise in inflation resulting from the negative technology shock, and
hence the policy rate rises. Under R2, the augmented Taylor rule implies that the interest rate
is lower (due to credit reduction) than with respect to R1. Under R3, the macroprudential
instrument reacts to the reduction in credit, so the interest rate is lower than in the baseline
case (the regulation premium $\tau_t$ drops because credit in the economy is below the steady state). This behavior implies different dynamics for nominal credit growth and capital inflows
in each regime. Additionally, policy-makers become less aggressive under macroprudential
regimes compared with the scenario in which they do not respond to financial imbalances
under productivity shocks.

4.2 A Negative Financial Shock: A Reduction in Perceived Lending Risk

Figure 4 shows the interactions between macroprudential and monetary policies due to a
negative financial shock (note that if it were the case of a positive financial shock, the
magnitude of the results would be the same but in the opposite direction). Following the
baseline scenario (R1 scenario), when lenders become more optimistic about the ability
of economic agents to pay their debt, which means a negative change in the exogenous
variable $f_t$, lending to domestic entrepreneurs becomes less risky, which provides easier credit
conditions and hence triggers capital inflows. As financing costs decline, firms borrow and
invest more. Stronger demand for goods and higher asset prices boost firms' balance sheets,
and the default premium $S_t$ declines. As the cost of borrowing declines, entrepreneurs
increase their use of external financing by undertaking more projects. Thus, the nominal
The abscissa axis shows the time, and the ordinate axis represents percent deviations from the steady state.
credit growth increases further.\textsuperscript{28} Higher borrowing also increases the future supply of capital and hence brings about a rise in investment, output, and consumption, along with a credit growth boom. Overall, following the capital inflows surge, which implies a reduction in net exports, the economy experiences higher demand and inflation pressures, and the exchange rate depreciates. Asset prices also increase after the shock. The higher policy rates partially offset the impact of the lower default premium on lending rates and stabilize output as consumption becomes more costly. Eventually, the stabilization of demand helps to reduce inflation, and the economy returns to normalcy. On the other hand, the R2, R3, and R4 regimes, which directly counteract easing in lending standards, mitigate the impact of the financial shock. Therefore, these regimes improve macroeconomic and financial stability. For example, in regime R3, the macroprudential policy rule (that responds to nominal credit growth) entails higher costs for financial intermediaries that are passed on to borrowers in the form of higher lending rates. In regimes R2 and R4, policy-makers also adopt a macroprudential tool that directly counteracts the easing of lending standards. In those cases, both domestic and foreign debt increase less than in the baseline scenario. Thus, the credit growth is lower compared with regime R1, when the policy regime is the standard Taylor rule. This setup captures the notion that such measures make it harder for firms to borrow during boom times and hence make the subsequent bust less dramatic. The increase in capital inflows, policy rate, capital, output and inflation is also lower in the presence of macroprudential measures. It is also noteworthy that under policy regimes R2 and R4, in which there is an augmented Taylor rule, inflation does not increase when financial shock occurs, as is observed in regime R1. In R2, forward-looking agents take the potential rise in the policy interest rate and reduce their borrowing accordingly. This soothes the responses in output and inflation, and the ex-post policy rate rises less than in the R1 regime. Inflation and output are stabilized in response to a reduction in perceived lending risk under both the regime that combines a standard Taylor rule with the regulation premium (regime R3) and the regime with the augmented Taylor rule with the regulation premium (regime R4). Note that one of the main results when policy-makers considered R2, R3 and R4 is that,

\textsuperscript{28}This result is consistent with Unsal (2011).
while macroprudential policies exert a direct stabilizing effect, they also have an indirect destabilizing effect, which works through the depression of economic growth. In other words, prudential policies slow down the economy and lead to inefficient factor utilization during a boom.

5 Welfare Analysis

5.1 The Optimal Policy Regime

In this Section, I compare welfare cost across equilibria with different policy rules.\(^{29}\) Let be

\[ W(\phi_c, \rho, \phi_\pi | \Theta) = E_t \left\{ \sum_{t=0}^{\infty} \beta^t u \left( c_t(\phi_c, \rho, \phi_\pi | \Theta), h_t(\phi_c, \rho, \phi_\pi | \Theta) \right) \right\}, \quad (46) \]

where \( W(\phi_c, \rho, \phi_\pi | \Theta) \) is defined as the conditional expected discounted utility of the representative agent attained for a given parametrization vector \( \Theta \) and the policy rule parameters \( \phi_c, \rho, \phi_\pi \). The arguments of the \( u(\cdot) \) function represent equilibrium allocations for consumption and leisure for a given vector of parameters \( \Theta \) (according to the values of the parameters reported in Table 3 and the mode estimates reported in Table 4). Following the work of Carrillo et al. (2020), I use standard compensating lifetime consumption variations that make agents indifferent between the levels of expected lifetime utility attainable under a given policy regime and the deterministic steady state, as a reference level (which is Pareto efficient).\(^{30}\) Thus, it is necessary to account for the transitional effects from the deterministic steady state to the different stochastic processes implied by each alternative policy regime, conditional on the same distribution of shocks (I assume the same posterior parameters governing the exogenous shocks across all regimes). Therefore, welfare must be lower under

\(^{29}\)Faia and Monacelli (2007) stated that one cannot safely rely on standard first-order approximation methods to compare the relative welfare associated with each monetary policy arrangement. In the same way, Schmitt-Grohe and Uribe (2006) showed that first-order approximation techniques are not appropriate for welfare comparisons across different policy environments. Following these works, the estimation of welfare evaluation is based on a second-order approximation.

\(^{30}\)It is because the deterministic steady state neutralizes the long-run effects of price stickiness and monopolistic competition.
Figure 4: **Response of key variables to a reduction in perceived lending risk.**

The abscissa axis shows the time, and the ordinate axis represents percent deviations from the steady state.
the stochastic version of the model for any combination of policy rule parameters \( \phi_c, \rho_T, \phi_\pi \) implying that welfare measures are negative numbers, or welfare cost.

Let \( W_{ss} \) be the welfare at deterministic steady state and \( x_{ss} \) any endogenous variable in the steady state, so that:

\[
W_{ss} = \frac{u(c_{ss}, h_{ss})}{1 - \beta}.
\] (47)

The welfare effect of a particular pair \((\phi_c, \phi_\pi)\) or \((\rho_T, \phi_\pi)\) of policy parameters is defined as the percent change in consumption, \( \Omega \), relative to the reference consumption levels (i.e. at the deterministic steady state), such that the following condition holds:

\[
W(\phi_c, \rho_T, \phi_\pi | \Theta) = \frac{u(c_{ss}(1 - \Omega), h_{ss})}{1 - \beta}.
\] (48)

Under this specification one can solve for \( \Omega \) and obtain:

\[
\Omega(\phi_c, \rho_T, \phi_\pi | \Theta) = 1 - \exp \left\{ (1 - \beta)(W(\phi_c, \rho_T, \phi_\pi | \Theta) - W_{ss}) \right\}.
\] (49)

Thus, \( \Omega \) is a welfare cost relative to the deterministic steady state. In the next step, to evaluate the relevance of Tinbergen’s rule, I compare the standard Taylor rule, the augmented Taylor rule (which implies one instrument for two objectives), and the macroprudential regime with a standard Taylor rule (which implies two instruments for two objectives), that is R1, R2 and R3 regimes, in terms of the welfare cost \( \Omega \) for a set of policy parameters. My target is to find the policy parameters \( \phi_c, \rho_T, \phi_\pi \) that yield the lowest welfare cost.

Figure 5 shows the surface plots of welfare costs for a set of policy coefficient pairs under the augmented Taylor rule (the first plot, which implies one instrument for two objectives) and the macroprudential regime plus the standard Taylor rule (the second plot, which implies two instruments for two objectives). This figure shows the differences in welfare costs \( \Omega \) across policy parameters for regimes R2 and R3. The results for the standard Taylor rule R1 regime are also included.\(^{31}\) The first graph, which corresponds to the case of one policy instrument

\(^{31}\)They correspond to the cases with \( \phi_c = 0 \) in the R2 case or \( \rho_T = 0 \) in the R3 regime.
to tackle price and financial stability, shows a $U$ shape as $\phi_c$ varies: for high values of $\phi_c$, for a given value for $\phi_\pi$, the welfare cost rises; for low values of $\phi_c$ the welfare cost $\Omega$ also rises. On the other hand, in the second graph with the two instruments active, the shape of $U$ does not hold. This result is consistent with Carrillo et al. (2020). The authors argue that these differences in the curvature of the surface plots indicate the relevance of Tinbergen’s rule because the R3 regime, which corresponds to the case of two policy instruments to tackle objectives, can “avoid sharply increasing welfare costs as $\rho_r$ rises for a given $\phi_\pi$, which is possible because it has separate instruments to tackle price and financial stability". In other words, dual policy rules, one aimed at the financing premium and one aimed at inflation, are more likely to succeed because they adjust two instruments to target two variables. Table 5 compares the optimal regimes, which implies the policy parameters $\phi_c, \rho_r, \phi_\pi$ that yield the lowest welfare cost such that

for the R2 regime:

$$\left\{ \hat{R}_t \right\} \in \arg \min_{\phi_\pi>1, \phi_c>0} \Omega (\phi_c, \phi_\pi | \Theta, \rho_r = 0)$$

subject to

$$\frac{R_t}{R} = \left[ \left( \frac{\Pi_t}{\Pi} \right)^{\phi_\pi} \left( \frac{Y_t}{Y} \right)^{\phi_y} \left( \frac{cg_t}{cg} \right)^{\phi_c} \right]^{1-\phi_r} \left[ \frac{R_{t-1}}{R} \right]^{\phi_r} e_t.$$ 

for the R3 regime:

$$\left\{ \hat{R}_t, \hat{\tau}_t \right\} \in \arg \min_{\phi_\pi>1, \rho_r>0} \Omega (\rho_r, \phi_\pi | \Theta, \phi_c = 0)$$

subject to

$$\frac{R_t}{R} = \left[ \left( \frac{\Pi_t}{\Pi} \right)^{\phi_\pi} \left( \frac{Y_t}{Y} \right)^{\phi_y} \right]^{1-\phi_r} \left[ \frac{R_{t-1}}{R} \right]^{\phi_r} e_t,$$

$$\tau_t = \left( \frac{cg_t}{cg} \right)^{\rho_r}, \text{ where } cg_t = \frac{B_t + B_t^*}{B_t + B_t^*}.$$ 

Table 5 lists the optimized parameters and the difference in welfare cost in the R2 and R1 regimes relative to the R3 regime. R3 is the “best policy” scenario because it yields the
Figure 5: Welfare cost of varying the response to inflation ($\phi_\pi$) and macroprudential-policy parameters ($\phi_c$ and $\rho_T$).

best welfare outcome of the three regimes. The R3 regime optimized policy parameters are $\rho_T = 1$, $\phi_\pi = 1.42$. Note that violations of Tinbergen’s rule entail large welfare costs $\Omega$: the R2 and R1 are 131 and 209 basis points larger than in R3, respectively, and 78 basis points lower in the R2 than in R1. Hence, allowing the Taylor rule to respond to the credit spread is better than not; however, the use of separate financial and monetary rules is significantly better.

<table>
<thead>
<tr>
<th>Regime</th>
<th>Optimal coeff.</th>
<th>Welfare Cost</th>
<th>Difference in $\Omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Taylor Rule (R1)</td>
<td>$0$</td>
<td>$0$</td>
<td>$1.76$</td>
</tr>
<tr>
<td>Augmented Taylor Rule (R2)</td>
<td>$0.3$</td>
<td>$0$</td>
<td>$1.45$</td>
</tr>
<tr>
<td>Macroprudential Regime (R3)</td>
<td>$0$</td>
<td>$1$</td>
<td>$1.42$</td>
</tr>
</tbody>
</table>

Policy coefficients from the sets used in Figure 5 that produce the lowest welfare cost under each policy regime. The difference in welfare cost under the R1 or R2 is relative to the R3 in basis points.

Welfare is higher when the economy is under economic regime R3. Why is this so? The volatility of consumption partially explains the answer. Table 6 shows that the introduction of a macroprudential measure could help in reducing macroeconomic volatility. As the table indicates, the standard deviation of both consumption and income is smaller under policy regime R3 and R4, that is, under regimes that consider a scheme with the macroprudential
tool as an exogenous component of the external finance premium. This supports the results shown in Table 5 where a macroprudential stance is found to be welfare improving, and monetary policy has only one objective, promoting lower consumption volatility in response to shocks.

Table 6: Second Moments in the Model and the Data.

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_c$</th>
<th>$\sigma_\pi$</th>
<th>$\sigma_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Taylor Rule (R1)</td>
<td>0.0330</td>
<td>0.0066</td>
<td>0.0367</td>
</tr>
<tr>
<td>Augmented Taylor Rule (R2)</td>
<td>0.0291</td>
<td>0.0063</td>
<td>0.0342</td>
</tr>
<tr>
<td>Macroprudential Regime (R3)</td>
<td>0.0260</td>
<td>0.0012</td>
<td>0.0267</td>
</tr>
<tr>
<td>Augmented Macroprudential Regime (R4)</td>
<td>0.0280</td>
<td>0.0019</td>
<td>0.0287</td>
</tr>
<tr>
<td>Data</td>
<td>0.0250</td>
<td>0.0100</td>
<td>0.0240</td>
</tr>
</tbody>
</table>

$\sigma_x$ refers to the standard deviation of the endogenous variable $x$.

5.2 The Dynamics on Welfare

I study how the impulse response function of welfare reacts to a financial and productivity shock across different policy parameters $\rho_\tau$ and $\phi_c$.

**Case 1: A reduction in perceived lending risk.** In this scenario, lending to entrepreneurs becomes less risky, which provides easier credit conditions and hence triggers capital inflows. Higher borrowing also increases the future supply of capital and hence brings a rise in investment, output and consumption, along with a credit growth boom. Thus, welfare rises. Now, however, with different values for $\phi_c$ and $\rho_\tau$ the dynamics of welfare change. The first graph in Figure 6 shows the effect on welfare when $\phi_c$ changes, keeping $\rho_\tau$ fixed at 1. As can be observed, while $\phi_c$ is close to 0, the initial impact on welfare is greater. On the other hand, when this parameter converges with 1, the positive impact on welfare decreases over time. Observe that this impulse response function returns to a steady-state in each scenario. The second graph in Figure 6 shows the dynamics on welfare when $\rho_\tau$ changes, keeping $\phi_c$ fixed at 0. In this scenario, while $\rho_\tau$ converges with zero, there is a greater initial impact on welfare; nevertheless, this greatly decreases in the following 20 periods compared to other cases when $\rho_\tau \geq 1$. Note that when this parameter rises, the positive impact in the first 20 periods decreases.
Figure 6: **Case 1. Dynamics on welfare of various policy coefficients during a financial shock.**

![Figure 6](image)

Figure 7: **Case 2. Dynamics on welfare of various policy coefficients during a technology shock.**

![Figure 7](image)

**Case 2: A positive TFP shock.** Suppose there is a positive technology shock. According to the results in Section 4 (inverse direction, same magnitude), output, capital, consumption, and investment all rise, and inflation declines. Thus, welfare rises. Nevertheless, compared to when a financial shock hits the economy, welfare dynamics are different when a productivity shock occurs. According to Figure 7, the first graph shows the dynamics on welfare when $\phi_c$ changes while keeping $\rho_\tau$ fixed at 1. A similar dynamic is observed in the second graph when $\rho_\tau$ changes (keeping $\phi_c$ fixed at 0) with a slight change in the convergence of welfare for values of $\rho_\tau$ close to zero. In this case, the dynamics of the IRFs under a productivity shock are very similar in both schemes.
6 Alternative Macroprudential Policy Rules and Experiments

6.1 Experiment I: Capital Controls

Capital controls are a subset of macroprudential regulation that consist of interventions in the capital account (Erten et al., 2019). It can be said that the defining feature of capital controls regarding other macroprudential and capital flow management measures (CFMs) regulations is that they discriminate based on the residency of the parties involved in a financial transaction.\(^{32}\)

The literature on the effects of capital controls has proliferated over the past two decades. However, the most interesting policy and academic debate on this issue has taken place since the GFC. After that event, the IMF undertook the most important multilateral effort to rethink the role of these regulations was in 2012, leading to what came to be called the Institutional View (IV) on capital account liberalization and management (IMF, 2012). The IV recommends that capital flows be managed primarily through macroeconomic policies. However, in certain circumstances, CFMs can be useful in supporting macroeconomic adjustment and safeguarding financial stability (IMF, 2018). More recently, the IV has evolved into an Integrated Policy Framework (IPF) for guiding policy-makers on the joint configuration of monetary policy, capital controls, foreign exchange intervention, and macroprudential policies. The IPF states that country characteristics, the nature of shocks, and the actions of trading partners will determine optimal policy combinations (IMF, 2020a).

How is a policy intervention in private borrowing decisions justified in economic terms? A growing number of theoretical studies have provided micro-foundations for why capital controls may enhance welfare. The common theme of this strand of literature is related to negative externalities that arise because agents do not internalize the effects of their individual decisions, which are distorted toward excessive borrowing.\(^{33}\) According to Erten et

\(^{32}\) Some authors use the terms CFMs and capital controls in an indistinct manner.

\(^{33}\) The main categories of such externalities emphasized by the literature are pecuniary externalities, associated with financial instability and aggregate demand externalities associated with unemployment.
al. (2019), a novel strand of literature demonstrates that these externalities lead private agents to over-borrow and take on excessive risks. Therefore, these externalities can be corrected through counter-cyclical capital account interventions. There is vast literature on this front. Cespedes et al., (2004), Bianchi (2009), Jeanne and Korinek (2010), Mendoza (2010), and Bianchi and Mendoza (2011) focus on overborrowing and consequent externalities. In these papers, regulations induce agents to internalize their externalities and thereby increase macroeconomic stability. However, overborrowing is a model-specific feature. For example, Benigno et al. (2011) find that in normal times, under-borrowing is much more likely to emerge rather than overborrowing. Following the same argument, Unsal (2011) assesses the stabilization performance of macroprudential measures that discriminate against foreign liabilities, as in her model, entrepreneurs borrow from both domestic and foreign resources rather than focusing on externalities. This paper fits into the latter standard of research.

The literature also provides some guidance on the effectiveness of macroprudential policies and capital controls, the complementarities, substitutability, and interactions among the different alternatives. Authors like Ostry et al. (2011) mention that when foreign borrowing occurs bypassing the regulated financial sector, capital controls may be a better alternative than macroprudential policies. The empirical evidence of Acosta-Henao et al. (2020) shows that when the use of macroprudential instruments increases, the intensity in capital controls falls, signaling that the two are substitutes. Similarly, Basu et al. (2020) mention that capital controls and macroprudential regulations on consumers of financial services could be substitutes under some conditions. These authors argue that domestic macroprudential taxes on consumer debt are perfect substitutes for capital controls when macroprudential taxes cover the entire economy.

How effective are macroprudential measures on foreign liabilities? This subsection assesses the stabilization performance of macroprudential measures that discriminate against foreign liabilities (or macroprudential capital controls) versus broad macroprudential measures, as economic agents borrow from both domestic and foreign resources in the model. In the case of capital controls, the regulation premium only applies to foreign borrowing, and
the macroprudential policy instrument $\tau_t$ is defined only in terms of nominal foreign credit growth. Thus, policy-makers use deviations in foreign credit growth from its steady state and monetary and financial policy only consider the nominal credit growth $\hat{c}g_t$ as follows:

$$\hat{c}g_t = \frac{B^*_t}{B^*_t-1},$$  \hspace{1cm} (50)  

where the regulation premium is $\hat{\tau}_t = \left(\frac{\hat{c}g_t}{c\hat{g}_t}\right)^{\rho_r}$, with $\rho_r > 0$ defined as a function of nominal foreign credit growth. As before, the broad macroprudential policy is:

$$c\hat{g}_t = \frac{B_t + B^*_t}{B_{t-1} + B^*_{t-1}}.$$  \hspace{1cm} (51)

Under macroprudential capital controls, the effect of the financial shock on foreign borrowing is less pronounced, and the exchange rate appreciates less. Nevertheless, the policy measure fails to achieve its primary objective of promoting financial stability: GDP and inflation have better performance in the broad macroprudential scenario under all regimes (see Figure 8) when financial shocks hit the economy. Similar results on GDP stability occur when there is a negative productivity shock (see Figure 9). As before, both macroprudential capital controls and broad macroprudential measures are not as useful in helping macroeconomic stability under productivity shocks. The reason behind these results is that capital flow management, as macroprudential policy, only brings a shift from foreign loans $B^*_t$ to domestic loans $B_t$, leaving the aggregate credit growth nearly unchanged compared to the baseline scenario. Thus, broad macroprudential measures are more effective than macroprudential capital controls, as the latter only bring a shift from foreign debt to domestic debt and hence affect the composition of economic agents’ debt, rather than the total volume.\textsuperscript{34}

This result is similar to the work of Unsal (2011), which highlights that capital controls are likely to bring a shift in the source of borrowing from domestic to foreign markets, causing only a limited change in aggregate credit growth.

\textsuperscript{34}Macroprudential measures could also be applied to domestic borrowing only. Nevertheless, similarly to the case of capital controls, such a measure is likely to bring a shift in the source of borrowing from domestic to foreign markets, causing only a limited change in the aggregate credit growth.
Figure 8: **Response of GDP and inflation to a reduction in perceived lending risk** with Broad Macroprudential Policy (BMP) and Capital Controls (CC).

The abscissa axis shows the time, and the ordinate axis represents percent deviations from the steady state.

Figure 9: **Response of GDP and inflation to a negative productivity shock** with Broad Macroprudential Policy (BMP) and Capital Controls (CC).

The abscissa axis shows the time, and the ordinate axis represents percent deviations from the steady state.
6.2 Experiment II: Welfare Gains Under an Open Economy Versus a Closed Economy

Much of the existing literature considers the potential gains from complementing monetary policy rules with macroprudential rules. However, most of these studies analyze closed economies in their framework. In this regard, I study the relevance of the Tinbergen rule in a closed economy, and I examine the welfare cost differences between an open economy and a closed economy with a monetary policy that *leans against the wind*. To answer this question, this section addresses: (1) whether the Tinbergen rule still complies within the case of a closed economy; (2) what are the optimal policy coefficients that produce the lowest social welfare cost; (3) how the welfare cost compares between an open and closed economy.

My hypothesis is that the welfare cost is lower in a closed economy than in an open economy because in an open economy, the monetary authority—through the reference rate—has less power in controlling the dynamics of credit since economic agents can save or borrow from the rest of the world. This affects the dynamics of consumption and income of the domestic economy, and ultimately of welfare and social cost in the face of various exogenous shocks.

Let us see how the main equations of the model change, adapting it to a closed economy. According to the model presented in Section 4, in the case of a closed economy the households’ borrowing constraints can be represented as

\[
P_t c_t + B_{t+1} = W_t h_t + R_t B_t + \Upsilon_t, \forall t. \tag{52}
\]

As before, households deposit \( B_t \) at the financial intermediaries. However, households do not have access to international financial markets. On the other hand, retailers produce an intermediate good \( d_{a,t} \) only with domestic insumes. Thus, \( \alpha_d = 0 \) and \( \theta_d = 0 \), which implies that \( d_{a,t} = y_{j,t} \), exports \( x_t = 0 \) and the marginal cost is equal to \( mc_t = \frac{p_{j,t}}{P_t} \). Then, the resource constraint for final goods is:

\[
y_t = \left[ c_t + c_{e,t} + \xi + \frac{\xi}{2} \left( \frac{i_t}{k_t} - \delta \right) k_t \right] \Gamma_t, \forall t. \tag{53}
\]
As defined in Section 5, $\Omega$ is the welfare cost relative to the deterministic steady state, thus:

$$\Omega(\phi_c, \rho_T, \phi_\pi | \Theta) = 1 - \exp\{(1 - \beta)(W(\phi_c, \rho_T, \phi_\pi | \Theta) - W_{ss})\}. \quad (54)$$

Given the vector $\Theta$ of parameters calibrated and estimated according to Table 3 and Table 4, Figure 10 shows surface plots of the welfare costs for a set of policy coefficient pairs $(\phi_\pi, \phi_c)$ and $(\phi_\pi, \rho_T)$. The right figure shows the macroprudential regime plus the standard Taylor rule (which implies two instruments for two objectives), and the left figure the augmented Taylor rule (which implies one instrument for two objectives) in the case of a closed economy. The most important result that the surface plots show is the relevance of Tinbergen’s rule because when policymakers have two instruments to tackle two objectives can avoid sharply increasing welfare costs (as $\rho_T$ rises for a given $\phi_\pi$). In contrast, when policymakers only have one instrument to tackle two targets (i.e., inflation and financial stability), the welfare cost rises. The second result is that the welfare cost is higher in an open economy. Comparing the surface plots in Figure 5 and Figure 10, the welfare cost $\Omega(\phi_c, \rho_T, \phi_\pi | \Theta)$ is always higher in an open economy. This means that it is more expensive for the open economy not to react to financial imbalances. Thus, allowing the Taylor rule to respond to the credit spread is better than not; however, using separate financial and monetary rules is significantly better in an open-economy than in a closed economy. In the third result, as can be seen in Figure 10, the welfare cost graph with only one instrument shows a less pronounced $U$ shape as $\phi_c$ varies. This implies that having an instrument that ensures two objectives is not as expensive in a closed economy.

Table 7 compares the optimal regimes, which implies that the policy parameters $\phi_c, \rho_T, \phi_\pi$ yield the lowest welfare cost, such that

for the $R2$ regime:

$$\left\{ \hat{R}_t \right\} \in \arg \min_{\phi_\pi > 1, \phi_c > 0} \Omega(\phi_c, \phi_\pi | \Theta, \rho_T = 0)$$

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Figure 10: **Welfare cost of varying the response to inflation (φ_π) and macroprudential-policy parameters (φ_c and ρ_τ) in a closed economy.**

\[ s.t. \left\{ \frac{R_t}{R} = \left( \begin{array}{c} \frac{\Pi_t}{\Pi} \\ \frac{Y_t}{Y} \\ \frac{c_{gt}}{c_g} \end{array} \right)^{\phi_\pi} \left( \begin{array}{c} \phi_y \\ \phi_c \end{array} \right)^{1-\phi_c} \left( \frac{R_{t-1}}{R} \right)^{\phi_r} e_t, \right. \]

for the R3 regime:

\[ \left\{ \hat{R}_t, \hat{\tau}_t \right\} \in \arg \min_{\phi_\pi > 1, \rho_r > 0} \Omega \left( \rho_r, \phi_\pi \mid \Theta, \phi_c = 0 \right) \]

\[ \left\{ \frac{R_t}{R} = \left( \begin{array}{c} \frac{\Pi_t}{\Pi} \\ \frac{Y_t}{Y} \\ \frac{c_{gt}}{c_g} \end{array} \right)^{\phi_\pi} \left( \begin{array}{c} \phi_y \\ \phi_c \end{array} \right)^{1-\phi_c} \left( \frac{R_{t-1}}{R} \right)^{\phi_r} e_t, \right. \]

\[ \left\{ \tau_t = \left( \frac{c_{gt}}{c_g} \right)^{\rho_r}, \text{ where } c_{gt} = \frac{B_t}{B_{t-1}}. \right. \]

Table 7 lists the optimized parameters and the difference in welfare cost in the R2 and R1 regimes relative to the R3 regime. Similar to the case of an open economy, R3 is the best policy scenario because it yields the best welfare outcome of the three regimes. The R3 regime optimized policy parameters are ρ_r = 1.5, φ_π = 1.25. Note that the violations of Tinbergen’s rule entail large welfare costs: R2 and R1 are 99 and 179 basis points higher,
respectively, than the R3, and 80 basis points lower in the R2 than in the R1. Hence, allowing the Taylor rule to respond to the credit spread is better than not; however, using separate financial and monetary rules is significantly better.

<table>
<thead>
<tr>
<th>Regime</th>
<th>Optimal coeff.</th>
<th>Welfare Cost</th>
<th>Difference in Ω</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Taylor Rule (R1)</td>
<td>0</td>
<td>1.61</td>
<td>5.95%</td>
</tr>
<tr>
<td>Augmented Taylor Rule (R2)</td>
<td>0.4</td>
<td>1.26</td>
<td>5.15%</td>
</tr>
<tr>
<td>Macroprudential Regime (R3)</td>
<td>0</td>
<td>1.5</td>
<td>4.16%</td>
</tr>
</tbody>
</table>

Policy coefficients from the sets used in Figure 10 that produce the lowest welfare cost under each policy regime. The difference in welfare cost under R1 or R2 is relative to R3 in basis points.

Table 8 summarizes the welfare cost derived from the optimal coefficients for an open and closed economy with one instrument and two objectives and two instruments with two objectives. Two key results are derived from this table. The first is that in both scenarios, the welfare costs are always higher for an open economy. This is because, for an economy in which economic agents can access international financial markets, domestic monetary policy is likely to have less impact on financial imbalances since individuals can borrow at the foreign interest rate. As a result, financial vulnerabilities can trigger or amplify negative shocks to economic activity. The second is that it is always better to respect the Tinbergen rule in each scenario. Moreover, violations of Tinbergen’s rule entail large welfare costs, but these costs are higher in an open economy than in a closed economy.

Why are welfare costs higher in an open economy if the Tinbergen rule is not followed? Intuitively, an open economy has more to lose than a closed economy: an open economy’s social welfare is greater than that of a closed economy due to the greater consumption available. For the same reason, the compensatory variation that the policy-maker has to carry out so that society is indifferent between any scenario of the monetary and macroprudential regime will be greater or equal in an open economy compared to a closed economy.
7 Conclusions

In this paper, I examined the interaction between monetary and macroprudential policies and whether policymakers should respond to financial imbalances in the context of a small-open model that features both financial frictions and sticky prices. The main results from this paper are as follows:

1. Macroprudential policies help monetary policy stabilize the economy in the face of a financial shock. In contrast, the responses of inflation, GDP and the credit markets are relatively similar in the four regimes under technological shocks, so macroprudential measures are not as effective. Hence, there is a trade-off between financial and macroeconomic stability objectives in the face of a productivity shock.

2. An additional objective for the monetary authority, like financial stability, should not be a task for the same instrument, the nominal interest rate. The results suggest that welfare is higher in regimes where policy-makers respond to financial imbalances using a macroprudential tool and inflation using the policy interest rate independently.

3. Broad macroprudential measures are more effective than macroprudential capital controls, as the latter only bring a shift from foreign debt to domestic debt and hence affect the composition of economic agents’ debt, rather than the total volume.

4. The violations of Tinbergen’s rule entail large welfare costs, but these costs are higher in an open economy than in a closed economy. This is because, in an economy in which firms can access international financial markets, domestic monetary policy is likely to have less impact on financial imbalances as firms can borrow at the foreign interest rate.
In practice, taking into account financial imbalances in the context of inflation targeting could require changes in how we think about monetary policy. According to Boivin, et al. (2010), giving monetary policy explicit responsibility for financial stability would result in a lack of clarity regarding monetary policy objectives and could possibly undermine the credibility of the inflation objective. Another potentially important cost of leaning against financial imbalances stems from the difficulty of identifying them and of calibrating an appropriate response. If financial imbalances are falsely identified, responding to them through monetary policy could induce undesirable economic results. In this article, I present two illustrations of these interactions in which financial imbalances stem from credit expansion. It is important to note that these examples should be seen merely as useful illustrations and not as the final word on the relationship between monetary policy and financial imbalances. Nonetheless, this model serves to illustrate a few fundamental principles that are of broader relevance. Finally, given the cyclical nature of credit, with phases of expansion and contraction, it is essential from the perspective of financial stability to monitor the evolution of financial assets in the economy. In this regard, asset prices, which are usually associated with excessive credit growth (and consequently with financial crises), are also important indicators that support the assessment of financial stability risks. However, the literature linking asset price bubbles, monetary policy, and macroprudential tools in business cycle models has been scant. Despite its potential importance, there is little formal analysis of how the asset price channel of macroprudential and monetary policies works. These issues seem like a promising avenue for future research.

References


Appendix

A Derivation of the First Order and Efficiency Conditions

A.1 Households’ Problem

The representative household chooses the optimal paths for \( \{c_t, h_t, B_t, B^*_t\}_{t=0}^{\infty} \) by solving:

\[
\max_{\{c_t, h_t, B_t, B^*_t\}_{t=0}^{\infty}} E_t \left[ \sum_{t=0}^{\infty} \beta^t \left( \log(c_t) - \theta \frac{h_t^{1+\eta}}{1+\eta} \right) \right]
\]

s.t. \[ c_t + \frac{B_t}{P_t} - \frac{ner_t B^*_t}{P_t} = \frac{W_t}{P_t} h_t + \frac{R_{t-1} B_{t-1}}{P_t} - \frac{R^*_{t-1} \Theta_{t-1} {ner}_t B^*_{t-1}}{P_t} + \frac{\Upsilon_t}{P_t}, \quad \forall t. \]

The Lagrangean of this problem can be written as:

\[
\mathcal{L} = E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left( \log(c_t) - \theta \frac{h_t^{1+\eta}}{1+\eta} \right) + \lambda_t \left[ W_t h_t + R_{t-1} B_{t-1} - R^*_{t-1} \Theta_{t-1} {ner}_t B^*_{t-1} + \Upsilon_t - P_t c_t - B_t + {ner}_t B^*_t \right] \right\}
\]

The First Order Conditions (FOC) of the problem are:

\[
\frac{\partial \mathcal{L}}{\partial c_t} = 0 : \quad \frac{\beta_t^t}{c_t} = \lambda_t P_t, \quad (A.55)
\]

\[
\frac{\partial \mathcal{L}}{\partial h_t} = 0 : \quad \beta^t \theta h_t^{\eta} = \lambda_t W_t, \quad (A.56)
\]

\[
\frac{\partial \mathcal{L}}{\partial B_t} = 0 : \quad \lambda_{t+1} R_t = \lambda_t, \quad (A.57)
\]

\[
\frac{\partial \mathcal{L}}{\partial B^*_t} = 0 : \quad \lambda_{t+1} R^*_t \Theta_{t+1} {ner}_{t+1} = \lambda_t {ner}_t, \quad (A.58)
\]

\[
\frac{\partial \mathcal{L}}{\partial \lambda_t} = 0 : \quad P_t c_t + B_t - {ner}_t B^*_t = W_t h_t + R_{t-1} B_{t-1} - R^*_{t-1} \Theta_{t-1} {ner}_t B^*_{t-1} + \Upsilon_t. \quad (A.59)
\]

The real exchange rate is defined as \( rer_t = \frac{ner P^*_t}{P_t} \) and the inflation as \( 1 + \pi_t = \frac{P_t}{P_{t-1}} \). Thus, using the equations (A.59) and (A.61), the optimal holding of deposits

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satisfies the following households’ Euler equation:

$$1 = \beta R^t E_t \left[ \frac{c_t}{c_{t+1}} \frac{1}{1 + \pi_{t+1}} \right]. \quad (A.60)$$

Using the equations (A.59) and (A.62), the optimal holding of foreign bonds (liabilities) satisfies the following households’ Euler equation:

$$1 = \beta R^* t \Theta_t E_t \left[ \frac{c_t}{c_{t+1}} \frac{rer_{t+1}}{rer_t} \frac{1}{1 + \pi_{t+1}} \right]. \quad (A.61)$$

Equations (A.59) and (A.60) characterize the optimal household’s labor supply:

$$\theta h_t^\eta c_t = \frac{W_t}{F_t}. \quad (A.62)$$

Excluding the possibility of arbitrage, suppose domestic and foreign returns on state-contingent claims must be the same, namely $F_t$. Following Funke, Paetz & Pytlarczyk (2011), when we assume complete markets, returns across countries should be equal while first order conditions similar to those of the domestic country should hold in any country. Without loss of generality, and assuming symmetric initial conditions (implying zero net foreign assets) $\Theta_t = 1$, to exclude arbitrage, the nominal gross-return $R_{t+1}$ on a safe one-period bond paying off a unit of currency in $t + 1$ must be defined as:

$$E_t [R_{t+1} F_{t+1}] = 1. \quad (A.63)$$

Assuming complete financial markets an analogous condition must be fulfilled in any foreign country:

$$E_t \left[ R^*_t F_{t+1} \frac{ner_{t+1}}{rer_t} \right] = 1. \quad (A.64)$$

Thus, combining (4) and (5), we get the Uncovered Interest rate Parity (UIP):

$$E_t \left\{ F_{t+1} \left[ R_{t+1} - \frac{R^*_t ner_{t+1}}{rer_t} \right] \right\} = 0. \quad (A.65)$$

Galí & Monacelli (2005) point out that the UIP follows from international risk sharing and can also be derived by combining the risk sharing condition with domestic and foreign Euler equations: Combining the equations (A.64) and (A.65), we get the Uncovered Interest rate Parity (UIP):

$$R_t = R^*_t \Theta_t E_t \left\{ \frac{1 + \pi_{t+1}}{1 + \pi^*_t} \left[ \frac{rer_{t+1}}{rer_t} \right] \right\}. \quad (A.66)$$

Using (A.67) equation and the definition of real exchange rate we get the following relation:
\[ R_t = R_t^* \Theta_t E_t \left[ \frac{ner_{t+1}}{ner_t} \right] = R_t^* \Theta_t E_t \left[ d_{t+1} \right], \tag{A.67} \]

that is, the domestic interest rate can be explained by the foreign interest rate, the country risk premium, and the expectation of the nominal depreciation rate \( E_t \left[ d_{t+1} \right] = E_t \left[ \frac{ner_{t+1}}{ner_t} \right] \).

### A.2 Capital Producers’ Problem

Capital producers maximize their profit choosing the optimal paths for \( \{i_t\}_{t=0}^{\infty} \) by solving:

\[
\max_{\{i_t\}_{t=0}^{\infty}} E_t \left\{ \chi_t Q_t i_t - i_t - \Phi \left( \frac{i_t}{k_t} \right) \right\}
\]

s.t. \[
\begin{align*}
\Phi \left( \frac{i_t}{k_t} \right) &= \frac{\xi}{2} \left( \frac{i_t}{k_t} - \delta \right)^2 k_t, \quad \forall t, \\
k_{t+1} &= \chi_t i_t + (1 - \delta) k_t, \quad \forall t, \\
\log(\chi_t) &= \rho_{\chi} \log(\chi_{t-1}) + \varepsilon_{\chi,t}, \quad \varepsilon_{\chi,t} \sim i.i.d. N(0, \sigma^2_{\chi}), \quad \forall t, \\
k_0 > 0 \text{ given.}
\end{align*}
\]

The Lagrangean of this problem can be written as:

\[
\mathcal{L} = E_t \left[ \chi_t Q_t i_t - i_t - \frac{\xi}{2} \left( \frac{i_t}{k_t} - \delta \right)^2 k_t + \lambda_t \left\{ \chi_t i_t + (1 - \delta) k_t - k_{t+1} \right\} \right].
\]

The FOC of the problem are:

\[
\frac{\partial \mathcal{L}}{\partial \lambda_t} = 0 : \quad k_{t+1} = \chi_t i_t + (1 - \delta) k_t, \quad \tag{A.68} \\
\frac{\partial \mathcal{L}}{\partial i_t} = 0 : \quad 1 = E_t \left[ \chi_t Q_t - \xi \left( \frac{i_t}{k_t} - \delta \right) \right]. \quad \tag{A.69}
\]

### A.3 Entrepreneurs’ Problem

Entrepreneurs maximize their profit choosing the optimal paths for \( \{k_{j,t}, h_{j,t}, y_{j,t}\}_{t=0}^{\infty} \) by solving:

\[
\max_{\{k_{j,t}, h_{j,t}, y_{j,t}\}_{t=0}^{\infty}} P_{j,t} y_{j,t} - w_t h_{j,t} - \tau r_t k_{j,t}
\]

s.t. \[
\begin{align*}
y_{j,t} &= z_t k_{j,t}^\alpha h_{j,t}^{1-\alpha}, \quad \forall t, \\
\log(z_t) &= \rho_{z_t} \log(z_{t-1}) + \varepsilon_{z,t}, \quad \varepsilon_{z,t} \sim i.i.d. N(0, \sigma^2_{z}).
\end{align*}
\]

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The Lagrangean of this problem can be written as:

\[ L = P_{j,t} y_{j,t} - w_t h_{j,t} - r r_t k_{j,t} + \lambda_t \left\{ z_t k_j \alpha_j h_j^{1-\alpha} - y_j \right\} . \]

The FOC of problem are:

\[
\frac{\partial L}{\partial \lambda_t} = 0 : \quad y_{j,t} = z_t k_j \alpha_j h_j^{1-\alpha}, \quad (A.70)
\]

\[
\frac{\partial L}{\partial k_{j,t}} = 0 : \quad P_{j,t} \alpha y_{j,t} k_{j,t} = r r_t, \quad (A.71)
\]

\[
\frac{\partial L}{\partial h_{j,t}} = 0 : \quad P_{j,t} (1-\alpha) y_{j,t} h_{j,t} = w_t. \quad (A.72)
\]

A.4 Retailers’ Problem

A.4.1 Cost Minimization Problem

\[
\min_{\{y_{j,t}, y^*_{i,t}\}} \left\{ P_{j,t} y_{j,t} + n r_t P^*_{t} y^*_{i,t} \right\}
\]

s.t. \quad da_{j,t} = \left[ (1-\alpha_d)^{1/\theta_d} (y_{j,t})^{1-1/\theta_d} + (\alpha_d)^{1/\theta_d} (y^*_{i,t})^{1-1/\theta_d} \right]^{\theta_d/\theta_d - 1}, \quad \forall t.

The Lagrangean of this problem can be written as:

\[ L = P_{j,t} y_{j,t} + n r_t P^*_{t} y^*_{i,t} + \lambda \left\{ (1-\alpha_d)^{1/\theta_d} (y_{j,t})^{1-1/\theta_d} + (\alpha_d)^{1/\theta_d} (y^*_{i,t})^{1-1/\theta_d} \right\}^{\theta_d/\theta_d - 1} - da_{j,t} \].

The FOC of problem are:

\[
\frac{\partial L}{\partial \lambda_t} = 0 : \quad da_{j,t} = \left[ (1-\alpha_d)^{1/\theta_d} (y_{j,t})^{1-1/\theta_d} + (\alpha_d)^{1/\theta_d} (y^*_{i,t})^{1-1/\theta_d} \right]^{\theta_d/\theta_d - 1}, \quad (A.73)
\]

\[
\frac{\partial L}{\partial y_{j,t}} = 0 : \quad P_{j,t} = -\lambda \frac{\theta_d}{\theta_d - 1} \left[ da_{i,t} \right]^{\theta_d/\theta_d - 1} \left[ (1 - \frac{1}{\theta_d}) \left( 1 - \alpha_d \right)^{1/\theta_d} (y_{j,t})^{-1/\theta_d} \right], \quad (A.74)
\]

\[
\frac{\partial L}{\partial y^*_{i,t}} = 0 : \quad n r_t P^*_{t} = -\lambda \frac{\theta_d}{\theta_d - 1} \left[ da_{i,t} \right]^{\theta_d/\theta_d - 1} \left[ (1 - \frac{1}{\theta_d}) \alpha_d^{1/\theta_d} (y^*_{i,t})^{-1/\theta_d} \right]. \quad (A.75)
\]

Equations (A.75) and (A.76) implies:

\[
\frac{y_{j,t}}{y^*_{i,t}} = \left[ \frac{1-\alpha_d}{\alpha_d} \right] \left[ \frac{r r_t}{P_{j,t}/P_{i,t}} \right]^{\theta_d}. \quad (A.76)
\]

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A.4.2 Calvo’s Mechanism of Price Adjustment

Each period, only a random fraction \((1 - \nu)\) of firms are able to reset their price; all other firms keep their prices unchanged. When firms do get to reset their price, they must take into account that the price may be fixed for many periods.

A firm reoptimizing in period \(t\) will choose the price \(P_{i,t}\) that maximizes the current market value of the profits generated while that price remains effective. Formally, it solves the problem

\[
\max_{\{P_{i,t}\}_{t=0}^{\infty}} E_t \left\{ \sum_{s=0}^{\infty} \nu^s \Delta_{t,t+s} \left[ \left( \frac{P_{i,t}}{P_{t+s}} - mc_{t+s} \right) da_{i,t+s} \right] \right\}
\]

s.t.

\[
da_{i,t+k|t} = \left( \frac{P_{i,t}}{P_{t+k}} \right)^{-\varepsilon} da_{t+1}, \forall i, \forall t,
\]

\[
mc_t = \left[ (1 - \alpha_d) \left( \frac{P_{j,t}}{P_t} \right)^{1-\theta_d} + \alpha_d \left( \frac{ner_t P^*_t}{P_t} \right)^{1-\theta_d} \right]^{\frac{1}{1-\nu_d}}, \forall i, \forall t.
\]

Where \(\Delta_{t,t+s} = \beta^s \frac{c_t}{c_{t+s}}\) is the stochastic discount factor for nominal payoffs (or the household intertemporal marginal rate of substitution), which the retailer takes as given, \(mc_t\) is the real marginal cost, and \(da_{i,t+k|t}\) denotes output in period \(t + k\) for a firm that last reset its price in period \(t\). Thus, the Lagrangean of this problem can be written as:

\[
\mathcal{L} = E_t \left\{ \sum_{s=0}^{\infty} \nu^s \Delta_{t,t+s} \left[ \left( \frac{P_{i,t}}{P_{t+s}} - mc_{t+s} \right) \left( \frac{P_{i,t}}{P_{t+s}} \right)^{-\varepsilon} da_{t+s} \right] \right\}
\]

\[
+ \lambda_t \left\{ (1 - \alpha_d) \left( \frac{P_{j,t}}{P_t} \right)^{1-\theta_d} + \alpha_d \left( \frac{ner_t P^*_t}{P_t} \right)^{1-\theta_d} \right\}^{\frac{1}{1-\nu_d}} - mc_t \}
\]

\[
\Leftrightarrow \mathcal{L} = E_t \left\{ \sum_{s=0}^{\infty} \nu^s \Delta_{t,t+s} \left[ \left( \frac{P_{i,t}}{P_{t+s}} \right)^{1-\varepsilon} - mc_{t+s} \left( \frac{P_{i,t}}{P_{t+s}} \right)^{-\varepsilon} \right] da_{t+s} \right\}
\]

\[
+ \lambda_t \left\{ (1 - \alpha_d) \left( \frac{P_{j,t}}{P_t} \right)^{1-\theta_d} + \alpha_d \left( \frac{ner_t P^*_t}{P_t} \right)^{1-\theta_d} \right\}^{\frac{1}{1-\nu_d}} - mc_t \}
\]

The FOC of problem are:

\[
\frac{\partial \mathcal{L}}{\partial \lambda_t} = 0 : \quad mc_t = \left[ (1 - \alpha_d) \left( \frac{P_{j,t}}{P_t} \right)^{1-\theta_d} + \alpha_d \left( \frac{ner_t P^*_t}{P_t} \right)^{1-\theta_d} \right]^{\frac{1}{1-\nu_d}}, \quad (A.77)
\]

\[
\frac{\partial \mathcal{L}}{\partial P_{i,t}} = 0 : \quad (A.78)
\]
(1 − ε)E_t \left\{ \sum_{s=0}^{\infty} \nu^s \Delta_{t,t+s} \left[ \frac{P_{i,t}}{P_{t+s}} \right]^{-\varepsilon} da_{t+s} \frac{1}{P_{t+s}} \right\} = \varepsilon E_t \left\{ \sum_{s=0}^{\infty} \nu^s \Delta_{t,t+s} \left[ \frac{P_{i,t}}{P_{t+s}} \right]^{-\varepsilon-1} mc_{t+s} da_{t+s} \frac{1}{P_{t+s}} \right\}.

Equation (A.79) implies that the retailer sets his price so that in expectation discounted marginal revenue equals discounted marginal cost, given the constraint that the nominal price is fixed in period \( s \) with probability \( \nu^s \). Equation (A.79) can be written as:

\[
P^{-\varepsilon} E_t \left\{ \sum_{s=0}^{\infty} \nu^s \Delta_{t,t+s} \left[ \frac{1}{P_{t+s}} \right]^{-\varepsilon} da_{t+s} \frac{1}{P_{t+s}} \right\} = \varepsilon E_t \left\{ \sum_{s=0}^{\infty} \nu^s \Delta_{t,t+s} \left[ \frac{1}{P_{t+s}} \right]^{-\varepsilon-1} mc_{t+s} da_{t+s} \frac{1}{P_{t+s}} \right\},
\]

\[\Leftrightarrow \hat{P}^{-\varepsilon+1}_{i,t} = \left[ \frac{\varepsilon}{1-\varepsilon} \right] E_t \left\{ \sum_{s=0}^{\infty} \nu^s \Delta_{t,t+s} \left[ \frac{1}{P_{t+s}} \right]^{-\varepsilon-1} mc_{t+s} da_{t+s} \frac{1}{P_{t+s}} \right\},\]

\[\Leftrightarrow \hat{P}_{i,t} = \left[ \frac{\varepsilon}{1-\varepsilon} \right] E_t \left\{ \sum_{s=0}^{\infty} \nu^s \Delta_{t,t+s} \left[ \frac{1}{P_{t+s}} \right]^{-\varepsilon} mc_{t+s} da_{t+s} \right\}. \quad (A.79)
\]

Note that in the limiting case of no price rigidities (\( \nu = 0 \)), the previous condition collapses to the familiar optimal price-setting condition under flexible prices:

\[\hat{P}_{i,t} = \left[ \frac{\varepsilon}{1-\varepsilon} \right],\]

which allows us to interpret \( (\varepsilon/1-\varepsilon) \) as the desired markup in the absence of constraints on the frequency of price adjustment. Henceforth, \( (\varepsilon/1-\varepsilon) \) is referred to as the desired or frictionless markup.

Given that the fraction \( \nu \) of retailers do not change their price in period \( t \), the aggregate price evolves according to:

\[P_t = \left( \nu P_{t-1}^{1-\varepsilon} + (1-\nu)(\hat{P}_{i,t})^{1-\varepsilon} \right) \frac{1}{1-\varepsilon}, \quad (A.80)\]

where \( \hat{P}_{i,t} \) satisfies Equation (A.80). By combining Equations (A.80) and (A.81), and then log-linearizing, it is possible to obtain the New Keynesian Phillips curve.
A.5 Final Goods Producers’ Problem

\[ \max_{\{a_{i,t}\}_{t=0}^\infty} \int_0^1 P_t da_t - \int_0^1 P_{i,t} da_{i,t} d_i \]

s.t. \( da_t = \left[ \int_0^1 da_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} d_i \right]^{\frac{\varepsilon}{\varepsilon-1}}, \forall t. \)

The Lagrangean of this problem can be written as:

\[ L = P_t \left[ \int_0^1 da_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} d_i \right]^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 P_{i,t} da_{i,t} d_i. \]

The FOC of the problem is:

\[ \frac{\partial L}{\partial da_{i,t}} = 0 : \frac{\varepsilon}{\varepsilon - 1} P_t \left[ \int_0^1 da_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} d_i \right]^{\frac{\varepsilon}{\varepsilon-1}} \frac{\varepsilon - 1}{\varepsilon} da_{i,t}^{\frac{\varepsilon}{\varepsilon-1}} = P_{i,t}, \tag{A.81} \]

\[ \Leftrightarrow P_t \left[ \int_0^1 da_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} d_i \right]^{\frac{1}{\varepsilon-1}} da_{i,t}^{\frac{1}{\varepsilon}} = P_{i,t}, \tag{A.82} \]

\[ \Leftrightarrow \left[ \int_0^1 da_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} d_i \right]^{\frac{\varepsilon}{\varepsilon-1}} da_{i,t} = \left( \frac{P_{i,t}}{P_t} \right)^{-\varepsilon}. \tag{A.83} \]

Making note of the definition of the aggregate final good, we have:

\[ da_{i,t} = \left( \frac{P_{i,t}}{P_t} \right)^{-\varepsilon} da_t. \tag{A.84} \]

To derive a price index, we use the assumption of zero-profits and equation (A.85):

\[ P_t da_t = \int_0^1 P_{i,t} da_{i,t} d_i, \tag{A.85} \]

\[ \Leftrightarrow P_t da_t = \int_0^1 P_{i,t} \left( \frac{P_{i,t}}{P_t} \right)^{-\varepsilon} da_t d_i, \tag{A.86} \]

Simplifying, we get an expression for the aggregate price level:
A stochastic competitive equilibrium for this economy is a set of:

1. contingent plans for allocations: \( \{c_t(z^t), h_t(z^t), B_t(z^t), B^*_t(z^t), i_t(z^t), k_t(z^t), k_{j,t}(z^t), h_{j,t}(z^t), y_{j,t}(z^t), y^*_{j,t}(z^t), d_{a,t}(z^t), d_{a^t}(z^t), y_t(z^t), n_t(z^t), c_g(z^t), c_{e,t}(z^t), \tau_t(z^t), L_{j,t}(z^t), L_{t}(z^t), o_a_t(z^t), x_t(z^t), \Theta_t(z^t) \}_{t=0}^{\infty}, z^t \in Z^T, j \in [0,1], i \in [0,1] \)

2. and prices: \( \{r_{t}(z^t), n_{r_t}(z^t), W_t(z^t), P_t(z^t), R_t(z^t), Q_t(z^t), P_{j,t}(z^t), m_{c_t}(z^t), R_{k,t}(z^t) \}

such that:

a) Given \( B_0(z^0), B^*_0(z^0), \Theta_0(z^0) \), the prices \( \{ner_t(z^t), W_t(z^t), P_t(z^t), R_t(z^t)\}_{t=0}^{\infty}, z^t \in Z^T \) and the stochastic process for \( R^*_t \), the contingent plans \( \{c_t(z^t), h_t(z^t), B_t(z^t), B^*_t(z^t)\}_{t=0}^{\infty}, z^t \in Z^T \) solve consumer’s problem:

\[
\max_{\{c_t(z^t), h_t(z^t), B_t(z^t), B^*_t(z^t)\}_{t=0}^{\infty}, z^t \in Z^T} \sum_{t=0}^{\infty} \beta^t \mathbb{U}(c_t(z^t), h_t(z^t)) = \sum_{z^t \in Z^T} \sum_{t=0}^{\infty} \beta^t \pi(z^t) \left\{ \log(c_t(z^t)) - \frac{\theta h_t(z^t)^{1+\eta}}{1+\eta} \right\}
\]

subject to:

\[
\begin{align*}
P_t(z^t)c_t(z^t) + B_t(z^t) - n_{r_t}(z^t)B^*_t(z^t) &= W_t(z^t)h_t(z^t) + R_{t-1}(z^t)B_{t-1}(z^t) - R^*_{t-1}(z^t)\Theta_{t-1}(z^t)n_{r_t}(z^t)B^*_{t-1}(z^t) + T_t(z^t), \quad \forall t, \forall z^t \in Z^T.
\end{align*}
\]

\[
c_t(z^t) > 0, \quad 0 \leq h_t(z^t) \leq 1, \quad \forall t, \forall z^t \in Z^T.
\]

\[
\Theta(B^*_t(z^t)) = \left( B^*_t(z^t)/B^*(z^t) \right)^{\theta}, \quad \forall t, \forall z^t \in Z^T.
\]

\[
\log(R^*_t) = \rho_{R^*} \log(R_{t-1}) + \epsilon_{R^*,t}, \quad \epsilon_{R^*,t} \sim i.i.d.N(0, \sigma_{R^*}^2), \quad \forall t, \forall z^t \in Z^T.
\]

\[
B_0(z^0), B^*_0(z^0), \Theta_0(z^0), \text{ given}.
\]

b) \( \forall z^t \in Z^T \) and \( \forall t, \) given the relative price \( \{Q_t(z^t)\}_{t=0}^{\infty}, z^t \in Z^T, \quad k_0(z^0) > 0 \) and the stochastic process for \( \chi_t, \) contingent plans \( \{i_t(z^t)\}_{t=0}^{\infty}, z^t \in Z^T \) solve capital producer’s problem:
\[
\max_{\{i_t(z^t)\}_{t=0}^\infty, z^t \in Z^T} E_t \left\{ Q_t(z^t) x_i i_t(z^t) - i_t(z^t) - \Phi \left( \frac{i_t(z^t)}{k_t(z^t)} \right) \right\} \\
\text{s.t.} \quad \left\{ \begin{array}{c}
\Phi \left( \frac{i_t(z^t)}{k_t(z^t)} \right) = \frac{\zeta}{2} \left( \frac{i_t(z^t)}{k_t(z^t)} - \delta \right)^2 k_t(z^t), \quad \forall t, \quad \forall z^t \in Z^T. \\
k_{t+1}(z^t) = \chi_t i_t(z^t) + (1 - \delta) k_t(z^t), \quad \forall t, \quad \forall z^t \in Z^T. \\
\log(\chi_t) = \rho_t \log(\chi_{t-1}) + \varepsilon_{t, i}, \quad \forall t, \quad \forall z^t \in Z^T. \\
k_0(z^0) > 0 \quad \text{given.}
\end{array} \right.
\]

c) \(\forall z^t \in Z^T\) and \(\forall t\), given the prices, \(\{Q_t(z^t), W_t(z^t), P_{j,t}(z^t), r_{j,t}(z^t)\}_{t=0, z^t \in Z^T}\) and the stochastic process for \(z_t\), the contingent plans \(\{y_{j,t}(z^t), k_{j,t}(z^t)\}_{t=0, z^t \in Z^T}\) solve entrepreneurs’ problem:

\[
\max_{\{y_{j,t}(z^t), k_{j,t}(z^t), h_{j,t}(z^t)\}_{t=0, z^t \in Z^T}} P_{j,t}(z^t) y_{j,t}(z^t) - w_t(z^t) h_{j,t}(z^t) - r_{j,t}(z^t) k_{j,t}(z^t)
\]

s.t. \(y_{j,t}(z^t) = z_t k_{j,t}(z^t) h_{j,t}(z^t), \quad \forall j, \quad \forall t, \quad \forall z^t \in Z^T.\)

\[
\log(z_t) = \rho_z \log(z_{t-1}) + \varepsilon_{z, t}, \quad \varepsilon_{z, t} \sim i.i.d. N(0, \sigma_z^2), \quad \forall j, \quad \forall t, \quad \forall z^t \in Z^T.
\]

d) \(\forall z^t \in Z^T\) and \(\forall t\), given the prices, \(\{P_{j,t}(z^t), \text{ner}_t(z^t)\}_{t=0, z^t \in Z^T}\) and the stochastic process for \(P_{t^*}\), the contingent plans \(\{y_{j,t}(z^t), y_{t^*}(z^t)\}_{t=0, z^t \in Z^T}\) solve retailers’ cost minimization problem:

\[
\min_{\{y_{j,t}(z^t), y_{t^*}(z^t)\}_{t=0, z^t \in Z^T}} P_{j,t}(z^t) y_{j,t}(z^t) + \text{ner}_t(z^t) P_{t^*}^* y_{t^*}(z^t)
\]

s.t. \(\text{da}_{j,t}(z^t) = \left[ (1 - \alpha_d)^{1/\theta_d} (y_{j,t}(z^t))^{1-1/\theta_d} + (\alpha_d)^{1/\theta_d} (y_{t^*}(z^t))^{1-1/\theta_d} \right]^{\theta_d}, \quad \forall i, \quad \forall t, \quad \forall z^t \in Z^T.\)

\[
\log(P_{t^*}^*) = \rho_{t^*} \log(P_{t-1}^*) + \varepsilon_{t^*, t}, \quad \varepsilon_{t^*, t} \sim i.i.d. N(0, \sigma_{t^*}^2), \quad \forall i, \quad \forall t, \quad \forall z^t \in Z^T.
\]

e) \(\forall z^t \in Z^T\) and \(\forall t\), given the prices, \(\{P_{t^*}(z^t), \text{mc}_t(z^t), P_{j,t}(z^t), \text{ner}_t(z^t)\}_{t=0, z^t \in Z^T}\), monopolistically retailers \(i \in [0, 1]\) sells retail good \(y_{i,t}(z^t)\) and choose the price \(P_{i,t}(z^t)\) in order to maximize its expected real total profit over the periods during which its price remains fixed subject to future demand:

\[
\max_{\{P_{i,t}(z^t)\}_{t=0, z^t \in Z^T}} E_t \left\{ \sum_{s=0}^\infty \nu^s \Delta_{i,t+s}(z^t) \left[ \left( \frac{P_{i,t}(z^t)}{P_{t^*}(z^t)} - \text{mc}_{t+s}(z^t) \right) \right] \right\}
\]

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Thus, the optimal contract implies that the risk premium follows:

\[ mc_t(z^t) = \left[ (1 - \alpha_d) \left( \frac{P_{l,t}(z^t)}{P_{l,s}(z^t)} \right)^{1-\theta_d} + \alpha_d \left( \frac{n_{\text{er},t}(z^t)P_r^*}{P_r(z^t)} \right)^{1-\theta_d} \right] \frac{1}{1-\theta_d}, \ \forall i, \ \forall t, \ \forall z^t \in Z^T. \]

\[ \Delta_{t,t+s}(z^t) = \beta_s \Delta_{t+s}(z^t), \ \forall i, \ \forall t, \ \forall z^t \in Z^T. \]

\[ \log(P_t^*) = \rho_{P^*} \log(P_{t-1}^*) + \varepsilon_{P^*,t}, \ \varepsilon_{P^*,t} \sim i.i.d. N(0, \sigma_{P^*}^2), \ \forall t, \ \forall z^t \in Z^T. \]

f) \ \forall z^t \in Z^T and \ \forall t, given the prices, \ \{P_t(z^t), P_{i,t}(z^t)\}_{t=0}^{\infty}, z^t \in Z^T, \ the \ contingents \ plans \ \{da_{i,t}(z^t), da_t(z^t)\}_{t=0}^{\infty}, z^t \in Z^T \ solve \ final \ goods \ producers’ \ problem:

\[ \max_{\{da_{i,t}(z^t), da_t(z^t)\}_{t=0}^{\infty}, z^t \in Z^T} P_t(z^t)da_t(z^t) - \int_0^1 P_{i,t}(z^t)da_{i,t}(z^t)di \]

s.t. \ \[ da_t(z^t) = \left[ \int_0^1 da_{i,t}(z^t)^{\frac{\varepsilon_{i,t}}{1-\varepsilon}} \right]^\frac{1}{\varepsilon_{i,t}}, \ \forall t, \ \forall z^t \in Z^T. \]

and the profits are zero.

g) For each history \ \forall z^t \in Z^T in each period t, given the prices, \ \{R_{k,t}(z^t), R_t(z^t), P_t(z^t)\}_{t=0}^{\infty}, z^t \in Z^T \ and \ the \ stochastic \ process \ for f_t, \ the \ spread \ between \ lending \ rate \ and \ policy \ rate \ is \ affected \ by \ both \ the \ regulation \ premium \ in \ the \ presence \ of \ macroprudential \ regulations \ and \ the \ default \ premium. \ Thus, \ the \ optimal \ contract \ implies \ that \ the \ risk \ premium \ follows:

\[ E_t \left[ \frac{R_{k,t+1}(z^t)}{R_t(z^t)P_{t+1}(z^t)} \right] = f_t \left( \frac{Q_t(z^t)k_{t+1}(z^t)}{n_{t+1}(z^t)} \right) \psi \left( \frac{c_{g_t}(z^t)}{c_{g}(z^t)} \right)^{\rho_r}, \ \forall t, \ \forall z^t \in Z^T \]

where, \ \log(f_t) = \rho_f \log(f_{t-1}) + \varepsilon_{f,t}, \ \varepsilon_{f,t} \sim i.i.d. N(0, \sigma_f^2) \ and \ c_{g_t}(z^t) = \frac{B_{t+1}(z^t)+B_t^*(z^t)}{B_{t-1}(z^t)+B_{t-1}^*(z^t)}.

h) Monetary and Financial policy obeys one of the four possible policy regimes:

i. Standard Taylor Rule (R1).

ii. Augmented Taylor Rule (R2).

iii. Macropreditural Regime with a Standard Taylor Rule (R3).

iv. Macropreditural Regime with the Augmented Taylor Rule (R4).

i) For each history \ \forall z^t \in Z^T in each period t, markets clear:

i. The total capital and the total labor in the economy is given by:

\[ k_t(z^t) = \int_0^1 k_{j,t}(z^t) dj, \ \forall j, \ \forall t, \ \forall z^t \in Z^T. \]
\[ h_t(z^t) = \int_{0}^{1} h_{j,t}(z^t) \, dj, \; \forall j, \; \forall t, \; \forall z^t \in Z^T. \]

ii. Household deposits in financial intermediaries are equal to total debt held by the entrepreneurs:

\[ B_t(z^t) = \int_{0}^{1} L_{j,t}(z^t) \, dj = L_t(z^t), \; \forall j, \; \forall t, \; \forall z^t \in Z^T. \]

iii. The aggregate entrepreneurial net worth is given by:

\[ n_{t+1}(z^t) = \zeta \left\{ R_{k,t}(z^t)Q_{t-1}(z^t)k_t(z^t) - E_{t-1} \left[ R_{k,t}(z^t) \right] (Q_{t-1}(z^t)k_t(z^t) - n_t(z^t)) \right\}, \]

and entrepreneurs going out of business will consume their residual equity:

\[ c_{e,t}(z^t) = (1 - \zeta)n_{t+1}(z^t), \; \forall t, \; \forall z^t \in Z^T. \]

iv. The budget constraints of the entrepreneurs is:

\[ Q_t(z^t)k_{t+1}(z^t) = n_t(z^t) + B_{t+1}(z^t), \; \forall t, \; \forall z^t \in Z^T. \]

v. The resource constraint for final goods is:

\[ da_t(z^t) = \left[ c_t(z^t) + c_{e,t}(z^t) + i_t(z^t) + \frac{\xi}{2} \left( \frac{i_t(z^t)}{k_t(z^t)} - \delta \right)^2 k_t(z^t) \right] \Gamma_t(z^t), \]

where, \( \Gamma_t(z^t) = (1 - \nu) \left[ \frac{\tilde{R}(z^t)}{P_t(z^t)} \right]^{-\varepsilon} + \nu \Pi_{t+1}(z^t) \Gamma_{t-1}(z^t), \; \forall t, \; \forall z^t \in Z^T. \)

vi. The relationship between total domestic demand and total supply of final goods is given by:

\[ da_t(z^t)\Gamma_t(z^t) = o\alpha_t(z^t), \]

where, \( o\alpha_t(z^t) \) is the aggregate supply of the composite goods, defined as:

\[ o\alpha_t(z^t) = \left[ (1 - \alpha_d)^{1/\theta_d}(y_t(z^t) - x_t(z^t))^{1-1/\theta_d} + (\alpha_d)^{1/\theta_d}(y^*_t(z^t))^{1-1/\theta_d} \right] \frac{\theta_d}{\theta_d - 1}, \]

and the foreign demand for exports is modeled as:

\[ x_t(z^t) = \left[ \frac{r_{er}(z^t)}{P_{j,t}(z^t)/P_t(z^t)} \right]^{\theta^*}, \; \forall t, \; \forall z^t \in Z^T. \]
C  Complete Set of Equilibrium Conditions

The equilibrium for the model economy, given macroeconomic policy rules for \( R_t \) and \( S_t \), is a sequence of:

i. quantities \( \{c_t, h_t, k_t, i_t, \Phi_t, y_t, n_t, B_t, \tau_t, cg_t, c_{e,t}, \kappa_t, da_t, oat_t, x_t, B^*_t, \Theta_t, y^*_t\}_{t=0}^{\infty}, \)

ii. and prices \( \{\Pi_t, w_t, Q_t, \Gamma_t, mc_t, R_{k,t}, P_t, \Omega_{1,t}, \Omega_{2,t}, P_{j,t}, rer_t, ner_t\}_{t=0}^{\infty}, \)

such that the following conditions are satisfied:

1. \( R_t = R^*_t \Theta_t \mathbb{E}_t \left\{ \frac{\Pi_{t+1}}{\Pi^*_{t+1}} \left[ \frac{rer_{t+1}}{rer_t} \right] \right\}. \)

2. \( \theta h^\eta_t c_t = w_t. \)

3. \( W_t h_t + R_{t-1} B_{t-1} - R^*_t \Theta_{t-1} ner_t B^*_t + \Upsilon_t - P_t c_t - B_t + ner_t B^*_t = 0. \)

4. \( k_t = \chi_t i_t + (1 - \delta) k_{t-1}. \)

5. \( \Phi_t = \xi^2 \left( \frac{i_t}{k_{t-1}} - \delta \right)^2 k_{t-1}. \)

6. \( w_t = (1 - \alpha) \frac{\Gamma^y_{t}}{k_t} P_{j,t}. \)

7. \( \mathbb{E}_{t-1} [R_{k,t}] = \frac{1}{Q_{t-1}} \mathbb{E}_{t-1} \left\{ P_{j,t}^\alpha \Gamma^y_{k_{t-1}} + (1 - \delta) Q_t \right\}. \)

9. \( y_t \Gamma_t = z_t k_{t-1}^{1 - \alpha} h_t. \)

10. \( Q_t k_t = n_t + B_t. \)

11. \( c_{e,t} = (1 - \zeta) n_t. \)

13. \( S_t = \mathbb{E}_t \left[ \frac{R_{k,t+1}}{R_t/\Pi_{t+1}} \right]. \)

14. \( S_t = f_t \left( \frac{Q_{t,k_t}}{\mu_t} \right)^\psi \tau_t. \)

15. \( \tau_t = \left( \frac{cg_t}{cg^*} \right)^{\rho_r}. \)

16. \( cg_t = \frac{B_t + B^*_t}{B_{t-1} + B^*_{t-1}}. \)
17. \( P_t = \frac{\varepsilon_1}{\varepsilon_1 - 1} \Omega_{z,t} \).

18. \( \Omega_{1,t} = y_t \, mc_t + \beta \nu \, \frac{c_t}{c_{t+1}} \Pi_{t+1}^{\varepsilon} \Omega_{1,t+1} \).

19. \( \Omega_{2,t} = y_t + \beta \nu \, \frac{c_t}{c_{t+1}} \Pi_{t+1}^{\varepsilon - 1} \Omega_{2,t+1} \).

20. \( P_t = \left( \frac{1 - \nu \Pi_{t+1}^{\varepsilon - 1}}{1 - \nu} \right)^{1 - \varepsilon} \).

21. \( \Gamma_t = (1 - \nu) \, P_t^{-\varepsilon} + \nu \Pi_t^\varepsilon \Gamma_{t-1} \).

22. \( da_t = c_t + c_{c,t} + i_t + \Phi_t \).

23. \( \kappa_t = \frac{k_t \Omega_t}{n_t} \).

24. \( R = \left( \begin{pmatrix} \phi_x \\ \frac{y_t}{y} \end{pmatrix} \right)^{1 - \phi_x} \left( \begin{pmatrix} R_{t-1} \\ R_t \end{pmatrix} \right) \phi_x \).

25. \( da_t \Gamma_t = oa_t \).

26. \( oa_t = \left( [1 - \alpha_d]^{1/\theta_d} (y_t - x_t)^{1 - 1/\theta_d} + (\alpha_d)^{1/\theta_d} (y_t^*)^{1 - 1/\theta_d} \right) \frac{\theta_d}{\theta_d + 1} \).

27. \( x_t = \left[ \frac{rer_t}{P_{j,t} / P_t} \right]^{\theta^*} \).

28. \( rer_t = ner_t P_t^* / P_t \).

29. \( mc_t = \left( [1 - \alpha_d] \left( \frac{P_{j,t}}{P_t} \right)^{1 - \theta_d} + \alpha_d \left( \frac{ner_t P_t^*}{P_t} \right)^{1 - \theta_d} \right) \frac{1}{\alpha_d} \).

30. \( \frac{y_t - x_t}{y_t^*} = \left[ \frac{1 - \alpha_d}{\alpha_d} \right] \left[ \frac{rer_t}{P_{j,t} / P_t} \right]^{\theta_d} \).

31. \( rer_t B_t^* = R_{t-1}^* \Theta_{t-1} rer_t B_{t-1}^* \left( \frac{B_{t-1}^*}{P_t} \right) - \frac{P_{j,t}}{P_t} x_t + rer_t y_t^* \).

32. \( \Theta_t = (B_t^* / B^*)^{\varepsilon} \).

### D  Real, Nominal and Financial Market Variables in the Model

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Table 9: Exogenous variables in the model.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Meaning</th>
<th>Stochastic AR(1) process</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R^*_t$</td>
<td>Foreign interest rate</td>
<td>$\log(R^<em><em>t) = \rho</em>{R^</em>} \log(R^<em><em>{t-1}) + \varepsilon</em>{R^</em>,t}, \varepsilon_{R^<em>,t} \sim \text{i.i.d.} N(0, \sigma^2_{R^</em>})$.</td>
</tr>
<tr>
<td>$P^*_t$</td>
<td>CPI for foreign economy</td>
<td>$\log(P^<em><em>t) = \rho</em>{P^</em>} \log(P^<em><em>t) + \varepsilon</em>{P^</em>,t}, \varepsilon_{P^<em>,t} \sim \text{i.i.d.} N(0, \sigma^2_{P^</em>})$.</td>
</tr>
<tr>
<td>$\chi_t$</td>
<td>Investment shock</td>
<td>$\log(\chi_t) = \rho_{\chi} \log(\chi_{t-1}) + \varepsilon_{\chi,t}, \varepsilon_{\chi,t} \sim \text{i.i.d.} N(0, \sigma^2_{\chi})$.</td>
</tr>
<tr>
<td>$z_t$</td>
<td>Total factor productivity shock</td>
<td>$\log(z_t) = \rho_z \log(z_{t-1}) + \varepsilon_{z,t}, \varepsilon_{z,t} \sim \text{i.i.d.} N(0, \sigma^2_z)$.</td>
</tr>
<tr>
<td>$f_t$</td>
<td>Financial shock</td>
<td>$\log(f_t) = \rho_f \log(f_{t-1}) + \varepsilon_{f,t}, \varepsilon_{f,t} \sim \text{i.i.d.} N(0, \sigma^2_f)$</td>
</tr>
<tr>
<td>$e_t$</td>
<td>Monetary shock</td>
<td>$\log(e_t) = \rho_e \log(e_{t-1}) + \varepsilon_{e,t}, \varepsilon_{e,t} \sim \text{i.i.d.} N(0, \sigma^2_e)$</td>
</tr>
</tbody>
</table>

Table 10: Endogenous variables in the model: the quantities.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_t$</td>
<td>Household labor.</td>
</tr>
<tr>
<td>$c_t$</td>
<td>Consumption.</td>
</tr>
<tr>
<td>$B_t$</td>
<td>Domestic holdings (deposits).</td>
</tr>
<tr>
<td>$B^*_t$</td>
<td>Foreign bonds.</td>
</tr>
<tr>
<td>$\Theta_t$</td>
<td>Risk premium for foreign bonds (liabilities).</td>
</tr>
<tr>
<td>$\Upsilon_t$</td>
<td>Dividend yields from retail firms and financial intermediaries.</td>
</tr>
<tr>
<td>$i_t$</td>
<td>Investment.</td>
</tr>
<tr>
<td>$k_t$</td>
<td>Aggregate capital stock.</td>
</tr>
<tr>
<td>$\Phi(\cdot)$</td>
<td>Quadratic capital adjustment cost.</td>
</tr>
<tr>
<td>$y_{j,t}$</td>
<td>Intermediate good $j$.</td>
</tr>
<tr>
<td>$n_t$</td>
<td>Entrepreneurial net worth.</td>
</tr>
<tr>
<td>$L_{j,t}$</td>
<td>Entrepreneurial nominal debt.</td>
</tr>
<tr>
<td>$\tau_t$</td>
<td>Macropreudential instrument.</td>
</tr>
<tr>
<td>$S_t$</td>
<td>Risk premium on domestic borrowing.</td>
</tr>
<tr>
<td>$\kappa_t$</td>
<td>Entrepreneurial leverage.</td>
</tr>
<tr>
<td>$V_t$</td>
<td>Entrepreneurial equity.</td>
</tr>
<tr>
<td>$c_{e,t}$</td>
<td>Aggregate entrepreneur’s consumption.</td>
</tr>
<tr>
<td>$\tau_t$</td>
<td>Macropreudential instrument.</td>
</tr>
<tr>
<td>$da_{i,t}$</td>
<td>Retailers’ intermediate composite good.</td>
</tr>
<tr>
<td>$x_t$</td>
<td>Exports of domestically produced goods.</td>
</tr>
<tr>
<td>$y_t$</td>
<td>Gross Domestic Product.</td>
</tr>
<tr>
<td>$da_t$</td>
<td>Aggregate demand for final goods.</td>
</tr>
<tr>
<td>$oa_t$</td>
<td>Aggregate supply of the composite goods.</td>
</tr>
<tr>
<td>$cg_t$</td>
<td>Growth rate of nominal credit.</td>
</tr>
</tbody>
</table>
Table 11: ENDOGENOUS VARIABLES IN THE MODEL: THE PRICES.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_t$</td>
<td>Nominal wage for household labor.</td>
</tr>
<tr>
<td>$w_t$</td>
<td>Real wage for household labor.</td>
</tr>
<tr>
<td>$P_t$</td>
<td>Aggregate consumption price index (CPI).</td>
</tr>
<tr>
<td>$R_t$</td>
<td>Domestic (gross) interest rate, where $R_t = \frac{(1+r_t)}{(1+\pi_t)}$.</td>
</tr>
<tr>
<td>$ner_t$</td>
<td>Nominal exchange rate (units of domestic currency per unit of foreign currency).</td>
</tr>
<tr>
<td>$rer_t$</td>
<td>Real exchange rate, where $rer_t = ner_tP^*_t/P_t$.</td>
</tr>
<tr>
<td>$d_t$</td>
<td>Nominal depreciation rate.</td>
</tr>
<tr>
<td>$\pi_t$</td>
<td>Domestic CPI inflation.</td>
</tr>
<tr>
<td>$\Pi_t$</td>
<td>Gross inflation, where $\Pi_t = (1 + \pi_t) = \frac{P_t}{P_{t-1}}$.</td>
</tr>
<tr>
<td>$Q_t$</td>
<td>Asset price.</td>
</tr>
<tr>
<td>$P_{j,t}$</td>
<td>Nominal price for intermediate goods ($y_{j,t}$).</td>
</tr>
<tr>
<td>$rr_t$</td>
<td>Rental rate of capital.</td>
</tr>
<tr>
<td>$R^K_t$</td>
<td>Gross return to holding a unit of capital.</td>
</tr>
<tr>
<td>$W_{e,t}$</td>
<td>Entrepreneurial wage.</td>
</tr>
<tr>
<td>$mc_t$</td>
<td>Real marginal cost.</td>
</tr>
<tr>
<td>$P_{a,t}$</td>
<td>Nominal price for intermediate goods ($da_{i,t}$).</td>
</tr>
<tr>
<td>$\Gamma_t$</td>
<td>Auxiliar variable in market clearing.</td>
</tr>
</tbody>
</table>