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Asymmetric Volatility Effects in Risk Management: An Empirical Analysis using a Stock Index Futures*

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Abstract: In this research paper ARCH-type models and option implied volatilities (IV) are applied in order to estimate the Value-at-Risk (VaR) of a stock index futures portfolio for several time horizons. The relevance of the asymmetries in the estimated volatility estimation is considered. The empirical analysis is performed on futures contracts of both the Standard and Poors 500 Index and the Mexican Stock Exchange. According to the results, the IV model is superior in terms of precision compared to the ARCH-type models. Under both methodologies there are relevant statistical gains when asymmetries are included. The referred gains range from 4 to around 150 basis points of minimum capital risk requirements. This research documents the importance of taking asymmetric effects (leverage effects) into account in volatility forecasts when it comes to risk management analysis.
Keywords: Asymmetric volatility, Backtesting, GARCH, TARCH, Implied volatility, Stock index futures, Value at Risk, Mexico.
JEL Classification: C15, C22, C53, E31, E37.

Resumen: En la presente investigación se aplican modelos de ARCH-tipo y volatilidades implícitas de opciones (IV) para estimar el valor en riesgo (VaR) de una cartera de futuros de índices bursátiles para varios horizontes temporales. Se considera la relevancia de las asimetrías en la estimación de la volatilidad. El análisis empírico es para los contratos de futuros de los Índices Standard and Poors 500 y el de la Bolsa Mexicana de Valores. De acuerdo con los resultados, el modelo IV es superior en términos de precisión. Si bien ambas metodologías muestran ganancias estadísticas relevantes cuando se incluyen asimetrías con respecto a cuando no se usan asimetrías, estas ganancias van de alrededor 4 a 150 puntos base de requerimiento mínimo de capital en riesgo. Se documenta la importancia de tener en cuenta los efectos asimétricos en los pronósticos de volatilidad en la gestión de riesgos.
Palabras Clave: Volatilidad asimétrica, Backtesting, GARCH, TARCH, Volatilidad implicíta, Futuros indices accionarios, Valor en Riesgo, México.

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I. Introduction

Measuring financial risks is the backbone of risk management and portfolio investments decisions. A well-known financial risk measure is the volatility of assets prices, as it allows the risk manager to assess potential risks associated with portfolio investments. Forecasts of price returns volatilities is perhaps the most used analytical toolkit for measuring financial risk, since are useful to make contemporaneous decisions based on expectations about the future level of prices. It is known in theory about the link between futures prices and expected future spot price. An advantage of using futures prices instead of spot prices, is that the former usually contain relevant information about the representative trader's future spot price expectation (Hull: 2013). Given the previously mentioned forward-looking component embedded in futures prices, it is relevant to consider these derivative instruments for financial decision making. Indeed, the price return volatility of futures prices has been used for risk management research in previous works (Bollerslev, Chou and Kroner: 1992, Engle: 2003).

In terms of volatility forecasts, part of the relevant research shows volatility estimates that are symmetric as they have the same volatility reaction (measure) when prices go up or down. However, statistical evidence shows or suggests that financial volatility may be asymmetric, i.e. the volatility reaction may be larger for price increases than for price decreases or vice versa. There are a few academic works that analyze price return volatilities with asymmetric effects (Poon and Granger: 2003; Giot and Laurent 2004). The reason for this may be that not all financial time series have asymmetric effects. Also, there is more complexity in the estimation methodology when asymmetric effects are included. When it comes to taking into account asymmetric volatility effects from a risk management perspective, i.e. within a Value at Risk (VaR) framework, there are even less documents (Chkili et. al.: 2014, Brooks and Persand: 2003). So far, there are more research that emphasizes in the monetary (not statistical) gains about VaR.

In the finance literature, volatility is seen as a risk variable, which captures all the uncertainty surrounding that financial variable. It is well-known that in the presence of asymmetric volatility it is important to adjust risk models to avoid possible under (over) quantification of risks. An unintended risk missestimation is undesirable for a financial institution involved in quantifying risks, given the potential costs associated with not being at an optimal risk quantification value (Brooks et. al.: 2000). On the one hand, handling an extreme event might be particularly costly to a firm with insufficient capital reserves, if its risk was underestimated in the first place. On the other hand, an overestimated risk may reserve excessive capital compared to its optimal level, which cause the manager to have more than is required (capital reserves), with a high opportunity cost of other uses of capital.

In this study, we aim to assess whether volatility is asymmetric and determine to what extent may impact stock market dynamics. The objective is to quantify if there are any statistically significant differences between taking and not taking into account possible asymmetric effects in volatility within a VaR framework for stock indices. The methodology involves backtesting techniques to validate VaR models that are relevant for risk management (Kupiec: 1995, Jorion: 2000, 2001, Nieppola 2009). The volatility measures used are GARCH, TARCH and option implied volatility (IV). These last two are able to capture volatility asymmetries. The ARCH-types are considered a 'backward-looking' forecast estimation methods, whilst the IV is considered a 'forward-looking' one. A contribution of the present research analysis is the combination of both types of techniques from a VaR perspective. For the purpose of the present study, asymmetric volatility is defined as the difference in the volatility level given positive or negative returns. In other words, the volatility may be higher when the returns are negative (or in some cases when they are positive).

The goal of the present research paper is twofold. First, provide optimal risk measures in order to avoid misestimation of risks, underlying the need to adjust for any volatility asymmetries. In doing that, we need to prevent unnecessary financial costs related to a non-optimal risk measure. By adjusting the model for volatility asymmetries, it is expected that a risk measure with higher accuracy can be obtained. Second, we propose a novel approach to compare two different types of asymmetric volatility forecasts: one that is backward-looking (ARCH-type) and one that is forward-looking (option-implied), within a VaR framework. Most research papers use one or the other measure but they do not compare them in terms of statistical performance in a VaR paradigm (Giot and Laurent: 2004, Chkili et. al.: 2014). We here assess each one of them and various combinations, including statistical inference. We start comparing volatility symmetries vs. asymmetries and also backward- vs. forward-looking within a VaR model. The comparison of the results for two different types of asymmetric volatility methodologies could give light about any qualitative differences that should be accounted for when deciding on VaR estimations.

The results show that asymmetric models provide more accurate VaR estimations. Among these the forward-looking forecast model is superior when compared to the backward-looking one. Based on these results, it is recommended to apply asymmetric IV volatility models, with VaR in particular, to perform risk management analysis. We argue that the present paper adds relevant information to the academic literature on of volatility forecast methodologies since our findings provide evidence about the accuracy gains of using asymmetric volatility models in the risk management sector. Also, the results can be useful for those involved in the risk management industry, i.e. portfolio-risk managers, to have better knowledge about superior (and close to optimal) estimation procedures.

The layout of this paper is as follows. The literature review is presented in Section II. Symmetric and asymmetric models are explained in Section III. Data is described in detail in Sections IV and V, respectively. The results of four volatility models are discussed in Section VI. Section VII concludes.

II. Literature Review

Historical (backward-looking) volatility is described by Brooks (2013) as the variance or standard deviation (σ) of returns over some long period of time (n). This unconditional variance or standard deviation may serve as a volatility forecast for all future periods (Markowitz: 1952). However, there is a drawback in this type of calculation. Unconditional volatility is assumed to be constant. Thus, for an n-days ahead forecast the unconditional standard deviation of the price-return series needs to be multiplied by the square root of the n-days considered in the forecast horizon, i.e. $\sigma \times \sqrt{n}$. With this product it is possible to obtain the time-adjusted volatility forecast for those n-days ahead. Nowadays, it is well known that financial volatility is time-varying, i.e. that volatility changes over time. This is also the case for the volatility of stock indices that are usually included in investment and portfolio analysis and investment-decision making.

It is well documented that non-linear Autoregressive Conditional Heteroskedasticity type models (ARCH-type models) can provide accurate in-sample estimates of time-varying price volatility. These type of volatility forecast models are also considered backward-looking, i.e. the estimates are obtained based on past time series data.[1] See for example, Engle (1982), Taylor (1986), Bollerslev, Chou and Kroner (1992), Wei and Leuthold (1998), Engle (2002), among many others.[2] However, the out-of-sample forecasting accuracy of this type of non-linear models is, in some cases, questionable (see Park and Tomek: 1989, Schroeder et. al.: 1993, Manfredo et. al.: 2001, Benavides: 2006, 2009, Pong et. al.: 2003).[3]

In addition, there is a related literature of the implications of non-linear dynamics of volatility forecasts for financial risk management (Hsieh: 1993). In light of this, some researchers have extended previous work on the application of time-varying volatility models, specifically ARCH- type models, and IV in VaR estimations (Brooks, Clare and Persand: 2000; Manfredo et. al.: 2001; Engle: 2003; Giot: 2005; Mohamed: 2005; among others). IV estimations are considered forward-looking since they are implied by derivative contracts (prices) and it is believed that these increase the performance of the volatility forecasts compared with the ARCH-type. Most of previously mentioned findings improved risk management applications using VaR, by proving to be more accurate in terms of quantifying risk. Although, there are several research papers that use these type of models for financial time series, there is, to the best of our knowledge, no research that carries out a rigorous comparison of volatility asymmetries using both ARCH-type (backward-looking) and option-implied (forward-looking) volatility within a risk management VaR perspective.

[1] The volatility forecast that it is considered here is the conditional volatility of a financial asset, which is estimated from an econometric model assuming a standard distribution for the estimated parameters.
[2] For an excellent survey about applications of ARCH-type models in finance the interested reader can refer to Bollerslev, Chou and Kroner (1992).
[3] All of them found that the explanatory power of these out-of-the-sample forecasts is relatively low. In particular, Pong et al. (2003) find that option implied volatility forecasts performed at least as well as forecasts from Autoregressive Fractional Integrated Moving Average Models (ARFIMA) for time horizons of one and three months. These were superior forecasts to those from ARCH-type models.

Furthermore, there are no research documents in which there is an empirical comparison between risk properties of two stock indices, one from an advanced economy (the SP500 stock index) and one from an emerging economy market (the IPC Mexican stock index), in which volatility asymmetries are accounted for.

Additionally, in terms of risk management applications in financial regulation, previous works have applied non-linear models within a VaR framework in order to estimate Minimum Capital Risk Requirements (MCRR) (Hsieh: 1991; Brooks, Clare and Persand: 2000). The MCRR are defined as the minimum amount of capital needed to successfully handle all but a pre-specified percentage of possible losses with a certain confidence level (Brooks, Clare and Persand: 2000). This concept is relevant to banks and bank regulators. For the latter, it is important to require banks to maintain enough capital so they can successfully cope with unforeseen losses. These regulatory practices go back to the original Basel Accord of 1988. Even though there is a broad consensus about the need of MCRR, there is significantly less agreement about the method to calculate them.[4] By estimating the VaR of their financial portfolios banks are able to calculate the amount of MCRR needed to meet bank supervisory requirements.[5] An additional contribution of the present research paper is the estimation and evaluation of MCRRs using both types of volatility forecasts with asymmetries (backward-looking vs. forward-looking).

Among the main objectives of the present research document is to extend the research of Hsieh (1991) and Brooks, Clare and Persand (2000) in two dimensions. One is that MCRRs are estimated for futures contracts of the S&P 500 and the IPC. The other one is a formal analysis of empirical applications of ARCH-type models and option implied volatilities, which both include asymmetric volatility effects. In addition, the present

[4] According to Brooks, Clare and Persand (2000) the most well-known methods are the *Standard*/*International Model Approach* of the Basel Accord (1988), the *Building-Block Approach* of the EC Capital Adequacy Directive (CAD), the *Comprehensive Approach* of the Securities Exchange Commission (SEC) of the US, the *Pre-commitment Approach* of the Federal Reserve Board (FED) and the *Portfolio Approach* of the Securities and Futures Authority of the UK.

[5] According to Basel Bank Supervision Requirements of 1988, banks have to hold capital (as a precautionary action) at least three times the equivalent to the VaR for a time horizon of 10 trading days at the 99% confidence level. There are no significant changes to this rule in the Basel II and III accords. The only change is that for repo-notes the time horizon must be 5 trading days. The interested reader can consult the previously mentioned information at the BIS webpage: http://www.bis.org/publ/bcbs107.htm

estimations do have implications for stock index level forecasts, given that simulations are carried out with certain statistical confidence levels. By considering a similar methodology as the one used in Hsieh (1991) and Brooks, Clare and Persand (2000), it is possible to have an idea of the future levels of both stock indices (S&P500 and IPC) with certain statistical confidence. For example, if a 95% confidence level VaR with a time horizon of one month is applied, it is possible to quantify the range of possible stock index level values one-month ahead, again, with 95% statistical confidence. Along the same lines, it is possible to quantify the probability of observing extreme values, i.e. those outside the 95% interval in a parametric and non-parametric distribution. The former is achieved with one-step ahead volatility forecasts from a parametric model (ARCH-type) whilst the latter is achieved by applying bootstrap simulation methods.

Furthermore, rigorous statistical accuracy tests for estimating VaR are carried out between ARCH-type models vs. IV following Kupiec (1995) backtesting tests. These will include an asymmetric volatility adjustment in them. The latter considers the number of violations or exceptions that occurred within the confidence intervals, i.e. the number of times the realized observed value was outside the relevant forecast range or confidence interval. Therefore, the null and alternative hypotheses to test are the following,

H_0: ARCH-type and IV asymmetric volatility are not accurate to estimate VaR.

H_1: ARCH-type and IV asymmetric volatility are accurate to estimate VaR.

Rejection of the null hypothesis will be in favor of asymmetric volatility modelling as being superior in terms of higher volatility forecasting accuracy for the VaR model. In order to test the null hypothesis, the results will be analyzed, again, according to the backtesting methodology (Kupiec: 1995; Jorion: 2001; Nieppola: 2009). These findings contribute new knowledge to the existing academic literature given that volatility asymmetries are included in estimation techniques in order to have a more accurate measure of financial risk. These results could be of interest of agents involved in risk management decisions related to stock index forecasts, i.e. private bankers, financial analysts, financial institutions management, policy makers, investors, futures traders, central banks, academic researchers, among others. In particular, this topic could be of interest to policymakers in countries that have relatively high stock market volatility, as is more common in emerging economies.

III. The Models

III.1 Volatility Models

III.1.1 ARCH-Type (GARCH-Symmetric) Specification

The volatility of the time series under analysis is estimated with historical data. A well-known model within the family of ARCH-type models is the univariate Generalized Autoregressive Conditional Heteroscedasticity, GARCH(p, q) model. This is an extension of the ARCH(q) model, in the sense that the ARCH model is nested in the GARCH model. The GARCH(p, q) model is estimated by applying the standard procedure as explained in Bollerslev (1986) and Taylor (1986).[6] The formulae for the GARCH(p, q) are presented below. For the model there are two main equations. These are the conditional mean equation and the conditional variance equation:

Conditional mean equation,

$$\Delta y_t = \mu + e_t \tag{1}$$

$$e_t \mid I_{t-1} \sim N(0, \sigma^2{}_t),$$

and the conditional variance equation,

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \ . \tag{2}$$

where Δy_t are the first differences of the natural logarithm (logs) of the series under analysis at time t (the futures-index), e_t is the error term at time t, I_{t-1} is the information set at time t-1, $\sigma^2{}_t$ is the conditional variance at time t. μ, ω, $\alpha_{i,}$ β_i are parameters and it is assumed that the log returns are normally distributed. In other words, assuming a constant mean μ (the

[6] The ARCH-type models presented in the present research paper were estimated using Eviews computer.

mean of the series Δy_t), the distribution of e_t is assumed to be Gaussian with zero mean and variance σ^2_t. The parameters are estimated using a maximum likelihood methodology applying the Marquardt algorithm.[7]

Considering that the assumption of normality of the residuals stated above usually does not hold, the Bollerslev and Wooldridge (1992) methodology is used in order to estimate consistent standard errors. The estimators under the previously mentioned procedure are then statistically robust and obtained from Quasi-Maximum Likelihood Estimation. Thus, the coefficients are robust even if the normality assumption is not met by the data.[8] The estimated coefficients can be used for statistical inference if they are statistically significant (statistically different from zero) and meet the conditions that the sum of the $\alpha + \beta < 1$ (otherwise, if the latter does not hold the series are considered explosive or, equivalently, non-mean reverting, which are undesirable properties when forecasting financial series, Taylor: 1986).

III.1.2 Threshold GARCH (GARCH-Asymmetric) Model

Another model used in this paper is the Threshold GARCH model, also known as TARCH. It was postulated by Glosten, Jaganathan, and Runkle (1993) and Zakoïan (1994). Compared with the GARCH(p, q) model, the specification of the TARCH model involves an additional term in the variance equation, that captures the asymmetric dynamics of the price-returns:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 + \sum_{k=1}^{r} \delta_k \varepsilon_{t-k}^2 I'_{t-k} \;, \tag{3}$$

where $I'_t = 1$ if $\varepsilon_t < 0$ and 0 otherwise. ε_t represents an innovation (error term). The intuition for this model is that bad news $\varepsilon_t < 0$ will have a different impact on the conditional variance compared to good news $\varepsilon_t > 0$.[9] In case of good news, the impact is on α_i for bad news the

[7] This algorithm modifies the Gauss-Newton algorithm by adding a correction matrix to the Hessian approximation. This allows handling numerical problems when the outer products are near singular, thus increasing the chance of improving the convergence of the parameters.
[8] For more details about Quasi-Maximum Likelihood Estimation the interested reader can refer to Bollerslev and Wooldridge (1992).
[9] Good news refers to news that increase financial assets returns. Bad news is the opposite.

impact is on $\alpha_i + \delta_i$. If $\delta_i > 0$ and is statistically significant, there will be a higher increase in volatility driven by bad news. If $\delta_i \neq 0$, then the news impact is asymmetric. This model is normally used for estimating stock price volatility considering the leverage effect on stocks.[10] For the case of the stock-index futures, the asymmetric TARCH model is applied.

III.1.3 Option-Implied Volatility (IV) Model

In this research paper, option implied volatilities provided by the Chicago Mercantile Exchange and MexDer are used. These are the VIX (option implied volatility index), which is an implied volatility for one-month ahead, and the VIMEX, which is the equivalent volatility index for the Mexican stock index for the same maturity. The option-implied volatility of an underlying asset is the market's forecast of its volatility and this is obtained with the options written on that underlying asset (Hull: 2013). To calculate an option implied volatility of an asset, an option valuation model is needed as well as the inputs for that model, such as the risk-free rate of interest, time to maturity, price of the underlying asset, the exercise price and the price of the option. An inappropriate valuation model will produce pricing errors and the option implied volatilities will be mismeasured (Harvey and Whaley: 1992). For example, a valuation model that does not consider the early exercise privilege of an American option to find the option implied volatilities from American options will produce errors in the calculations, i.e. using the Black and Scholes (1973) model to find the option implied volatilities from American options (henceforth, the BS model).[11]

To obtain the aforementioned implied volatility indices, the relevant derivatives exchanges use an approximation valuation method similar to the one widely known to price options, i.e. the BS. For completeness, an explanation of the BS is given next. The assumptions made for this model are: 1) Interest rates are non-stochastic, which means that

[10] The leverage effect on stocks refers to asymmetric volatility considering that a bear market sentiment has higher price volatility when compared with a bull market sentiment. In a bear market higher uncertainty about the cash flow stream could cause the stock price to decrease and the company increases its leverage ratio, which is undesirable (Brooks: 2013).
[11] The Black-Scholes option valuation model is for European-style options. These options do not have the early exercise privilege that American-style options do have.

the forward is equal to the futures price; 2) there are no-arbitrage profits, 3) all options are European; 4) agents are risk-neutral; 5) there are no transaction costs, and 6) the prices follow a Geometric Brownian Motion. The BS for exchange rates is stated formally in Equation 10 below.

$$c = Se^{-r_f T}N(d_1) - Xe^{-rT}N(d_2) \quad , \tag{4}$$

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r - r_f + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

$d_2 = d_1 - \sigma\sqrt{T}$, where c is the value of the European call option, T represents the time to maturity of the option, $N(x)$ is the cumulative probability distribution function, which is assumed to be normally distributed (in other words, the probability that a variable with a standard normal distribution, $\psi(0, 1)$ will be less than x). The exercise price is represented by X, $ln(\cdot)$ is the natural logarithm function and σ is the asset's volatility measured as its annualized standard deviation. The other variables are the same as previously defined. To find the relevant implied volatility the model is inverted to solve for σ given a market (observed) price for the option.

III.2. The Value-at-Risk (VaR) Model

The Value-at-Risk or VaR model is a useful measure of risk.[12] It was developed in the early 1990s by the JP Morgan Corporation. According to Jorion (2001) 'VaR summarizes the expected maximum loss over a target horizon with a given confidence interval.' Even though it is a statistical figure, most of the times VaR estimates are presented in monetary terms. The intuition is to have an estimate of the potential change in the value of a financial asset resulting from systemic market changes over a specified time horizon (Mohamed: 2005). It is also normally used to obtain the probability of losses for a financial portfolio of futures contracts. Assuming normality, the VaR estimate is relatively easy to obtain from GARCH models. For example, for a one trading day 95% confidence interval VaR, the

[12] Value at Risk is normally abbreviated as VaR. The lower case '*a*' letter differentiates this abbreviation to that of Vector Autoregressive Models, which are usually abbreviated as VAR (with a capital *A*).

estimated GARCH standard deviation (for the next day) is multiplied by ±1.645. If the standard deviation forecast is, say, 0.0065, the VaR is approximately 1.07%, looking at the positive tail of the distribution. To interpret this result, it could be said that an investor can be 95% sure that she will not lose more than 1.07% of asset or portfolio value in that specific day. However, a problem with the parametric approach is that if the observed asset returns depart significantly from a normal distribution the applied statistical model may be incorrect to use (Dowd: 1998).

As mentioned, when using VaR models it is necessary to make an assumption about the distribution of the returns. Although normality is often assumed for price returns series, it is known in practice that this assumption is highly questionable (Mandelbrot: 1963, Fama: 1965, Engle: 1982, 2003). If the daily returns are divided by the (adjusted) TARCH standard deviations, the new series will have a constant volatility with a non-normal distribution (Engle: 2003). For these 'standardized residuals' or 'de-volatized returns' the kurtosis must be above normal, thus a non-normal distribution is assumed in the VaR. The volatility asymmetries estimated within the TARCH model allow for this non-normality. This method will be considered here for the estimation of VaR for time horizons of one trading day. However, there is also another approach which will also be applied in this project for time horizons of more than one trading day. This is explained next.

For time horizons of more than one trading day (ten, and twenty trading days), the bootstrapping methodology of Efron (1982) is applied.[13] The fact that the returns of the series are non-normally distributed motivates the use of a non-parametric procedure such as bootstrapping. The procedure used in Hsieh (1993) and Brooks, Clare and Persand (2000) is considered here. In the latter, they empirically test the performance of that VaR model for futures contracts traded in the London International Financial Futures Exchange (LIFFE).[14]

[13] The bootstrap is a resampling method for inferring the distribution of a statistic, which is derived by the data in the population sample. This is normally estimated by simulations. It is said to be a nonparametric method given that it does not draw repeated samples from well-known statistical distributions. Alternatively, a Monte Carlo simulation draws repeated samples from assumed statistical distributions. In this research project the bootstrap methodology was implemented using Eviews.
[14] These futures contracts were the FTSE-100 stock index futures contract, the Short Sterling contract and the Gilt contract.

A similar paradigm is applied here for stock-indexed futures contracts. Thus, a hypothetical portfolio of stock-indexed futures is considered and MCRRs are estimated.[15] These estimated MCRRs values for the stock index futures portfolio are compared to the observed (historical) inflation. This analysis allows the evaluation of how accurate are the ARCH-type models in terms of estimating MCRRs for stock-indexed futures. Another objective is to analyze the performance of these in terms of how accurate they are for providing an upper threshold for the stock index, i.e. the statistical chances that the stock index will be high enough to be outside the upper (positive) confidence interval.

In order to calculate an appropriate VaR estimate it is necessary to find out the maximum loss that a position might have during the life of the futures contract. In other words, by replicating via the simulations (bootstrapping) the daily values of a long futures position it is possible to obtain the possible loss during the sample period. This will be given by the lowest replicated value. The same reasoning applies for a short position. But in that case the highest possible loss will be given by the highest replicated value.[16] Following Brooks, Clare and Persand (2000) and Brooks (2013) the formula is as follows. The maximum loss (L) is given by

$$L = (P_0 - P_1) * Number\ of\ contracts \tag{5}$$

where P_0 represents the price at which the contract is initially bought or sold; and P_1 is the lowest (highest) simulated price for a long (short) position, respectively, over the holding period. Without loss of generality it is possible to assume that the number of contracts held is one. Algebraically:

$$\frac{L}{P_0} = \left(1 - \frac{P_1}{P_0}\right). \tag{6}$$

[15] In finance textbooks it is common to see that the theoretical futures (forward) price is expressed in continuous time, (Hull: 2013, pg. 46): $F_0 = S_0 e^{rT}$. Where F_0 is the current futures (or forward) price, S_0 is the current spot price, e equals the $e(\cdot)$ function, r is the risk-less rate of interest per annum expressed with continuous compounding and T is the time to maturity in years. For the previous formula it is assumed that the underlying asset pays no income. For the research purposes of this project F_0 equals the observed stock-index futures price as reported by CME and MEXDER (in discrete time) and S_0 equals the observed stock index spot price, taken from a Bloomberg terminal.

[16] As it is well known in futures market payoffs that decreases in futures prices mean losses for long positions and increases in futures prices mean losses for short positions.

Given that P_0 is a constant, the distribution of L will depend on the distribution of P_1. It is reasonable to assume that prices are lognormally distributed (Hsieh: 1993), i.e. the log of the ratios of the prices are normally distributed. However, this assumption is not considered here given that empirical distributions of the series under study are not normal. However, the log of the ratios of the prices is transformed into a standard normal distribution following J.P. Morgan Risk-Metrics (1996) methodology. This is done by matching the moments of the log of the ratios of the prices' distribution to a distribution from a set of possible ones known (Johnson: 1949). Following Johnson (1949) a standard normal variable can be constructed by subtracting the mean from the log returns and then dividing it by the standard deviation of the series,

$$\frac{\ln\left(\frac{P_1}{P_0}\right) - \mu}{\sigma}. \tag{7}$$

The expression above is approximately normally distributed. It is known that the 5% lower (upper) tail critical value is -1.645 (+1.645).

From Equation 6 the following can be expressed as

$$\frac{L}{P_0} = 1 - \exp\left[-1.645\sigma + \mu\right] \tag{8}$$

when the maximum loss for the long position is obtained. For the case of finding the maximum possible loss for the short position the following formula applies:

$$\frac{L}{P_0} = \exp\left[1.645\sigma + \mu\right] - 1. \tag{9}$$

The MCRRs of the short position can be interpreted as an upper threshold for the stock index. By the same reasoning the MCRRs of the long position can be interpreted as a lower threshold for the stock index.

The simulations were performed in the following way. The GARCH and TARCH models were estimated with the bootstrap using the standardized residuals from the whole sample (instead of residuals taken from a normal distribution as was written in Equation 1). The stock index variable was simulated, for the relevant time horizon (10 and 30 trading days) with 10,000 replications. The formula used was $Y_{t+1} = Y_t e^{rT}$(where Y is the futures price, rT are the underlying asset's returns for time T, from an ARCH-type (GARCH, TARCH) model and the rest of the notation is the same as specified above). From the futures price indices simulations, the maximum and minimum values were taken in order to have the MCRRs for the short and long positions respectively.

IV. Data

IV.1. Data Sources

It is known that the US capital market is a relatively large and liquid market. In contrast the Mexican capital market is relatively smaller and less liquid. These different types of markets may give light about differences between capital markets (investments) between developed and developing stock markets within a VaR framework. In the present research project, stock index volatility for both the US Standard & Poors 500 Index (S&P500) and Mexico 'Índice Nacional de Precios y Cotizaciones' (IPC) are analyzed using their respective daily stock futures indices. The methodology is carried out for futures prices of both stock indices. The data consists of daily spot and futures closing prices of the IPC and S&P indices obtained from MEXDER and CME respectively.[17] Table 1 shows the contract specifics for each of the underlying assets under analysis. The sample period under analysis consists of more than two years of daily data for a time frame from 3rd January 2016 to 30th December 2019. The sample size consists of 889 daily observations. The sample period was chosen considering the most recent data available in Bloomberg, which for IPC futures is from January 2016. The sample size of 889 observations is considered large enough for the estimation task at hand. These types of derivative contracts have daily trading and daily data is commonly publicly available. Given that the time horizon for these simulations is relatively short (up to one month ahead) there is no need for a larger sample size. The futures contracts for the Mexican IPC have delivery dates for up one year and a half ahead. The periodicity of the maturities of the contracts is four times within one year and the delivery months are

[17] The MEXDER web page is http://www.mexder.com.mx/MEX/paginaprincipal.html
The CME webpage is https://www.cmegroup.com/trading/equity-index/us-index/sandp-500.html

March, June September and December. The MEXDER is relatively new compared to other derivatives exchanges around the world. It began operations in December 1998, whilst Chicago started in 1848.

IV.2. Data Transformation

When creating a time-series of futures prices a significant number of researchers use the prices of the futures contract closer to maturity or the one with higher trading volume.[18] These procedures have the inconvenience of creating a pattern of 'jumps' in the price series when switching prices from one futures contract to another.[19] This type of 'jumps' is unrealistic according to the market's price dynamics. Even though 'jumps' are observable in futures prices, there is usually no clear pattern. In order to avoid these unrealistic 'jumps' when creating a time-series of futures prices from different contracts (Pelletier, 1983; Wei and Leuthold: 1998), synthetic futures prices were created.[20] These were calculated by a 'roll-over' procedure that is basically an interpolation of futures prices from different maturity futures contracts (Herbst et. al. 1989, Kavussanos and Visvikis: 2005). This procedure creates a constant maturity weighted average futures price based upon the futures prices and the days to maturity of the two nearby expiration contracts. The formula used to obtain the synthetic futures price is shown in Equation 10 below:[21]

$$SYN_T = F_j\left[\frac{(T-T_i)}{(T_j-T_i)}\right] + F_i\left[\frac{(T_j-T)}{(T_j-T_i)}\right] \quad , \tag{10}$$

where SYN_T is the synthetic futures price for delivery at T, F_j is the contract j futures price expiring at T_j, F_i is the contract i futures price expiring at T_i, T equals 30, the chosen constant

[18] Even though futures contracts can be used to hedge financial risk it is common to observe that, in some cases, there is not an optimal demand for them. For example, see Benavides and Snowden (2006) for details.

[19] For a good reference about the mechanics of futures markets the reader could refer to Fink and Feduniak (1988).

[20] The synthetic futures prices were calculated using the Visual Basic for Applications computer language.

[21] The terms synthetic futures price and futures price are taken to be synonymous for the rest of this paper.

maturity in number of days, T_i is the contract i expiration in days remaining, T_j is the contract j expiration in days remaining, $j = i + 1$, with $T_i \leq T \leq T_j$.

The time to expiration of the synthetic futures prices calculated is T equals 30 days. This means that a constant 30-day maturity synthetic futures price was calculated. This is considered an appropriate time-to-expiration given that a shorter time-to-expiration could have higher expected volatility. This situation is observed in empirical research papers, which have found that volatility in futures prices increases as a contract gets closer to expiration (Samuelson: 1965). This could be the case for futures contracts of less than 30 days remaining. A higher expected volatility due to time-to-expiration could bias the results of this analysis. It is also possible to increase the maturity of the futures price if needed and we could always have a greater maturity contracts available for comparison.

V. Descriptive Statistics

This section presents the descriptive statistics for the daily (observed) volatilities of the IPC and S&P500 spot and futures returns. The volatility forecast from the models is also presented. Prior to fitting the GARCH and TARCH models shown in the graphs, an ARCH-effects test was conducted for the series under analysis. This was done in order to see if these types of models are appropriate for the data (Brooks: 2002). The test conducted was the ARCH-LM following the procedure of Engle (1982).[22] According to the results both series under study have ARCH effects. Under the null of homoscedasticity in the errors the *F*-statistics were 8.04 for the spot and 4.00 for the futures prices of the IPC (the critical value is 2.21 for 5 restrictions, 877 degrees of freedom). Both statistics clearly reject the null in favour of heteroscedasticity on those errors. For the S&P 500 the results were qualitatively

[22] These tests were conducted by using ordinary least squares, regressing the logarithmic returns of the series under analysis against a constant. The ARCH-LM test is performed on the residuals of that regression. The test consists on regressing, in a second stage regression, the square residuals against constant and lagged values of the same square residuals. The square residuals are a proxy for the variance. The null hypothesis is that the errors are homoscedastic. An *F*-statistic was used in order to test the null. The test was carried out with different lags 2 to 10. All have the same qualitative results. Only the cases for 5 lags are reported in the main text above, given the common practice in the literature for daily data, in which there could be seasonality effects or 'day of the week effects' with that frequency of the data. Thus, five trading days can take into consideration the previously mentioned situation.

similar, indicating heteroscedasticity on those errors dynamics (*F*-statistics were 21.75 for the spot and 21.79 for the futures prices of the S&P500). Therefore, it is consistent to apply ARCH-type models to the data.

Figure 1 presents the logs of the spot and futures prices of the IPC and their respective daily volatilities for the time frame under analysis.[23] It can be observed that the futures price is usually above the spot price. This could be an indication of the expected inflation reflected in futures prices (Working: 1958). Also, it can be observed that the futures volatility is considerably higher than the spot volatility (Samuelson: 1965). Figure 2 presents the logs of the spot and futures prices of the S&P500 and their respective daily volatilities for the relevant sample period. The graph is qualitatively different to that one for the IPC, given that it is not clear that futures price is usually above the spot price, which gives insights of a 'normal backwardation' market (the spot price being above the futures price). It is sometimes the case that markets show 'normal backwardation' and it is related mostly to random events (Working: 1958). The difference between both indices may be related to the liquidity and trade volume, which is significantly larger for the S&P500. Also, there is a clear trend in the S&P500 series, which is not seen for the IPC.

According to Zivot (2009) it is possible to test for asymmetric effects analyzing the returns of the sample series. If r_t^2 and r_{t-1} have a negative correlation coefficient (statistically different from zero) then there are asymmetric effects (also known as 'leverage effects'). For the series under study these are -0.0634 and -0.0989 for the spot and futures respectively IPC series and -0.1570 and -0.2089 for the spot and futures respectively S&P500 series.[24] Given that there are asymmetric effects the TARCH model explained above will be applied in the following estimations. Tables 2 and 3 show the parsimonious specifications GARCH(1,1) and TARCH(1,1) for the IPC and S&P500, respectively. These models were chosen according to results obtained from information criteria (Akaike Information Criterion and Schwarz Criterion tests). The model parameters were positive and most of them statistically

[23] The daily volatility is simply defined as the absolute value of the log-return.
[24] The relevant *t*-statistics for these estimated coefficients are -4.95 and -7.61 for the IPC and -9.33 and -6.80 for the S&P500, which clearly rejects the null hypothesis of the estimated coefficients being equal to zero at the 1% significance level.

significant at the 5% level. The sum of $\alpha_1 + \beta_1$ was less than one. Diagnostic tests on the models were applied to ensure that there were no serious misspecification problems. The Ljung-Box statistic in the Autocorrelation Function was applied on the standardized residuals obtained from the forecast models (white noise test). This shows that these residuals are white noise by analysing the test statistic at the twelve lag, so these are *i.i.d.*, which show no serious specification problems with the estimated models, considering that Portmanteau test.

Table 4 shows the descriptive statistics for the daily volatility and the volatility from the forecasting models for the IPC and S&P500. As it can be observed, the means of the futures IPC series are the ones with higher values (the daily volatilities and the volatility forecasts). These findings are consistent with Figure 1 where the daily volatility of the futures was normally seen higher than the spot's volatility. The distributions in that table are highly skewed and leptokurtic, indicating non-normality of the returns and the forecast estimates. This is consistent with the work of Wei and Leuthold (1998) that analyzed volatility in futures markets and had similar findings with daily futures price volatility for agricultural commodities. In terms of the S&P500, it can be observed that there is also clear evidence of time series (either for spot and futures prices) with a distribution different than the standard normal, given that the skewness and kurtosis values are different to zero and three respectively, which are the values for a standard normal distribution. In terms of comparing the IPC vs S&P500, it can be observed that the kurtosis for the latter is relatively larger to that of the former. This is an indication that for the time period under analysis the S&P500 had more extreme (tails) events, compared with the IPC.

Lastly, Figure 3 presents the observations of the daily IPC (top line) and the estimates of the volatility forecast models for the futures and spot series respectively (bottom lines). It can be observed in both graphs that the models captured the volatility clustering shown for the daily volatility. The implications of these forecasts are that they capture fairly well the dynamics of the IPC levels for both series under study. It is worthwhile to mention that the peaks observed in that graph coincide with financial volatility periods: the FED increase of its target interest rates, Brexit and the 2016 US elections. That is, the GARCH(1,1) and TARCH(1,1) models show forecasts that predict high volatility when in fact the actual IPC level was low and predict low volatility when the actual IPC level was high. The forecasts are relatively consistent in terms of capturing the dynamics for basically all the days in the sample. Similarly, Figure 4 shows the same type of information for the S&P500 series. As it can be observed in Figure 4 the results are qualitatively similar the ones obtained for the IPC series. The peak values coincide for both series in relation with the events of the FED increasing its target interest rate.

VI. Results

VI.1 Parametric Method

Once the next-day volatility estimate is obtained, the 95% confidence intervals are created by multiplying ±1.96 by the forecasted conditional standard deviation (from the GARCH and TARCH model). An analysis is made about the number of times the observed IPC spot return was above that 95% threshold (a violation or an exception). Figure 5 shows the spot IPC returns and the futures confidence intervals constructed with the GARCH model. It can be observed that the IPC spot returns were mostly within the 95% confidence level for the daily forecasts. However, for the GARCH model there were violations in 14 days, which represent 2.75% of the total number of observations. Considering that a 95% confidence level is applied, the model should not exceed the VaR more than 5% (Jorion: 2001) and should not be far below from 5% or it will be overestimating the VaR. Figure 6 shows the same IPC spot returns but with confidence intervals constructed with the asymmetric volatility model (TARCH model). For this case the number of violations is 27, which represents 5.29% of the total number of observations.

In order to analyse the accuracy of both methodologies the Kupiec test (1995) is applied. This test chcks if the number of exceptions is consistent with the chosen confidence level. The null hypothesis is in favour of the model ‘being accurate’, by having statistically-speaking relevant number of exceptions considering the confidence level. As explained in Dowd and Blake (2006), in order to carry out the Kupiec test, only 3 inputs are needed. Using the same notation as in Dowd and Blake (2006) these are (c) the confidence level chosen, (x) the number of exceptions or violations and (T) the total number of observations. The null hypothesis for the aforementioned Kupiec test is $H_0: \hat{p} = p = \frac{x}{T}$, where $\hat{p}$ is the relevant exception rate (observed with the estimated model) and p is the suggested failure rate according to the statistical table. This test follows a χ^2 distribution and it has a likelihood-ratio form (LR) as follows (Dowd and Blake: 2006),

$$LR_{Kupiec} = -2ln\left(\frac{(1-p)^{T-x}p^x}{\left[1-\left(\frac{X}{T}\right)\right]^{T-x}\left(\frac{x}{T}\right)^x}\right). \qquad (11)$$

The Kupiec test, as explained by Jorion (2000) and Dowd and Blake (2006), and having one degree of freedom for the χ^2 the critical value is 3.84 is applied. The non-rejection region (interpolating) for 889 observations (Kupiec: 1995) is 16 < x < 36. So the GARCH model is rejecting the null hypothesis of having a correct model. The asymmetric volatility model (TARCH model) is not rejecting the relevant null hypothesis in favour of a 'correct' model. According to Equation 11 the Kupiec test statistic for the GARCH model is 6.5284, which clearly rejects the null of the 'correct' model (6.5284>3.8401). The TARCH model (asymmetric model) has a Kupiec test statistic of 0.0851, which clearly does not reject the null of the 'correct' model (0.0851<3.8401). So it is possible to conclude that for these estimations the asymmetric model is superior to the symmetric model in terms of risk management analysis.

For the S&P500 the results are qualitatively similar. Figures 7 and 8 present the relevant estimations with the GARCH and TARCH models respectively. It can be observed that the S&P500 spot returns were mostly within the 95% confidence level for the daily forecasts. However, for the GARCH model there were exceptions for 15 days, which representing 2.94% of the total number of observations (Figure 7). In Figure 8 the same S&P500 spot returns but with confidence intervals constructed with the asymmetric volatility model (TARCH model) are presented. For this case the number of violations is 55, which represents 6.27% of the total number of observations. Again, according to Equation 11 the Kupiec test statistic for the GARCH (S&P500) model is 5.35, which clearly rejects the null of the 'correct' model (5.35>3.84). The TARCH model (asymmetric model) has a Kupiec test statistic of 1.59, which clearly does not reject the null of the 'correct' model (1.59<3.84).

So it is possible to conclude that for these estimations the asymmetric model is superior compared to the symmetric model in terms of risk management analysis.

The likelihood ratio (LR) type tests include the null in favour of the traditional non-asymmetric type models. Rejection of the null will be in favour of the asymmetric ARCH-type and models and option implied volatilities. The *n*-day ahead forecast horizon is also interpreted as the probability that the future stock market level will be within certain statistical confidence interval, i.e. the 95% confidence interval. It is then expected that these results can also give forecasts of the future (expected) U.S. and Mexican stock index level, which could also have implications for investment-decision making. According to the results we can see the LR in favour of the asymmetric modelling.

VI.2 Bootstrapping Simulations

The methodology to carry out the simulations was explained in Section III above. Tables 5 and 6 present the VaR for the bootstrap simulations performed in the IPC and S&P500 futures series respectively. The numbers of *n*-days ahead considered in the simulations were 10 and 30 trading days. The simulations were done applying the GARCH(1,1), TARCH(1,1) and an option implied volatility measures. The simulations were conducted for 10,000 replications. For each replication the lowest, highest and the average were taken from time series to separate matrices (low, average, high). In order to obtain the MCRR for the long position, the relevant observations from the low value matrix were considered. The same applies for the short position, but for that case, the matrix with the highest values were considered. This follows the logic that for long positions price decreases (low values) are risks for potential losses and for short positions price increases (high values) are risk for potential losses.

Considering the fact that the IPC spot returns have autocorrelation, it is necessary to do the bootstrap adjusting for this autocorrelated process. The procedure postulated by Politis and Romano (1994) is applied here. This is basically a method in which the autocorrelated returns are grouped into non-overlapping blocks. For this case the size of these blocks is fixed during the estimation.[25] With the bootstrap the blocks are resampled. During the simulation of the IPC spot prices the returns are taken from the resample blocks. The intuition is that if the autocorrelations are negligible for a length greater than the fixed size of the block, then this ‘moving block bootstrap’ will estimate samples with approximately the same autocorrelation structure as the original series (Brownstone and Kazimi: 2000). Thus, with this procedure the autocorrelated process of the residuals is almost replicated and it is possible to obtain a more accurate simulated IPC spot series. In addition, that standard procedure is common practice for similar types of estimations considering sample sizes and statistical dynamics of the series (Mader et. al.: 2013, Shao and Tu: 2012, Kosowski et. al. 2006, Brownstone and Valleta: 2001).[26]

From Table 5 it can be observed that for ten trading days long and short positions (third and fourth columns) the null hypothesis is rejected for the GARCH(1,1) model and not rejected for the TARCH(1,1) model during the simulated period from 19/12/2019 until 30/12/2019. Not rejecting the null is in favor of the ‘correct’ model, so it can be observed that the asymmetric volatility model is superior to its counterpart. For completeness Figure 9 show the relevant density obtained with the simulated process for the IPC. The expected value is 46,448.85, which refers to the level of the IPC expected for 30/12/19. Similar qualitative results to those of ten trading days are observed for S&P500 series. However, for that series the GARCH(1,1) estimations do reject the null. An explanation for these results is that the adjustment of the volatility forecast for asymmetric effects show a gain, statistically speaking, compared to not having the adjustment, as it can be observed for those estimations besides the GARCH(1,1). Figure 10 shows the relevant density for the S&P500, again, with the bootstrap simulation process. In this case the expected value for that stock index is

[25] It is also possible to have random size blocks. For a more detailed explanation please refer to Politis and Romano (1994).

[26] Some part of the literature evidenced that for *strongly* dependent returns the use of block-bootstrapping fails. However, there is no consensus about it. Parts of the literature in which they advocate the statistical robustness of that block-bootstrapping methodology for some autocorrelated process, that are not specifically *strongly* dependent (See for example, Shao and Tu: 2012, Kosowski et. al. 2006, Brownstone and Valleta: 2001). The basic idea behind the latter is that there is a problem of observations in the same block being dependent in the bootstrap samples, but observations in different blocks are independent. The procedure carried out in the present research paper applies a moving block bootstrap, which is in line with that part of the literature. I am thankful to an anonymous referee for pointing out the problems of block-bootstrapping estimation.

2,621.74 for the 30/12/19. We can observe that for the 30-day ahead time horizon, besides the GARCH(1,1) estimations for the remaining models the null is not rejected showing that they are qualitatively similar in terms capturing the relevance between asymmetric and symmetric models. This is true for 1-day and 10-day ahead VaR estimations. For both cases we can observe gains that range from 4 to around 150 basis points of minimum capital risk requirements (VaR). However, a word of caution must be made. Clare et. al (2002) argue about the possibility of the volatility persistence in the series, that are sometimes observed in ARCH-type, estimations may overestimate the VaR. Further research for other financial assets, i.e. exchange rate, interest rates, commodity prices in addition to the possibility of expanding the present analysis to include stochastic volatility estimations as well as for different periods is encouraged.

VII. Conclusions

In the present research we analyze volatility asymmetries in stock indices with superior performance and more Kupiec-accurate forecasts than those obtained through symmetric models within a Value-at-Risk (VaR) framework. We estimate three main volatility forecasts models: a backward looking ARCH-type model, a forward-looking option-implied volatility model, and VaR models with volatility asymmetries. The results show that VaR models with asymmetries provide superior estimates relative to the same model without asymmetries. The empirical case is for the Mexican Stock Index and the S&P 500 Index daily future prices from 2016 to 2019. According to the estimated results, the null hypothesis about ARCH-type and Option-implied asymmetric volatility being not accurate to estimate VaR was rejected in favor of modelling VaR models with volatility asymmetries. Thus, there is a statistical gain in terms of applying an asymmetrical volatility model within a VaR (Risk Management) framework. Thus, it is concluded that it is important to carry out asymmetric volatility forecasts within VaR models in order to obtain more accurate risk measures. Our findings are in line with the literature, regarding the relevance of taking into account volatility asymmetries as we show sizeable improvements when comparing estimates of symmetric volatility VaR models to those obtained in both backward and forward- looking forecasts models. The referred gains range from 4 to around 150 basis points of minimum capital risk requirements (VaR). It is documented the relevance of taking into account volatility asymmetries for both broad volatility estimation methodologies, backward- vs. forward-looking.

Bibliography

Benavides, G. (2006). Volatility Forecasts for the Mexican Peso – U.S. Dollar Exchange Rate: An Empirical Analysis of GARCH, Option Implied and Composite Forecast Models. *Series de Documentos de Investigación del Banco de México*, No. 2006-04.

Benavides, G. (2009). Price Volatility Forecasts for Agricultural Commodities: An Application of Historical Volatility Models, Option Implieds and Composite Approaches for Futures Prices of Corn and Wheat. *Journal of Management, Finance and Economics*. Vol. 3, Issue 2, pp. 40-59.

Benavides, G. and Snowden P. N. (2006). Futures for Farmers: Hedging Participation and the Mexican Corn Scheme. *Journal of Development Studies*. Vol. 42, No. 4, pp. 698 – 712.

Bollerslev, T. P. (1986). Generalized Autoregressive Conditional Heteroscedasticity. *Journal of Econometrics*. Vol. 31, pp. 307-327.

Bollerslev, T. P., Chou, R. Y. and Kroner, K. F. (1992). ARCH Modeling in Finance: A Review of the Theory and Empirical Evidence. *Journal of Econometrics* Vol. 52, pp.5-59.

Bollerslev, T. and Wooldridge, J. M. (1992) "Quasi-Maximum Likelihood Estimation and Inference in Dynamic Models with Time Varying Covariances," *Econometric Reviews*, Vol. 11, pp. 143–172.

Brooks, C. (2013). Introductory Econometrics for Finance. *Cambridge University Press*.

Brooks, C., Clare, A.D. and Persand, G. (2000). A Word of Caution on Calculating Market-Based Minimum Capital Risk Requirements. *Journal of Banking and Finance*. Vol. 24, pp. 1557-1574.

Brooks, C. and Persand, G. (2003). The Effect of Asymmetries on Stock Index Return Value-at-Risk Estimates. *Journal of Risk Finance*. Vol. 4, No. 2, pp. 29-42.

Brownstone, D. and Kazimi, C. (2000). Applying the Bootstrap, Dept. of Economics, Working Paper, *University of California*, Irvine.

Brownstone, D. and Valletta, R. (2001). The Bootstrap and Multiple Imputations: Harnessing Increased Computing Power for Improved Statistical Tests. *Journal of Economic Perspectives*, Vol. 15, Issue 4, pp. 129-141.

Chkili, W., Hammoudeh, S. and Nguyen, D. K. (2014). Volatility Forecasting and Risk Management for Commodity Markets in the presence of Asymmetry and Long Memory. *Energy Economics*. Vol. 41, pp. 1-18

Dowd, K. (1998). Beyond Value at Risk: The New Science of Risk Management. Chichester and New York: *Wiley and Sons*.

Dowd, K. and Blake, D. (2006). After VaR: The Theory, Estimation, and Insurance Applications of Quantile-Based Risk Measures. *Journal of Risk and Insurance*. Vol. 73, Issue 2, pp. 193-229

Efron, B. (1982). The Jacknife, the Bootstrap, and other Resampling Plans. *Society for Industrial and Applied Mathematics*. Philadelphia, PA, USA.

Engle, R. F. (1982) Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of U.K. Inflation, *Econometrica*. Vol. 50, pp. 987–1008.

Engle, R. F. (2002). Dynamic Conditional Correlation – A Simple Class of Multivariate GARCH Models. *Journal of Business & Economic Statistics*. Vol. 20, No. 3, pp. 339-350.

Engle, R. F. (2003). Risk and Volatility: Econometrics Models and Financial Practice. *Economics Nobel Prize Lecture*. New York University, Department of Finance, New York, USA. December.

Fama, E. 1965. The Behavior of Stock Market Prices. *Journal of Business.* Vol. 38. pp. 34-105.

Fink, R. E. and Feduniak, R. B. (1988). Futures Trading: Concepts and Strategies. *NYIF Corp*.

Giot, P. (2005). Implied Volatility Indexes and Daily Value at Risk Models. *Journal of Derivatives*. Vol. 12. pp.54-64

Giot, P. and Laurent, S. (2004). Modelling daily Value-at-Risk using realized volatility and ARCH type models. *Journal of Empirical Finance*. Vol. 11, Issue 3, pp. 379-398

Glosten, L. R., Jaganathan, R. and Runkle, D. (1993). On the Relation between the Expected Value and the Volatility of the Normal Excess Return on Stocks. *Journal of Finance*. Vol. 48, pp. 1779-1801.

Herbst, A. F., Kare, D. and Caples, S. (1989). Hedging Effectiveness and Minimum Risk Hedge Ratios in the Presence of Autocorrelation: Foreign Currency Futures. *Journal of Futures Markets*. Vol. 9, pp. 185-197.

Hsieh, D. A. (1991). Chaos and Nonlinear Dynamics: Application to Financial Markets. *Journal of Finance*. Vol. 46. pp. 1839-1877.

Hsieh, D. A. (1993). Implications of Nonlinear Dynamics for Financial Risk Management. *Journal of Financial and Quantitative Analysis*. Vol. 28. pp. 41-64.

Hull, J. 2013. Options, Futures and Other Derivatives. 9th. Edition. Prentice Hall.

Johnson, N. L. (1949) Systems of Frequency Curves Generated by Methods of Translations. *Biometrika*. pp. 149-175.

Jorion, P. (2000). The Value at Risk Field Book: The complete Guide to Implementing VaR. *McGraw-Hill*.

Jorion, P. (2001). Value at Risk: The Benchmark for Managing Market Risk. *McGraw-Hill*.

J.P. Morgan/Reuters Risk Metrics Technical Document. (1996). Available in the following Web page: http://www.riskmetrics.com/rmcovv.html

Kavussanos, M. G. and Visvikis, I. D. (2005) The Predictability of Non-Overlapping Forecasts: Evidence from the Derivatives Market in Greece. *Athens University of Economics and Business, Greece*. Working paper presented at the European Financial Management Association Milan Meetings June 2005.

Kosowski, R., Timmermann, A., Wermers, R. and White, H. (2006). Can Mutual Fund 'Stars' Really Pick Stocks? New Evidence from a Bootstrap Analysis. *The Journal of Finance*. Vol. 61, Issue 6, pp. 2551-2596.

Kupiec, P. H. (1995). Techniques for Verifying the Accuracy of Risk Measurement Models. *Finance and Economics Discussion Series 95-24*. Board of Governors of the Federal Reserve System. USA.

Mader, M., Mader, W., Sommerlade, L., Timmer, J. and Schelter, B. (2013). Block-bootstrapping for Noisy Data, *Journal of Neuroscience Methods*, Vol. 219, Issue 2, pp 285-291.

Mandelbrot, B. 1963. The Variation of Certain Speculative Prices. *Journal of Business*. Vol. 36. pp. 394-419.

Manfredo, M. Leuthold, R. M. and Irwin, S. H. (2001). Forecasting Cash Price Volatility of Fed Cattle, Feeder Cattle and Corn: Time Series, Implied Volatility and Composite Approaches. *Journal of Agricultural and Applied Economics*. Vol. 33. Issue 3, pp. 523-538.

Markowitz, H. (1952). Portfolio Selection. *The Journal of Finance*. Vol. VII, No. 1.

Mohamed, A. R. (2005). Would Student's *t*-GARCH Improve VaR Estimates? Master's Thesis. *University of Jyväskylä, School of Business and Economics*.

Park, D. W. and Tomek, W. G. (1989). An Appraisal of Composite Forecasting Methods. *North Central Journal of Agricultural Economics*. Vol. 10, pp.1-11.

Pelletier, R. (1983). Contracts that Don't Expire Aid Technical Analysis. *Commodities*, pp. 71-75.

Politis, D.M. and Romano, J. P. (1994). The Stationary Bootstrap. *Journal of the American Statistical Association*. Vol. 89. No. 428, pp. 1303-1313.

Poon, S.H., Granger, C.W.J., 2003. Forecasting Volatility in Financial Markets: A Review. *J. Econ. Lit*. Vol. 41, pp. 478–539.

Pong, S., Shackleton, M., Taylor, S. and Xu, X. (2003) Forecasting Currency Volatility: A Comparison of Implied Volatilities and AR(FI)MA models *Journal of Banking and Finance.* Vol. 28, Issue 10, pp. 2541-2563, Samuelson, P. (1965). Proof that Properly Anticipated Prices Fluctuate Randomly. *Industrial Management Review,* Vol. 6, pp. 41-49.

Schroeder, T. C., Albright, M. L., Langemeier, M. R. and Mintert, J. (1993). Factors Affecting Cattle Feeding Profitability. *Journal of the American Society of Farm Managers and Rural Appraisers.* Vol. 57, pp. 48-54.

Shao, J. and Tu, D. (2012) The Jackknife and Bootstrap. *Springer Finance Publishers*.

Taylor, S. J. (1986). The Behaviour of Futures Prices Overtime. *Applied Economics,* Vol. 17, Issue 4, pp. 713-734.

Wei, A. and Leuthold, R. M. (1998). Long Agricultural Futures Prices: ARCH, Long Memory, or Chaos Processes. *OFOR Paper Number 98-03.* University of Illinois at Urbana – Champaign. Available at SSRN: https://ssrn.com/abstract=126951 or http://dx.doi.org/10.2139/ssrn.126951

Working, H. (1958). A Theory of Anticipatory Prices (in Economic Projections and a Statistical Contribution to Price Theory) *The American Economic Review,* Vol. 48, Papers and Proceedings of the Seventieth Annual Meeting of the American Economic Association. May. pp. 188-199.

Zakoïan, J. M. (1994). Threshold Heteroskedastic Models. *Journal of Economic Dynamics and Control.* Vol. 18, pp. 931-944.

Zivot, E. (2009) Practical Issues in the Analysis of Univariate GARCH Models. In: Mikosch T., Kreiß JP., Davis R., Andersen T. (eds) *Handbook of Financial Time Series*. Springer, Berlin, Heidelberg.

Appendix

TABLE 1 IPC AND S&P 500 FUTURES CONTRACT SPECIFICATIONS

Underlying asset	**IPC (Índice de Precios y Cotizaciones).**	**S&P 500 Index**
Exchange	MexDer.	CME Globex.
Settlement	The value of the IPC is multiplied by $100 MXN	$250 USD x S&P 500 Index
Symbol	IPC.	SP
Maturity months	Every month for the following twelve months and every quarter afterwards.	CME Globex: One month in the March Quarterly Cycle (Mar, Jun, Sep, Dec).
Price limits	There are no price limits.	7%, 13%, and 20% price limits are applied to the futures fixing price and are effective from 8:30 a.m. CT – 3:00 p.m. CT. Mondays through Fridays.
Negotiations mechanics	Electronically through the MEXDER's electronic trading system.	Electronically through CME Globex (Electronic Platform:
Trading hours	Weekdays from 7:30 until 15:00 hrs Mexico City time.	CME Globex: Sunday – Friday: 6:00 pm – 5:00 pm New York Time/ET .
Marking-to-market	Applies according to the rules established by MEXDER. Daily profit/losses are daily by the clearinghouse.	Applies according to the rules established by CME. Daily profit/losses are daily by the clearinghouse
Minimum price fluctuation	First trading day (non-holiday) following the last trading day.	0.10 index points=$25

This table presents detail information about the IPC and S&P500 futures contracts. MXN = Mexican pesos (Mexican currency). The source of the information is MEXDER and CME. The web page where this information was obtained is: http://www.mexder.com.mx/MEX/Contratos_Futuros.html (the information is also available in English) and https://www.cmegroup.com/

TABLE 2 VOLATILITY ESTIMATES (VARIANCE EQUATION) OF THE DAILY SPOT AND FUTURES PRICES OF THE IPC

GARCH(1, 1)	Spot	Futures	TARCH(1,1) Spot	Futures
α_0	1.49 x 10^{-4} (3.06 x 10^{-4})	6.13 x 10^{-6} (2.25 x 10^{-6})**	1.25 x 10^{-5} (3.75 x 10^{-6})***	5.01 x 10^{-6} (1.75 x 10^{-6})**
α_1	0.1899 (4.42 x 10^{-2})***	0.1322 (3.28 x 10^{-2})***	0.2097 (6.09 x 10^{-2})***	0.0689 (2.76 x 10^{-2})**
β_1	0.5802 (0.0915)*	0.7731 (0.0398)**	0.5830 (0.0957)**	0.823 (0.0379)**
δ	*N/A*	*N/A*	0.00045 (0.0007)**	0.1194 (0.0415)**
L	1,802.52	1,686.40	1,806.95	1,690.67
$Q(12)$	18.26	12.08	19.18	14.82
$Q^2(12)$	14.87	12.83	13.69	48.65
N	889	889	889	889

This table reports parameter values of the GARCH(1,1) and TARCH(1,1) models. Standard errors are shown in brackets. L represents the log likelihood of the estimation. The rows showing $Q(12)$ and $Q^2(12)$ are the Ljung-Box statistic for standardized residuals and standardized residuals squared respectively, which has a χ^2 distribution with 5 degrees of freedom. The critical value is 21.02 at the 5% level. N represents the sample size. The sample size consists of daily data from June 2016 to December 2019. Source: Bloomberg and Banco de México.

TABLE 3 VOLATILITY ESTIMATES (VARIANCE EQUATION) OF THE DAILY SPOT AND FUTURES PRICES OF THE S&P500

GARCH(1, 1)	Spot	Futures	TARCH(1,1) Spot	Futures
α_0	4.58×10^{-6} (7.93×10^{-4})***	4.38×10^{-6} (5.75×10^{-7})**	4.29×10^{-6} (6.19×10^{-7})***	3.65×10^{-6} (4.43×10^{-7})***
α_1	0.1751 (0.0288)*	0.2470 (0.0280)**	-0.0235 (0.0246)	0.0027 (0.0281)*
β_1	0.7356 (0.0389)*	0.6771 (0.0337)**	0.7589 (0.0369)**	0.7314 (0.0255)**
δ	*N/A*	*N/A*	0.3184 (0.042)**	0.1301 (0.0495)**
L	1,877.20	1,912.22	1,919.93	1,919.51
$Q(12)$	8.23	8.613	7.23	7.73
$Q^2(12)$	3.87	3.83	2.45	3.433
N	889	889	889	889

This table reports parameter values of the GARCH(1,1) and TARCH(1,1) models. Standard errors are shown in brackets. *L* represents the log likelihood of the estimation. The rows showing $Q(12)$ and $Q^2(12)$ are the Ljung-Box statistic for standardized residuals and standardized residuals squared respectively, which has a χ^2 distribution with 5 degrees of freedom. The critical value is 21.02 at the 5% level. *N* represents the sample size. The sample size consists of daily data from June 2016 to December 2019. Source: Bloomberg, Banco de México, Fred Database.

TABLE 4 DESCRIPTIVE STATISTICS FOR THE DAILY VOLATILITY OF THE SPOT AND FUTURES IPC THE FORECASTING MODELS

Model/Series IPC/S&P500	Mean	Std. Deviation	Skewness	Kurtosis	*N*
Spot daily volatility series	5.54×10^{-3}	4.99×10^{-3}	2.3350	14.33	889
	4.34×10^{-3}	1.28×10^{-3}	2.7614	14.75	889
Futures daily volatility series	6.88×10^{-3}	6.32×10^{-3}	2.1365	10.15	889
	4.53×10^{-3}	5.65×10^{-3}	3.4085	23.36	889
GARCH(1,1) model for the spot series	5.44×10^{-5}	3.79×10^{-5}	6.9702	79.36	889
	4.36×10^{-5}	3.69×10^{-5}	6.6691	79.23	889
GARCH(1,1) model for the futures series	9.11×10^{-5}	5.87×10^{-14}	2.3384	9.15	889
	9.13×10^{-5}	5.29×10^{-14}	2.3719	11.39	889
TARCH(1,1) model for the spot series	5.57×10^{-5}	3.89×10^{-5}	6.9191	79.87	889
	4.92×10^{-5}	2.24×10^{-5}	4.7726	31.63	889
TARCH(1,1) model for the futures series	9.15×10^{-5}	5.86×10^{-5}	2.4319	11.98	889
	5.18×10^{-5}	2.36×10^{-5}	5.7714	46.84	889

This table reports the descriptive statistics of the daily volatility and the volatility forecasting models for the daily IPC and S&P 500spot and futures returns. The sample size is 889 daily observations (adjusted sample 888 daily observations) from 19th June 2016 to 30th December 2019. *N* = Number of observations. Source: Bloomberg, Banco de México, Fred Database.

TABLE 5 VaR FOR THE IPC FUTURES PORTFOLIO OBTAINED WITH BOOTSTRAPPING SIMULATIONS

Model	VaR *t*-day horizon (trading days)	Minimum capital risk requirement *long* position	Minimum capital risk requirement *short* position	Kupiec-test outcome
GARCH(1,1)	10 trading days (from 19/12/2019 until 30/12/2019).	8.23%	3.74%	Reject the null
TARCH(1,1)		8.27%	3.85%	Do not reject the null
GARCH(1,1)	30 trading days (from 30/11/2019 until 30/12/2019).	15.85%	5.34%	Do not reject the null
TARCH(1,1)		15.24%	5.47%	Do not reject the null
Implied volatility (VIMEX)		9.83%	4.96%	Do not reject the null

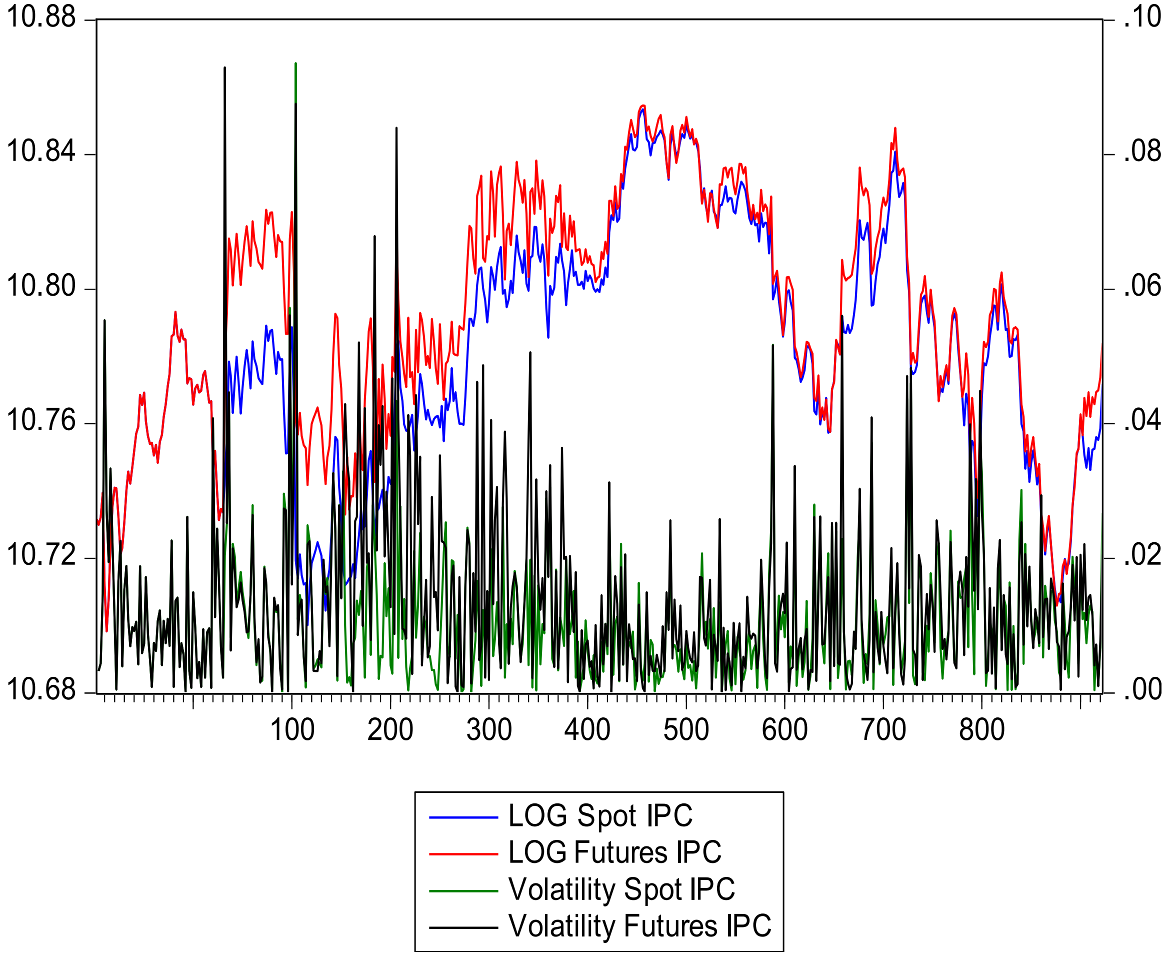
This table presents the results of the bootstrap simulations. 10,000 replications were applied to simulate the IPC price. The time horizons are 10 and 30 trading days. IPC futures prices are used for this table. The models applied are GARCH(1,1), TARCH(1,1) and the Implied Volatility (VIMEX). The Kupiec-test outcome refers to the statistical test (result) as explained in Kupiec (1995). The sample size is 889 observations from 17th June 2016 to 30th December 2019.
Source:Bloomberg and Banco de México.

TABLE 6 VaR FOR THE S&P500 FUTURES PORTFOLIO OBTAINED WITH BOOTSTRAPPING SIMULATIONS

Model	VaR *t*-day horizon (trading days)	Minimum capital risk requirement *long* position	Minimum capital risk requirement *short* position	Kupiec-test outcome
GARCH(1,1)	10 trading days (from 19/12/2019 until 30/12/2019).	6.25%	11.35%	Reject the null
TARCH(1,1)		5.92%	11.71%	Do not reject the null
GARCH(1,1)	30 trading days (from 30/11/2019 until 30/12/2019).	11.99%	12.84%	Reject the null
		10.26 %		
		10.43%		
TARCH(1,1)			13.51%	Do not reject the null
Implied volatility (VIX)			11.44%	Do not reject the null

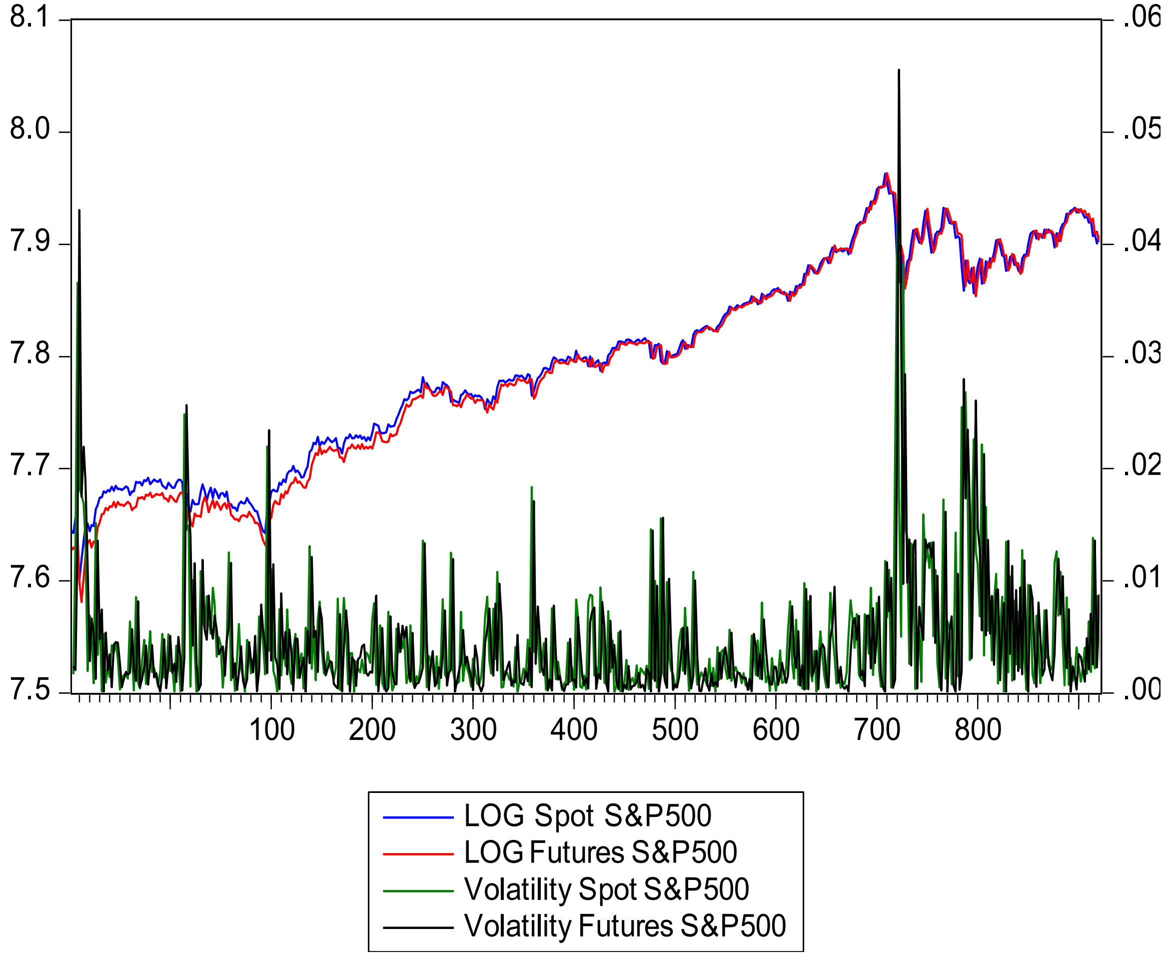
This table presents the results of the bootstrap simulations. 10,000 replications were applied to simulate the S&P500 price. The time horizons are 10 and 30 trading days. S&P500 futures prices are used for this table. The models applied are GARCH(1,1), TARCH(1,1) and the Implied Volatility (VIX). The Kupiec test outcome refers to the statistical test (result) as explained in Kupiec (1995). The sample size is 889 observations from 17th June 2016 to 30th December 2019. Source: Bloomberg and Banco de México.

FIGURE 1 LOG FUTURES AND SPOT IPC AND THEIR DAILY VOLATILITIES (RIGHT AXIS CORRESPONDS TO THE DAILY VOLATILITIES)



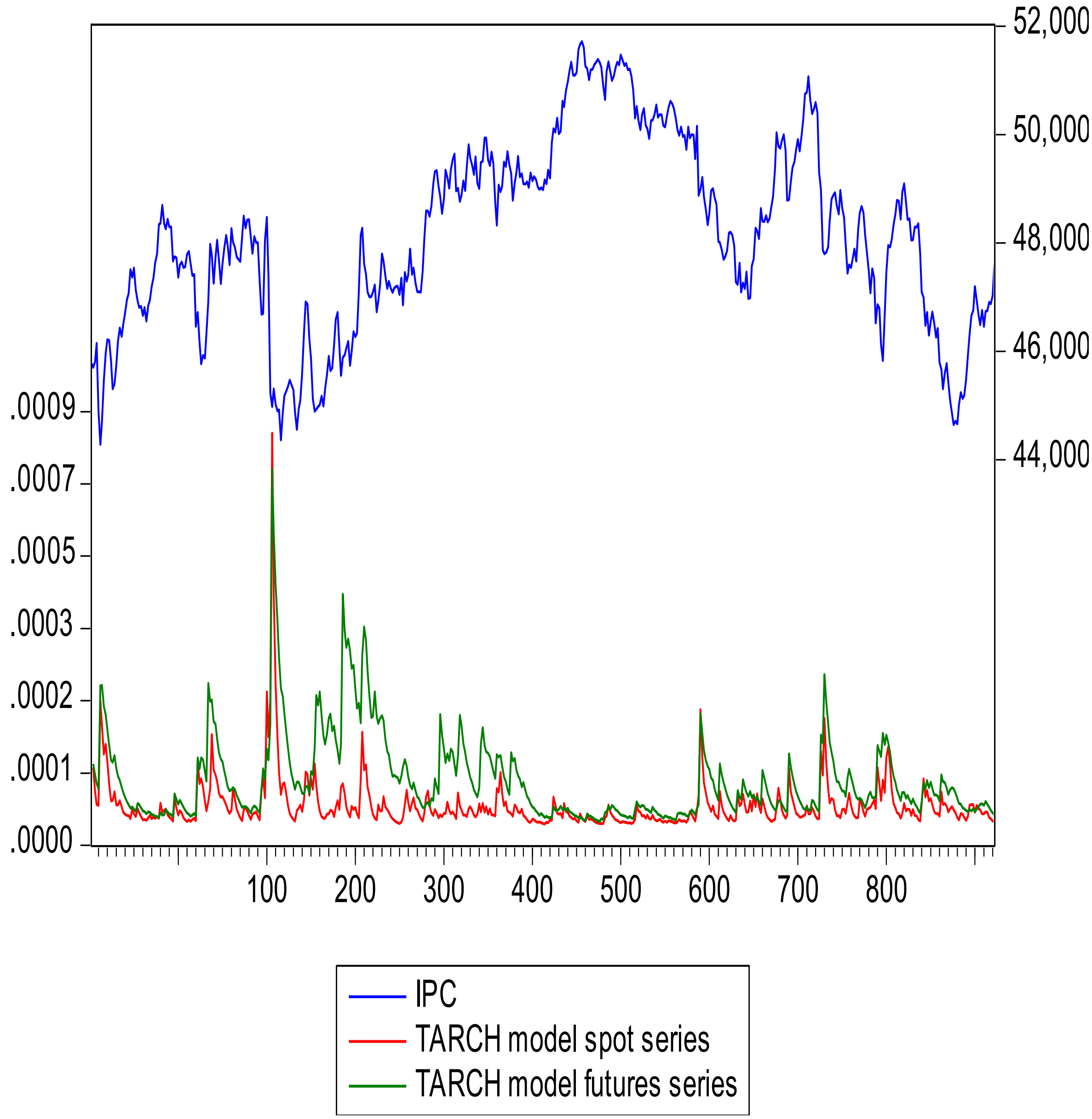
Source: Own estimation with data from Bloomberg and Banco de México.

FIGURE 2 LOG FUTURES AND SPOT S&P500 AND THEIR DAILY VOLATILITIES (RIGHT AXIS CORRESPONDS TO THE DAILY VOLATILITIES)



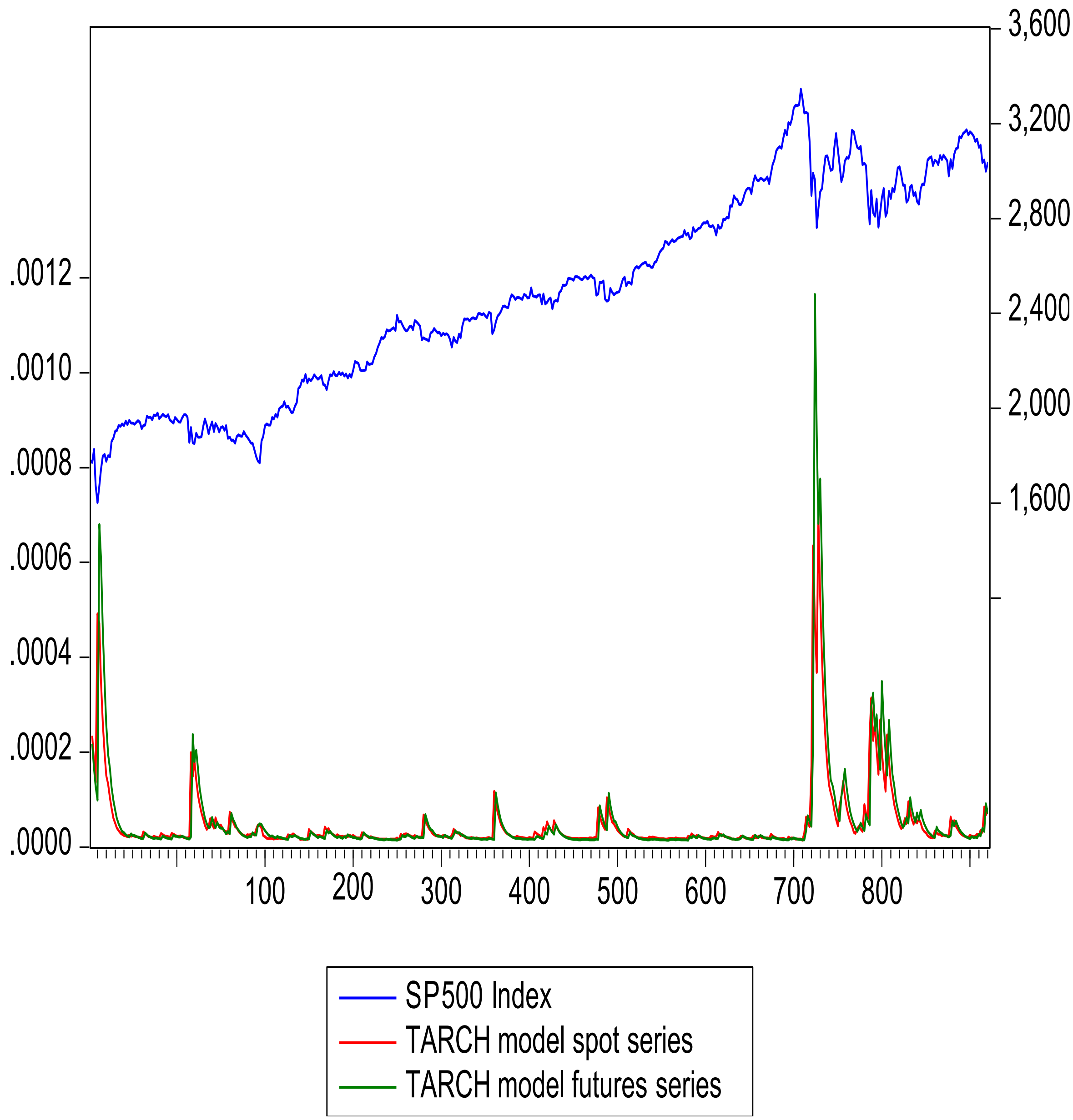
Source: Own estimation with data from Bloomberg and Banco de México.

FIGURE 3 IPC INDEX AND SPOT AND FUTURES DAILY VOLATILITIES
(LEFT AXIS CORRESPONDS TO THE DAILY VOLATILITIES)



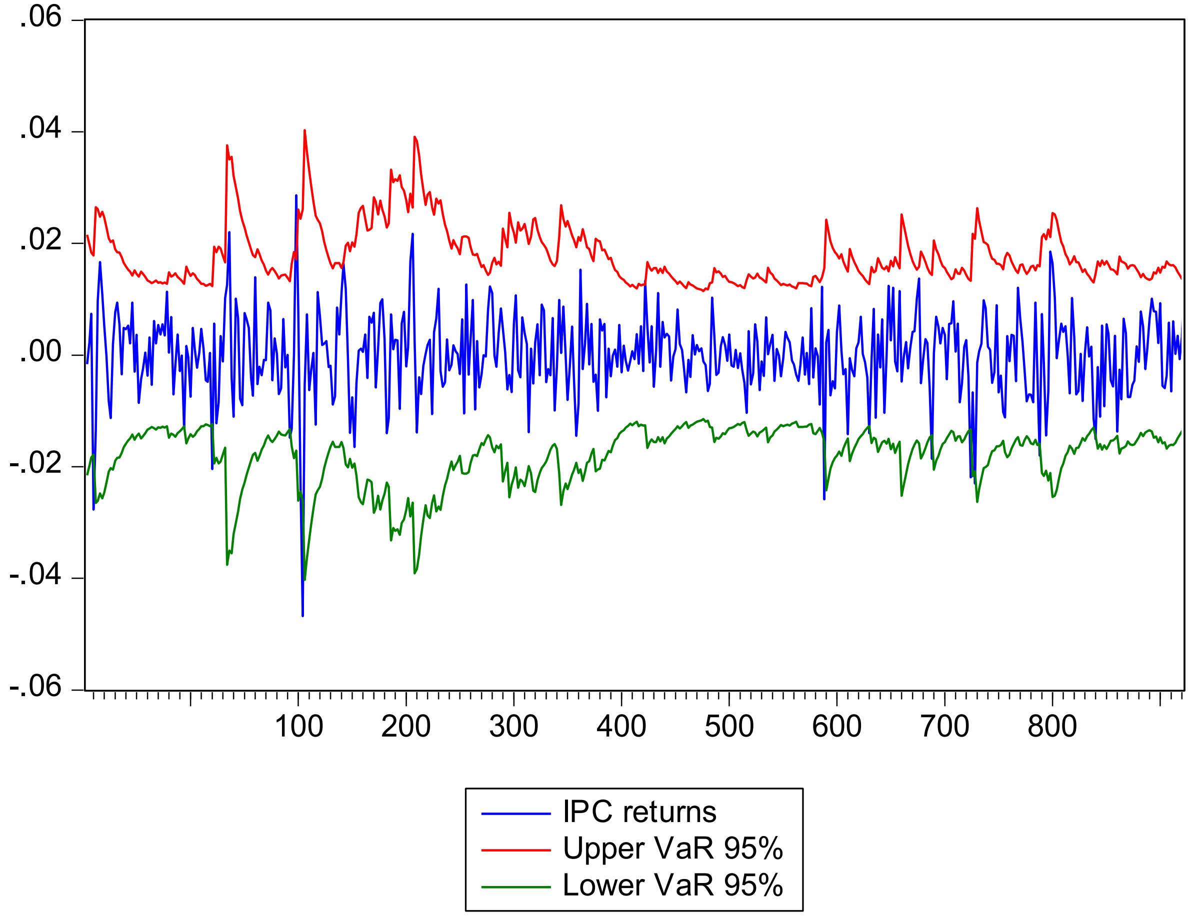
Source: Own estimation with data from Bloomberg and Banco de México.

FIGURE 4 S&P500 INDEX AND SPOT AND FUTURES DAILY VOLATILITIES (LEFT AXIS CORRESPONDS TO THE DAILY VOLATILITIES)



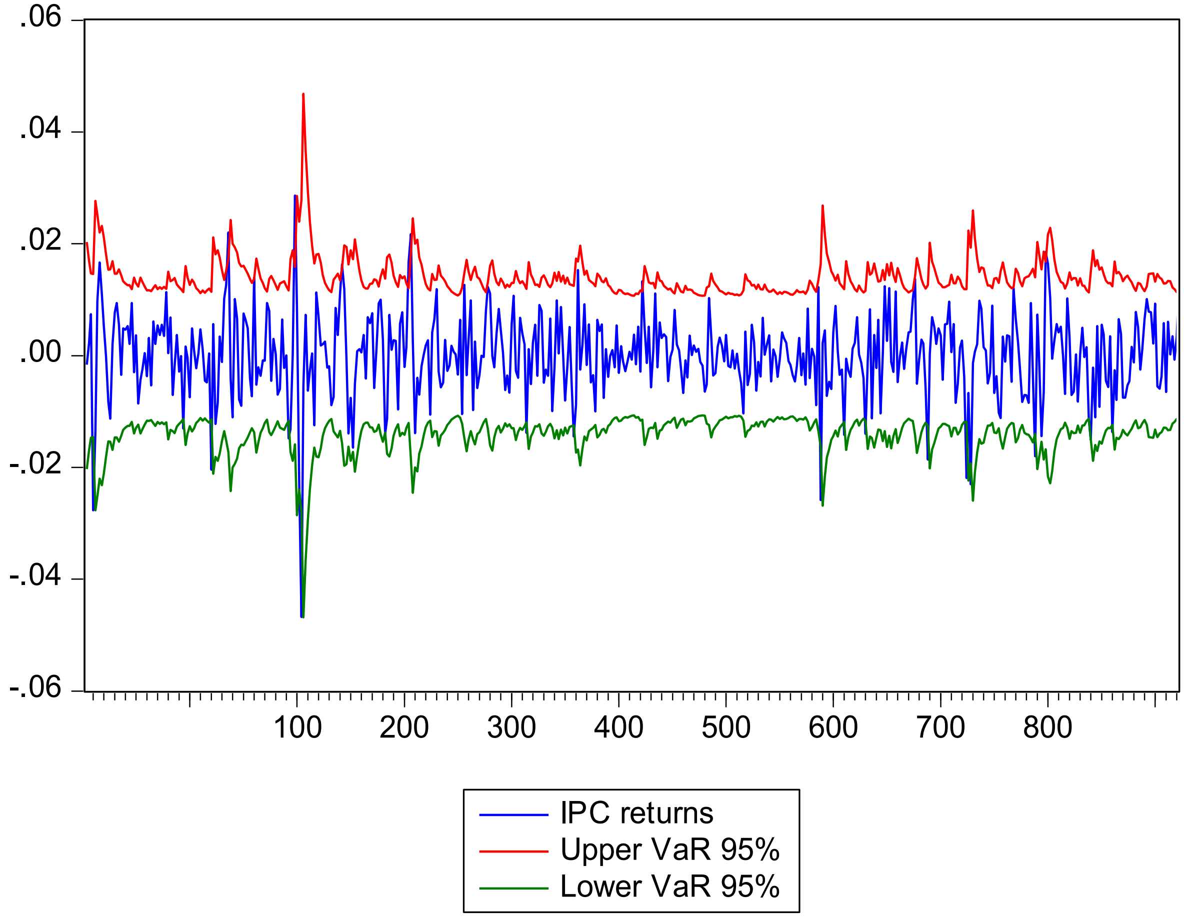
Source: Own estimation with data from Bloomberg and Banco de México.

FIGURE 5 IPC SPOT RETURN AND 95% CONFIDENCE LEVEL OF THE VaR CONSTRUCTED WITH IPC FUTURES PRICES – GARCH (1,1) MODEL



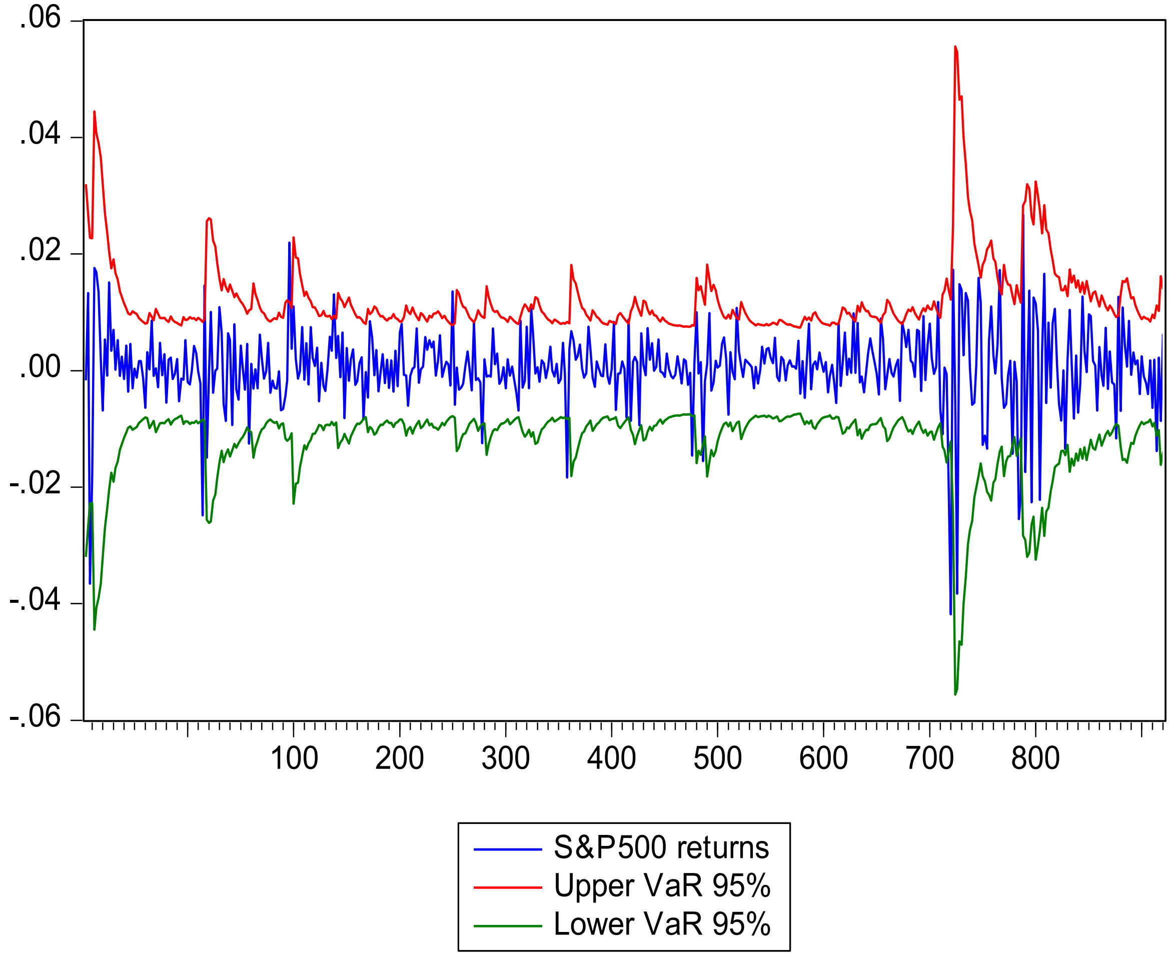
Source: Own estimation with data from Bloomberg and Banco de México.

FIGURE 6 IPC SPOT RETURN AND 95% CONFIDENCE LEVEL OF THE VaR CONSTRUCTED WITH IPC FUTURES PRICES – TARCH (1,1) MODEL



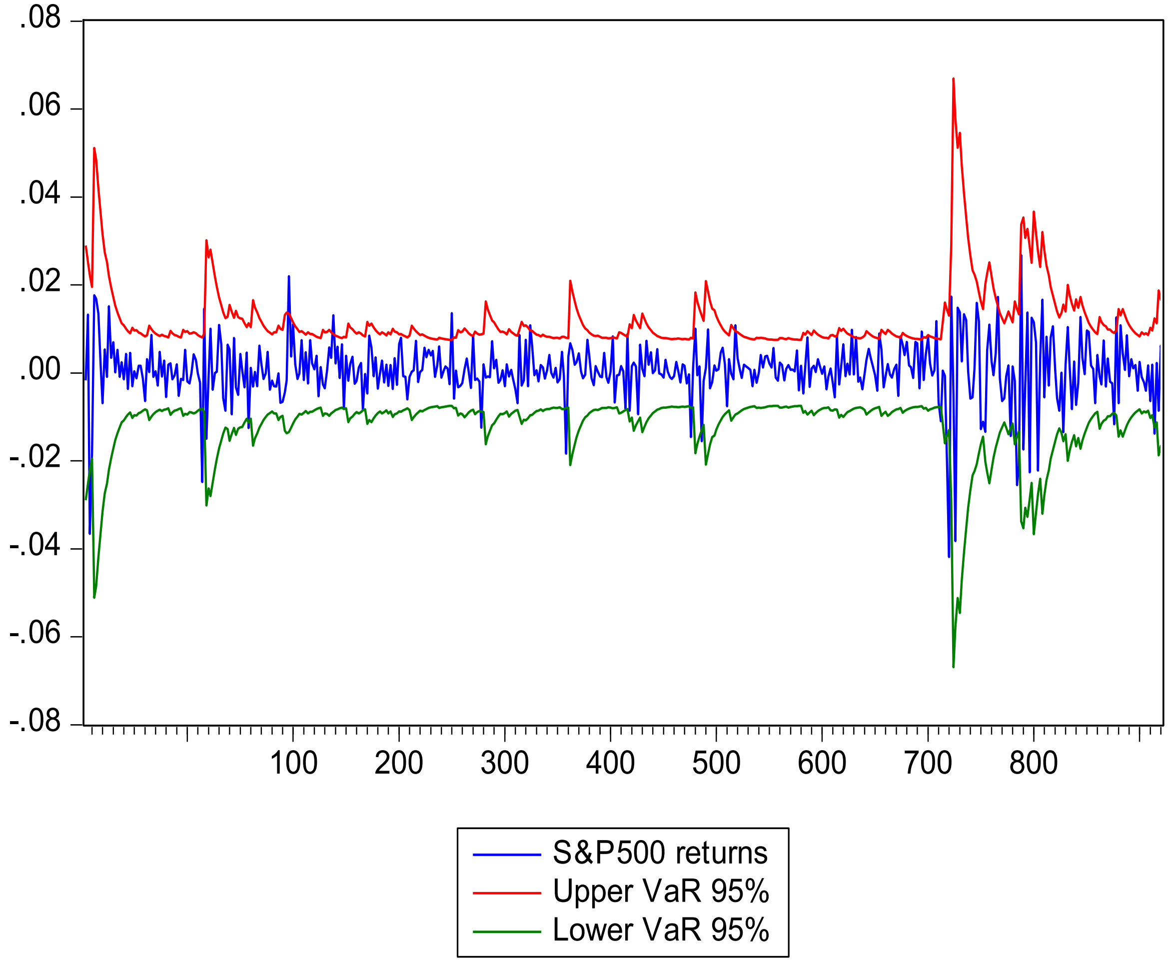
Source: Own estimation with data from Bloomberg and Banco de México.

FIGURE 7 S&P500 SPOT RETURN AND 95% CONFIDENCE LEVEL OF THE VaR CONSTRUCTED WITH S&P FUTURES PRICES – GARCH (1,1) MODEL



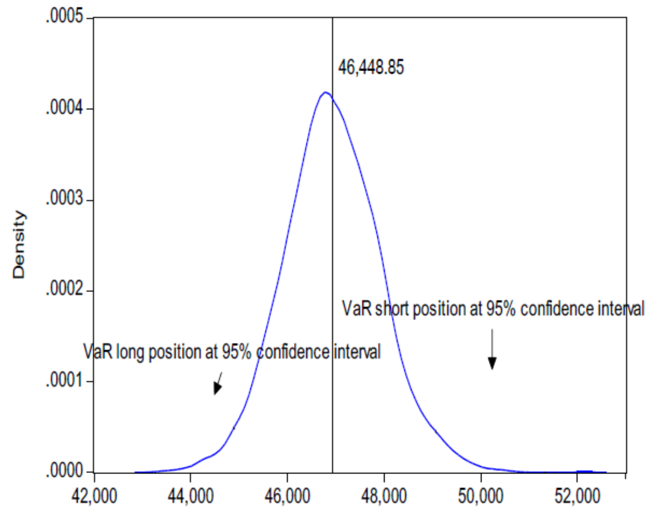
Source: Own estimation with data from Bloomberg and Banco de México.

FIGURE 8 S&P500 SPOT RETURN AND 95% CONFIDENCE LEVEL OF THE VaR CONSTRUCTED WITH S&P FUTURES PRICES – TARCH (1,1) MODEL



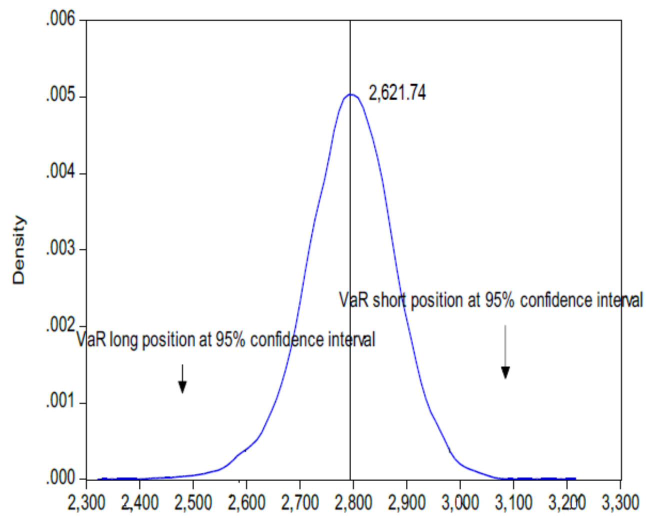
Source: Own estimation with data from Bloomberg and Banco de México.

FIGURE 9 IPC FUTURES RETURN BOOTSTRAPPING AND 95% CONFIDENCE LEVEL OF THE VaR CONSTRUCTED WITH THE TARCH (1,1) MODEL (Kernel, Bandwith=309.7)



Source: Own estimation with data from Bloomberg and Banco de México.

FIGURE 10 S&P500 FUTURES RETURN BOOTSTRAPPING AND 95% CONFIDENCE LEVEL OF THE VaR CONSTRUCTED WITH THE TARCH (1,1) MODEL (Kernel, Bandwith=25.55)



Source: Own estimation with data from Bloomberg and Banco de México.