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Measuring the cost of U.S. housing policy*

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Abstract: Unlike other developed countries, the U.S. has a high proportion of long-term fixed-rate mortgages (30 years). This is partly because the Government Sponsored Enterprises (GSE), which operate in the secondary mortgage market, reduce the interest rate of these contracts. This document measures the cost and studies the consequences of such policy. GSE's actions are modeled as an interest rate subsidy applied directly to 30-year mortgages, in the context of a general equilibrium model with two types of agents, housing and default. The cost of this policy is measured as the minimum subsidy that makes households choose 30-year fixed-rate contracts over one-year contracts, in equilibrium. The resulting subsidy is 36 basis points. Finally, I investigate how the results vary with the duration of the fixed-rate mortgage contract, and I find that mortgage terms under 30 years require smaller subsidies.
Keywords: mortgage contracts, housing policy
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Resumen: A diferencia de otros países desarrollados, EUA presenta una alta proporción de hipotecas de tasa fija a largo plazo (30 años) debido, en parte, a que las Empresas Patrocinadas por el Gobierno (GSE), que operan en el mercado hipotecario secundario, reducen la tasa de interés de estos contratos. En este documento se mide el costo y estudia las consecuencias de dicha política. Las acciones de las GSE son modeladas como un subsidio de tasa de interés aplicado directamente sobre las hipotecas a 30 años, en el contexto de un modelo de equilibrio general con dos tipos de agentes, viviendas e impago. El costo de dicha política es medido como el mínimo subsidio que hace que los hogares elijan contratos de tasa fija a 30 años en lugar de contratos de un año, en equilibrio. El subsidio resultante es de 36 puntos básicos. Finalmente, investigo cómo varían los resultados con la duración del contrato hipotecario de tasa fija, y encuentro que los plazos hipotecarios menores a 30 años requieren subsidios más pequeños.
Palabras Clave: Contratos Hipotecarios, políticas de vivienda

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1 Introduction

The U.S. mortgage market has an unusually high market share of long-term fixed-rate mortgages (LFRMs), mostly in the form of 30-year contracts. The fraction of newly issued LFRMs has been 85% on average, with peaks of almost 100% in the aftermath of the 2007-2008 financial crisis. In other developed countries, short- to medium-term fixed-rate mortgages and adjustable-rate mortgages (ARMs) have the highest market shares.

Different studies[1] suggest that the substantial dominance of LFRMs in the US is in part the result of goverment policies that favor these contracts.[2] Specifically, Fannie Mae and Freddie Mac—government-sponsored enterprises (GSEs) that operate in the secondary mortgage market—drive down the interest rate of 30-year fixed-rate mortgages relative to other mortgage products.[3] This is possible because of an implicit (now made explicit) government guarantee on GSE debt.[4] The objective of these policies is to increase the home ownership rate, especially among low income households.[5]

This paper measures the cost and studies the implications of such policies. Specifically, I model the implicit government guarantee on GSE debt as a tax-financed interest rate subsidy directly set on 30-year FRM contracts, in the context of a general equilibrium model with housing, lack of commitment, and default. To reflect the high market share of LFRMs in the U.S., the subsidy-equivalent cost is defined as the minimum interest rate subsidy that makes households choose 30-year contracts over a short-term (1-year) contract.[6] Non-contingent 1-year contracts are chosen as an approximation to ARMs because of their simplicity and the fact that most ARMs in the U.S. have an interest-rate fixation period of one year.[7]

[1] See Vickery (2007); Moech, Vickery and Aragon (2010); Lea (2010); and Campbell (2013).

[2] Campbell (2013) also points out to the historically stable inflation rate in the US as a second factor for the high market share of LFRMs.

[3] GSEs also buy ARMs and issue adjustable-rate mortgage-backed securities (ARMBS). However, Vickery (2007) argues that their pricing is less attractive to depositary institutions than FRMs, and that ARMBSs are less liquid than those linked to FRMs.

[4] In September 2008, the Federal Housing Finance Agency (FHFA) decided to place Fannie Mae and Freddie Mac into conservatorship. As a result, GSE debt has been explicitly backed by the US government since then.

[5] See, for instance, the Fannie Mae 2015 Annual Housing Activities Report and Annual Mortgage Report and Freddie Mac's statutory purposes as set out in the Federal Home Loan Mortgage Corporation Act (Freddie Mac Act).

[6] The subsidy arising from this "corner portfolio solution" represents an upper bound of the subsidy that would generate a market share lower than 100% (85% for the US).

[7] The typical ARM has a 30-year term, with interest rate adjustments made every year following a market interest rate such as the LIBOR or the Fed's funds rate. How this approximation impacts the measurement of the subsidy is discussed later in this section.

The model follows the tradition of Kiyotaki and Moore (1997), in that it has two types of agents with different levels of patience. In equilibrium, impatient households borrow (Borrowers) whereas patient households save (Savers). In addition, and departing from Kiyotaki and Moore, households cannot commit to honor their mortgage obligations: If they choose to default, they lose their housing stock. Additionally, I introduce an idiosyncratic house depreciation shock which, along with the lack of commitment, results in a fraction of households defaulting every period. Finally, mortgage contracts have endogenous interest rates that depend on both the total amount borrowed and the size/value of the housing stock (collateral).[8]

Borrowers can only choose one type of mortgage: either a 30-year fixed-rate contract or a one-year contract;[9] thus, they compare the cost and benefits of each type. For a given debt level, one-period mortgages are cheaper than LFRMs, because changes in the default probability and the funding cost of financial institutions are fully reflected in the interest rate at which the mortgage debt is rolled over.[10] One-period contracts also provide an incentive to reduce the default probability.[11] On the other hand, LFRMs render insurance against negative shocks that increase interest rates, as only the additional debt is charged with a high interest rate.

Using a calibration that matches U.S. data, I find that the subsidy is around 36 basis points, financed with a labor income tax of 0.60% on average. The resulting subsidy is slightly lower than most estimates of the GSE interest rate advantage,[12] and is consistent with a scenario in which most of the implicit subsidy on GSE debt is passed to households.[13] The fact that the subsidy is positive implies that, for the main calibration, the incentives provided by one-period contracts dominate the insurance rendered by FRMs.[14] That is, in the absence of the

[8]In other words, a full schedule of interest rate/loan amount/size of the collateral is available.

[9]Thus, this paper abstracts from the use of maturities for hedging.

[10]In the case of ARMs like the ones in the U.S., only changes the funding cost of financial institutions are considered when periodically updating the interest rate. This makes ARMs more expensive than one-period contracts, for a given level of debt. However, they are still cheaper than FRMs because, by definition, the interest rate in the latter is invariant to both changes in the funding cost and the default probability.

[11]This is because any reduction in the default probability translates into a lower interest rate when the mortgage is rolled-over. Notice that this incentive is absent in ARMs contracts.

[12]Lucas and McDonald (2010) estimate an interest rate advantage of 20 to 30 basis points. The Congressional Budget Office (CBO, 2001) reports an estimate of 41 basis points. Passmore (2005) estimates the subsidy to be 40 basis points.

[13]General equilibrium effects are essential when measuring the size of the government intervention, as any subsidy affects prices and the utility levels reached by Borrowers when choosing either short-term contracts or long-term contracts, which ultimately affects the size of the minimum subsidy needed so that Borrowers choose long-term contracts

[14]As ARMs are more expensive than one-period contracts, for any given level of debt, the computed subsidy to sustain FRMs in equilibrium would be smaller if ARMs were the alternative mortgage contract.

subsidy, LFRMs are *too expensive* in equilibrium.[15]

This policy implies a welfare gain of 0.31% in consumption-equivalent terms for Borrowers, and welfare loss of 1.05% for Savers, when compared to the case with no subsidy. Borrowers' housing consumption is 4.1% higher in the steady state compared to the case in which the subsidy is equal to zero. I also investigate how the results vary with the term of the long-term mortgage contract. I find that the required subsidy is smaller if the government is to promote mortgage contracts with shorter terms. In other words, under the main calibration, the subsidy is positive increasing in the term of the long-term contract.

Finally, I perform a sensitivity analysis, focusing on the level of income volatility. I find that, the lower the volatility parameter, the higher the subsidy needed to sustain longer-term contracts. Also, in the context of this model, if volatility is *too low*, the equilibrium subsidy will be large enough so that the resulting interest rate on 30-year FRMs will be smaller than that on one-year contracts, which has not been observed in the data.

On the other hand, for volatility values above a certain level, the subsidy becomes negative, at least for short terms. This result arises because, when volatility is too high, Borrowers value more the insurance given by longer-term contracts than the incentive one-period contracts provide. In fact, there exists a high enough value of income volatility such that even 30-year contracts do not need to be subsidized. However, quantitatively, it is difficult to justify using such extreme values.

Related Literature

This paper relates to several strands of the literature on housing and financial macroeconomics.[16] The first covers the impact and implications of US housing policy. Early work focused on the preferential tax treatment of owner-occupied housing. In the context of fixed house prices, Gervais (2002) studies the elimination of tax-deductibility and the taxation of imputed rents from owner-occupied housing, and finds that both policies would be welfare improving. Floetotto, Kirker, and Stroebel (2012) endogenize house prices and find that removing mortgage interest deductions improves welfare in both the short and long run, and that taxing imputed rents improves welfare in the long run, but not in the short run.

[15] Aguiar et al. (2016) have a similar result for a sovereign's maturity choice, in an environment that also abstracts from using maturities for hedging. In equilibrium, the sovereign remains passive on existing long-term bonds, and any new issuance consist of short-term bonds only.

[16] See Davis and Van Nieuwerburgh (2015) for a recent review of the literature.

The most recent strand of the literature focuses on the effects of GSEs on the housing market. Jeske, Krueger, and Mitman (2013) study the effects of eliminating government bailout guarantees for GSEs, which are modeled as an exogenous interest-rate subsidy on mortgages financed by income taxes. They assume that mortgage contracts last one period, in the context of a heterogeneous agent model with idiosyncratic risk, exogenous house prices, endogenous rents, and equilibrium default. They find that eliminating the subsidy increases aggregate welfare, benefits low-income low-asset households, and has no significant effect on foreclosure rates or the home-ownership rate.

Elenev, Landvoigt, and Van Nieuwerburgh (2015) allow the government to sell mortgage insurance to the private sector (like GSEs do) as it also provides deposit insurance to the financial sector. They study the effects of having (exogenously) underpriced government mortgage insurance, in a model with long-term fixed-rate mortgages contracts in which the interest rate is given,[17] but nonetheless incorporates the aggregate risk of default. They find that increasing the price of such government guarantees reduces the equilibrium default rate and financial sector leverage, lowers house prices, and is Pareto improving.

My contribution, relative to these two works, is to endogenize the size of government intervention, measured in subsidy-equivalent terms as in Jeske et al. (2013), and exploits the fact that LFRMs have a considerable market share—which, in my framework, is an equilibrium result rather than assumed. Like Elenev et al. (2015), reducing the size of the government subsidy reduces default rates, because it decreases households debt levels. There is, however, an additional channel in this paper that is absent in their work: Following a decrease in the size of the government intervention, equilibrium default rates are also reduced due to a decrease in the equilibrium mortgage term.

This paper is also related to the strand of literature that discusses household mortgage choice, pioneered by Campbell and Cocco (2003). Their work studies the choice between FRMs and ARMs, in a partial equilibrium framework.[18] Van Hemert (2010) studies the household's balance sheet in a model with stochastic interest rates, and finds that households prefer to finance housing consumption with ARMs to avoid paying the risk premium present in long-term FRMs. In Van Hemert's framework, ARMs are one-year rollover contracts, which

[17] That is, borrowers do not choose the interest rate from a schedule of interest-rate/loan amount/size of the collateral.

[18] They find that the preference for ARMs is greater among Borrowers with rapidly increasing income and large houses relative to income.

makes this result consistent with the ones in this paper in the absence of the interest-rate subsidy.

This paper also complements work that studies the time variation of the long-term FRM market share. Koijen, Hemert, and Van Nieuwerburgh (2009) find that the aggregate FRM share depends on the difference between the FRM (long-term yield) and the expected average future ARM rate (short-term yield). Moech, Vickery, and Aragon (2010) report that this forward-looking FRM-ARM spread can account for the low ARM share between 2007 and 2010. Krainer (2010) finds that the simple FRM-ARM spread is sufficient to explain the share of FRMs. Badarinza, Campbell, and Ramadorai (2015) empirically study the share of ARMs across countries and over time, and find that both spreads are relevant. In this line, this paper contributes to previous work by studying the implications of government policies that affect the FRM-ARM spread by means of an interest-rate subsidy.

More generally, this paper is also related to works that use models featuring long-term contracts and equilibrium default. In particular, Chatterjee and Eyigungor (2011) model LFRMs in a way similar to this paper, and also have endogenous house prices. Corbae and Quintin (2014) and Garriga and Schlagenhauf (2009) also work with long-term mortgages, in the context of exogenous house prices. All of these authors study possible reasons for and consequences of the recent foreclosure crisis. Unlike these three papers and most of the literature, lenders in this paper (Savers) are risk averse, which implies that the mortgage interest rate depends on the covariance between Savers' intertemporal marginal rate of substitution and the mortgage payoff.[19]

The rest of the document is organized as follows. Section 2 briefly compares the mortgage market in the US to that in other developed countries, and discusses how the GSEs operate. The model is presented in Section 3, while details on the calibration are presented in Section 4. Section 5 reports the main results, while Section 6 discusses the role of income volatility in the results. Section 7 concludes. Details on computation methods can be found in the Appendix.

[19]Elenev, Landvoigt, and Van Nieuwerburgh (2015) also consider risk-averse lenders.

2 The U.S. mortgage market vs other countries

LFRMs are the most popular mortgage product in the US. According to the Monthly Interest Rate Survey (MIRS), LFRMs had an average market share of 85 percent during the period 1991-2017, for yearly new residential mortgages. Also using data from the MIRS, Badarinza, Campbell, and Ramadorai (2015) report an initial interest rate fixation period of 22.8 years on average.

On the other extreme, in countries like Australia, Ireland, Korea, Spain and the UK, the dominant products are adjustable-rate mortgages. Lea (2010) documents two different types of ARMs. Spain and Korea have ARMs similar to the ones in the US: the interest rate is adjusted based on an underlying index, which reflects the cost to the lender of borrowing on the credit markets. Meanwhile, ARMs in Australia and Ireland are reviewed by the lender at its discretion. These reviewable ARMs used to be the standard mortgage product in the UK, but recently, indexed (also called tracker) ARMs have become popular.[20]

Finally, there is a third set of countries where short-to-medium-term fixed rate mortgages are the dominant mortgage instruments. Lea (2010) includes Canada, Germany, and the Netherlands in this group. The interest rate is fixed for a period of one to ten years, after which the loan is rolled over – 5 years in Canada, and 10 years in Germany and the Netherlands.

Fannie and Freddie's operations

How exactly do the GSEs, Fannie Mae and Freddie Mac, reduce the interest rate on LFRMs? Basically, by providing liquidity in the secondary mortgage market and charging underpriced guarantee fees. Banks do not want to keep LFRMs on their balance sheets because they expose them to both the interest rate risk and the debt dilution risk.[21] Therefore, banks sell them to the GSEs; Fannie and Freddie pool them into mortgage-backed securities that they guarantee against losses from default (and for doing this they charge banks a guarantee fee), and they finally sell these securities to investors or keep them in their balance sheets. This process is called securitization, and banks typically resort to it with LFRMs.

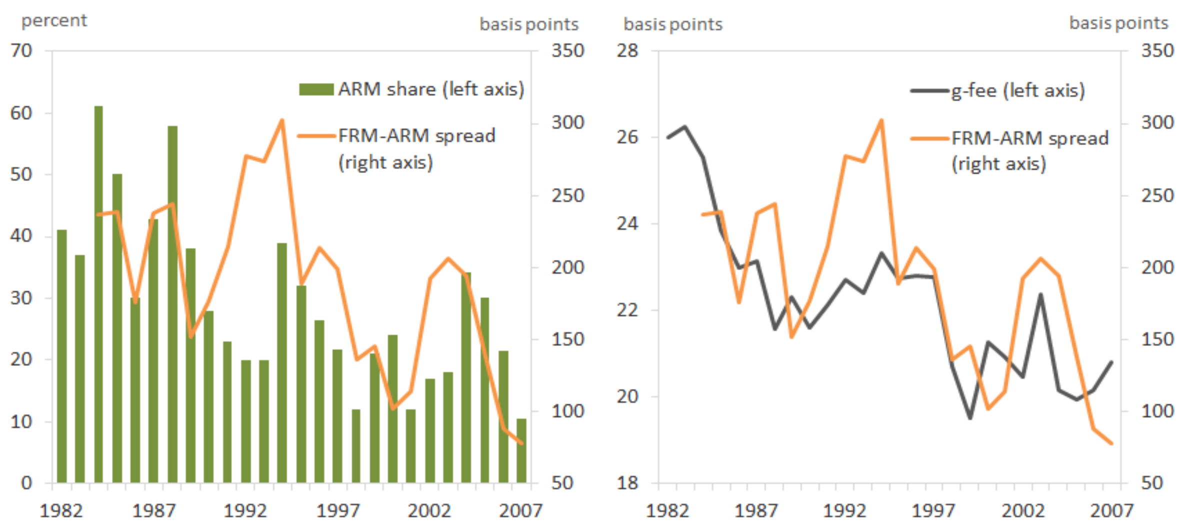
Notice that the liquidity provided by Fannie and Freddie, and the low guarantee fees that

[20]Badarinza et al. (2015) document that the share of ARMs in Spain and the UK varies considerably through time. However, fixed-rate contracts in these countries have short fixation periods, typically lower than 3 years.

[21]**Interest rate risk**: Banks that finance LFRMs with deposits and retain them in their balance sheets are exposed to losses when interest rates rise (maturity mismatch), and when they fall (prepayment risk). **Debt dilution**: When households take on additional debt and increase the default probability on the existing mortgage, but they do not internalize it.

they charge, are equivalent to a reduction in the cost of funding of banks (a subsidy), which essentially translates into lower interest rates on LFRMs. On this regard, and using data from the MIRS, Vickery (2007) finds that FRM interest rate spreads in the U.S. are relatively low when compared to those in the U.K. With the estimated elasticities of substitution between FRMs and ARMs, he finds that this interest rate differential can explain up to 50 percent of FRMs higher market share in the U.S. with respect to the U.K. Vickery (2007) also argues that a more liquid secondary market in the U.S. could be behind mortgage price differences between these two countries.

Figure 1: GSEs Guarantee fees, ARM market share, and FRM-ARM interest rate spread (1984-2007)



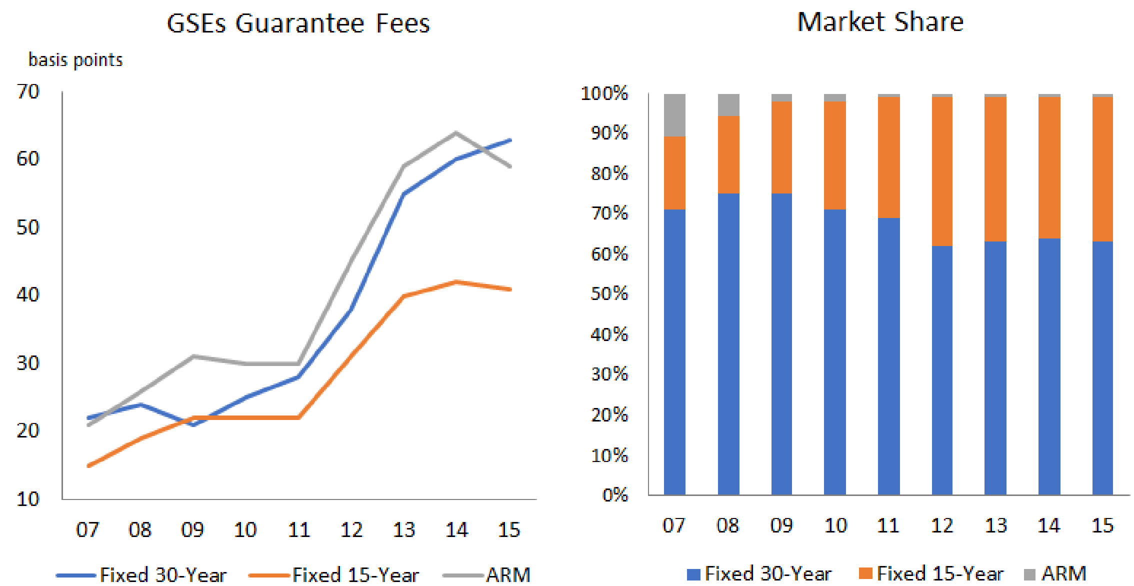
Source: Monthly Interest Rate Survey and Federal Housing Finance Agency

Low guarantee fees might have also played a role in explaining the high market share of LFRMs. In fact, during the 80's, when ARMs became available in the U.S.[22], the market share of ARMs reached levels above 50 percent of newly issued residential mortgages, but gradually declined as the FRM-ARM interest rate spread dropped. This process was driven, in part, by lower guarantee fees charged by the GSEs (see Figure 1).[23]

[22] In 1979, ARMs became available nationwide with severe restrictions, which were lifted in April 1981. See Peek (1990) for more details.

[23] A linear regression analysis for the period 1984-2007 shows a statistically significant positive effect of guarantee fees on both the FRM-ARM interest rate spread and on ARMs market share, even after controlling

Figure 2: GSEs Guarantee fees and market share by type of Mortgage Product (2007-2015)



Source: Monthly Interest Rate Survey and Federal Housing Finance Agency

Finally, since 2008, Fannie and Freddie have been increasing their guarantee fees on all mortgage products, an action that is equivalent to reducing the size of the subsidy on FRMs. As a result, as shown in Figure 2, the average term of newly originated mortgages has decreased, as more people take 15-year fixed-rate mortgages as opposed to 30-year fixed-rate ones.[24]

3 The Model

3.1 Enviroment

Endowments. The economy has two types of goods: an aggregate endowment of a non-durable good y which follows an AR(1) process; and a perfectly divisible durable good (housing) in fixed supplied normalized to H_s. The endowment of the non-durable good can be interpreted as labor income with fixed labor supply. The government sets a proportional

for inflation and/or the FED funds interest rate. The sample in the regression exercise ends in 2007 to exclude the effects of regulation implemented in the aftermath of the financial crisis that severely restricted the use of ARMs.

[24] The Federal Housing Finance Agency (FHFA) reports the GSEs' guarantee fees by type of mortgage product since 2007 only. Before that year, only the average fee is available.

tax τ on labor income to finance any interest rate subsidy ϕ.

Preferences. There are two types of households, a measure ψ of Impatient households ("Borrowers") with discount factor β; and a measure $(1-\psi)$ of Patient households ("Savers") with discount factor $\widetilde{\beta}$, where $\beta < \widetilde{\beta}$. Throughout the paper, for decision variables common to both types of households, x denotes Borrowers choices while $\widetilde{x}$ represents Savers choices.

Both types of households derive period utility $u()$ from nondurable consumption (c for Borrowers, $\widetilde{c}$ for Savers) and housing consumption which is proportional to the housing stock owned in that period (h for Borrowers, $\widetilde{h}$ for Savers).[25] The housing good can be purchased every period at price p (relative to the nondurable good).

There is also a competitive bank, which is owned by Patient households ("Savers").

Assets. Households can buy, from the competitive bank, one-period deposits d' that pay a risk-free rate r^d. Households can also purchase houses at price p, set in terms of the nondurable good. Houses are risky assets, subject to both aggregate risk (given by the endowment y) and idiosyncratic depreciation shock ω. At the beginning of each period, each household faces a realization of ω so that the effective housing stock is ωh_{-1}. The depreciation shock ω is i.i.d. across households, has lognormal cumulative distribution $F(\omega)$, $E(\omega) = 1$, and $\sigma = var(\ln \omega)$.

Mortgages. Households have access to a mortgage contract offered by the competitive bank. The mortgage contract is modelled as a perpetuity with geometrically declining payments governed by a parameter λ. Let Q denote the price schedule of such contract.[26] If a household takes a new mortgage, she gets Qm' in the current period and agrees to make sequential payments $\{m', \lambda m', \lambda^2 m', ...\}$ starting on the next period. Notice that all we need to keep track of is the next period's total coupon payment m'.

Notice that, when $\lambda = 0$, the contract collapses to a one-period contract, whereas higher values of λ correspond to longer maturities.[27] Every period, besides the long-term fixed- rate (LFR) contract with parameter $\lambda > 0$, there is also a one-period contract ($\lambda = 0$) available. While LFR contracts benefit from the interest rate subsidy ϕ, one-period contracts are not

[25] u must meet the minimum requirements for utility functions: nondecreasing and quasi-concave.

[26] The mortgage interest rate r^m is then given by $(1 + r^m) = 1/Q + \lambda$

[27] For an annual frequency, a maturity of N years is equivalent to $\lambda = (N - 1)/N$. One-year contracts then have $\lambda = 0$, ten-year contracts imply $\lambda = 0.9$ and thirty-year $\lambda = 0.967$.

subsidized. For simplicity, and in order to abstract from the potential use of maturities for hedging, I assume that households can only choose one type of mortgage contract.

Finally, every household has the option to default on its mortgage obligations after observing the realization of its depreciation shock ω. When default is chosen, a household loses her entire housing stock which is seized by the bank. There are no other costs for the household after default.[28] The bank then sells the house incurring in a proportional cost μ.

Big Families. For the sake of tractability, it is also assumed that households of each type belong to large representative families of each type,[29] as in Lucas (1990), so that they can diversify away any idiosyncratic risk. As a result, each household inside a family consumes exactly the same amount of the durable good c and housing services h. Also, at the end of each period, the family pools all its assets among its members.

Aggregate state. The aggregate state is given by the beginning-of-period distribution of housing stock and deposit/mortgages among the two types of households, along with the realization of the non-durable good y. However, given the big family assumption, it suffices to use the aggregate housing and asset/mortgage positions of one of the types. I choose the aggregate positions of Borrowers (H_{-1}, M), where H_{-1} is the initial aggregate housing stock and M is the aggregate promised mortgage payments for the period. Let $X = \{H_{-1}, M, y\}$ be the aggregate state of the economy.

Additionally, in order to simplify the exposition of the model, I assume that the subsidy ϕ on LFR contracts is already set high enough, in every state of world, so that households always choose the LFR contract with parameter $\lambda > 0$. In the following sections, the condition to find the minimum interest rate subsidy that makes this result hold will be defined.[30]

[28] No market exclusion and no recourse.

[29] Borrowers belong to a large representative family of Borrowers, whereas Savers belong to a large representative family of Savers.

[30] Formally, if households can choose either LFRMs or one-period contracts every period, then they will compare the utility they get from each contract, given the current level of the subsidy ϕ. However, when considering future values of ϕ, they will also take into account the possibility that, in later periods, they may choose to switch to a different contract. Given the goal of the main exercise of this paper -namely, finding the minimum interest rate subsidy that sustains LFRMs-, both the more general framework and the simplified version presented in the section deliver the same results.

3.2 Borrower's Problem

Inside the representative family of Borrowers, each household starts the period with the same portfolio $x = \{h_{-1}, m\}$ of housing stock and promised mortgage payments for that period. Each household also gets the (same) aggregate non-durable good y and learns what her idiosyncratic depreciation shock ω is.

The family of Borrowers makes default/payment decisions regarding current period's mortgage payments m, then chooses consumption (c, h) along with next period's total mortgage obligations m' taking as given the mortgage contract with $\lambda > 0$ and price schedule Q. After all these decisions have been made, the new portfolio is pooled among all members and consumption is made equally by all households inside the family.

I guess and later verify that the default decision at the family level is characterized by a threshold $\overline{\omega}$. That is, the family honors the promised payment of mortgages associated with values of $\omega > \overline{\omega}$ and defaults otherwise. Specifically, the family of Borrowers makes the following mortgage payments:

$$\int_{\overline{\omega}}^{\infty} dF(\omega)\, m$$

and, accordingly, keeps the houses associated with values of $\omega > \overline{\omega}$:

$$\int_{\overline{\omega}}^{\infty} \omega dF(\omega)\ ph_{-1}$$

Given the mortgage contract with parameter $\lambda > 0$, interest rate subsidy $\phi(h, m', X)$, and price schedule $Q(h, m', X;)$; the house price $p(X;)$, an income tax $\tau(X;)$, and future decision rules, the recursive problem of a representative family of Borrowers consists of choosing nondurable consumption c, housing stock h, total promised mortgage payments m' and a default threshold $\overline{\omega}$ to solve

$$V(h_{-1}, m, X; \lambda, \phi) = \max_{c,h,m',\overline{\omega}} u(c, h) + \beta E_y V(h, m', X'; \lambda, \phi)$$

$$c + p(X; \lambda, \phi)h + \int_{\overline{\omega}}^{\infty} dF(\omega)\, m = (1 - \tau(X; \lambda, \phi))y + \int_{\overline{\omega}}^{\infty} \omega dF(\omega)\ ph_{-1} + Q(h, m', X; \lambda, \phi) \left[m' - \int_{\overline{\omega}}^{\infty} dF(\omega)\lambda m\right]$$

Notice how functions explicitly depend on both the contract parameter λ and the subsidy function ϕ. The left hand side of the budget constraint consists of nondurable consumption and housing consumption, as well as the non-defaulted fraction ($\int_{\overline{\omega}}^{\infty} dF(\omega)$) of the promised mortgage payments m. The right hand side includes the endowment of the nondurable good y, the value of houses kept after default $\int_{\overline{\omega}}^{\infty} \omega dF(\omega)\ ph_{-1}$, and the resources from additional mortgages taken in the current period, which are determined by tomorrow's additional coupon payments $m' - \int_{\overline{\omega}}^{\infty} dF(\omega)\lambda m$.[31]

Finally, I define the problem of an atomistic family of Borrowers that switches forever to a one-period contract, in the context of the equilibrium in which *all other families* stay with the λ-contract.[32][33] In this case, this atomistic family starts the period with a portfolio $x_0 = \{h_{-1}, m, b\}$ of housing stock and promised payments from both long-term and one-period contracts. For simplicity, I assume that once the family decides to switch, it remains passive on its long-term holdings.

The recursive problem of an atomistic family of Borrowers, in this case, consists of choosing nondurable consumption c, housing stock h, one-period mortgage level b' and a default threshold $\overline{\omega}^0$ to solve

$$V^0(h_{-1}, m, b, X; \lambda, \phi) = \max_{c,h,b',\overline{\omega}^0} u(c, h) + \beta E_y V^0(h, m', b', X'; \lambda, \phi)$$

$$c + p(X; \lambda, \phi)h + \int_{\overline{\omega}^0}^{\infty} dF(\omega)\,(m + b) = (1 - \tau(X; \lambda, \phi))y + \int_{\overline{\omega}^0}^{\infty} \omega dF(\omega)\ ph_{-1} + Q^0(h, m', b', X; \lambda, \phi)\, b'$$

$$m' = \int_{\overline{\omega}^0}^{\infty} dF(\omega)\lambda m$$

[31]Recall that the in the absence of new borrowing, next period's coupon payment would decrease according to λ, and also because only the fraction $\int_{\overline{\omega}}^{\infty} dF(\omega)$ of mortgages were not defaulted. Thus, with no additional borrowing, $m' = \int_{\overline{\omega}}^{\infty} dF(\omega)\ \lambda m$.

[32]Recall that the mass of Borrowers is ψ. "*all the other families*" refers to the fact that a mass ψ of Borrowers stay with the LFR contract, whereas only a set of families with mass zero switches to the one-period contract.

[33]Assuming that the family switches to the one-period contract forever, as oppose to having the option to switch back to the LFR contract at any time in the future, is not restrictive, given of nature of the exercise in this paper.

where $V^0(h_{-1}, m, b, X; \lambda, \phi)$ is the value function of a family of Borrowers that has switched to one-period contracts ($\lambda = 0$) but also keeps long-term liabilities m, and $Q^0(h, m', b', X; \lambda, \phi)$ is the price of the one-period mortgage.[34] Accordingly, the utility of a family which chooses to switch in the current period is given by $V^0(h_{-1}, m, 0, X; \lambda, \phi)$. This utility level will be used later to pin down $\phi(h, m', X)$.

It is important to highlight that, because the family does not take any additional LFR debt, the level of the long-term obligations will decline at a rate greater or equal to λ, as the repayment rate $\int_{\varpi^0}^{\infty} dF(\omega)$ is smaller or equal to 1.

3.3 Saver's Problem

Inside the representative family of Savers, each household starts the period with the same portfolio $(\widetilde{h}_{-1}, d)$ of housing stock and one-period deposits. Given the house price $p(X; \lambda, \phi)$, an income tax $\tau(X; \lambda, \phi)$, and the risk-free interest rate $r^d(X; \lambda, \phi)$, the recursive problem of a representative family of Savers consists of choosing nondurable consumption $\widetilde{c}$, housing stock $\widetilde{h}$, and new deposits d' to solve

$$\widetilde{V}(\widetilde{h}_{-1}, d, X; \lambda, \phi) = \max_{\widetilde{c}, \widetilde{h}, d'} u(\widetilde{c}, \widetilde{h}) + \widetilde{\beta} E_y \widetilde{V}(\widetilde{h}, d', X'; \lambda, \phi)$$

$$\widetilde{c} + p(X; \lambda, \phi)(\widetilde{h} - \widetilde{h}_{-1}) + \frac{d'}{1 + r^d} = (1 - \tau(X; \lambda, \phi))y + d + \text{div}(X; \lambda, \phi)$$

where div accounts for the dividends collected from the competitive bank. Notice that even though households in the representative family of Savers are also subject to idiosyncratic depreciation shocks, they are completely unaffected from this because, in equilibrium, they do not take any debt.[35]

[34] These functions depend on both λ and ϕ because, in this problem, p and τ are taken from the equilibrium in which a mass one families stays with the λ-contract, given the subsidy function ϕ.

[35] Given that $E(\omega) = 1$, the initial stock of housing, after all ω are realized, remains constant. Notice that, at this stage, there is heterogeneity at the member's level. However, the family pools its total housing stock among its members, and the heterogeneity disappears.

3.4 Banks and the mortgage price schedule

The competitive bank is owned by Savers, so when choosing a mortgage price schedule, they take into account Savers' stochastic discount factor. They also take as given Borrowers' future decision rules, including the default decision. In equilibrium, given administrative costs θ and the interest rate subsidy ϕ, the mortgage price schedule $Q(h, m', x, X)$ satisfies:

$$Q(h, m', X; \lambda, \phi) = \frac{E_y \left[\Lambda(X'; \lambda, \phi) \, \Gamma(h, m', X'; \lambda, \phi) \right]}{(1 + r^d(X;))(1 + \theta - \phi(h, m', X))}$$

where $\Lambda(X'; \lambda, \phi)$ is the Savers' stochastic discount factor and Γ satisfies

$$\Gamma(h, m', X')m' = \underbrace{\int_{\overline{\omega}'}^{\infty} dF(\omega) \, [1 + \lambda Q'] \, m'}_{\text{non-defaulted coupon payments + cont.value}} + \underbrace{(1 - \mu) \int_0^{\overline{\omega}'} \omega dF(\omega) \, p(X')h}_{\text{housing siezed from defaulting members}}$$

The function Γ accounts for the state-contingent resources the bank gets for every unit of next period's promised coupon payment, given the household's total collateral h and the total promised coupon m'. It consists of two parts. The first one accounts for the non-defaulted fraction $\int_{\overline{\omega}'}^{\infty} dF(\omega)$ of next period's coupon payment m' plus its continuation value $\lambda Q' m'$. The second part is the value of the houses associated with defaulted mortgages $\int_0^{\overline{\omega}'} \omega dF(\omega) \, p(X')h$, net of the foreclosure cost μ.[36]

Note that the interest rate subsidy ϕ is reducing the bank's effective cost of financing one dollar of mortgage. To see why, notice that, in equilibrium, $\Lambda(X'; \lambda, \phi) = \widetilde{\beta} u_{\widetilde{c}'}(X'; \lambda, \phi)/u_{\widetilde{c}}(X; \lambda, \phi)$. Also, in equilibrium, $u_{\widetilde{c}}(X) = \widetilde{\beta} E_y u_{\widetilde{c}'}(X')$. After some algebra, $Q(h, m', X)$ reads:

$$Q(h, m', X) = \frac{E_y \left[\frac{u_{\widetilde{c}'}(X')}{E_y u_{\widetilde{c}'}(X')} \, \Gamma(h, m', X') \right]}{(1 + r^d(X))(1 + \theta - \phi(h, m', X))}$$

where $(1 + r^d)(1 + \theta - \phi)$ accounts for the bank's cost of financing. Finally, let $s = \mathbb{R}_+ \times \mathbb{R}_+$

[36] One way alternative way to interpret this payoff function is by assuming that banks live for two periods. In the first period, they get deposits from Savers to buy a diversified portfolio of mortgages. In the second period, banks meet their deposit obligations with funds collected from non-defaulted coupon payments, from selling the non-defaulted mortgages (continuation value) and from selling the sized houses.

denote the individual state space of Borrowers. Then, dividends "per Saver" are given by:

$$\text{div}(X) = \frac{\psi}{(1-\psi)} \int_s \Gamma(h, m', X)\, m\, ds - d$$

where ψ is the mass of Borrowers and $(1-\psi)$ is the mass of Savers.[37]

3.5 Government

The government collects revenues from a labor tax τ to finance the interest rate subsidy ϕ on LFRMs. In order to reflect the high market share of LFRMs in the US, the government in this model sets the minimum $\phi(h_{-1}, m, X)$ such that Borrowers always choose LFR contracts ($\lambda > 0$). Any subsidy $\widetilde{\phi}$ that sustains those contracts satisfies the following condition:

$$V(h_{-1}, m, X; \lambda, \widetilde{\phi}) \geq V^0(h_{-1}, m, 0, X; \lambda, \widetilde{\phi})$$

for every $\{h_{-1}, m, X\}$. Thus, the minimum subsidy ϕ satisfies the previous condition with equality. Evidently, this minimum subsidy can be negative, in which case, Borrowers value more the insurance benefit of entering a long-term contract than the incentives benefit of one-period contracts.

The total minimum subsidy $\Phi(X; \lambda, \phi)$ is then given by

$$\Phi(X; \lambda, \phi) = \psi \int_s \phi(h, m', X) Q m' ds$$

Finally, the government balance its budget every period by setting $\tau(X;)$ such that $\tau(X;)y = \Phi(X;)$.

3.6 Equilibrium

Let $s = \mathbb{R}_+ \times \mathbb{R}_+$ denote the individual state space of Borrowers, $\widetilde{s} = \mathbb{R}_+ \times \mathbb{R}_+$ the individual state space for Savers, and $S = \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}$ be the aggregate state space.

[37]This equation reflects the fact that banks generate positive dividends when the resources they collect from mortgages ($\frac{\psi}{1-\psi} \int \Gamma m$) are higher than their deposits obligations (d).

A recursive competitive equilibrium is a collection of decision rules of Borrowers c, m', h, $\overline{\omega} : s \times S \to \mathbb{R}$; decision rules of Savers $\widetilde{c}$, $\widetilde{h}$, $\widetilde{d} : \widetilde{s} \times S \to \mathbb{R}$; associated value functions $V : s \times S \to \mathbb{R}$ and $\widetilde{V} : \widetilde{s} \times S \to \mathbb{R}$, future decision rules g^c, g^m, g^h, $g^{\overline{\omega}} : s \times S \to \mathbb{R}$; prices $p, r^d : S \to \mathbb{R}$, mortgage price schedule $Q : s \times S \to \mathbb{R}$, subsidy function $\phi : s \times S \to \mathbb{R}$ along with the associated tax function $\tau : S \to \mathbb{R}$, and a law of motion for the aggregate state $T^X : S \to S$ such that:

1. Decision rules and value functions solve both households' problems, taking future decision rules, p, r^d, Q and T^X as given.

2. All markets clear

$$\psi \left[c + \mu \int_0^{\overline{\omega}} \omega dF(\omega) ph_{-1} + \theta Qm' \right] + (1 - \psi)\widetilde{c} = y$$

$$\psi h + (1 - \psi)\widetilde{h} = H_s$$

$$\psi Qm' = (1 - \psi)\frac{d'}{(1 + r^d)} + \Phi$$

3. The subsidy function is, at every state, the minimum interest rate subsidy on λ- contracts such that these contracts are chosen

$$V = V^0(h_{-1}, m, 0, X)$$

4. Balanced government budget constraint:

$$\tau y = \Phi = \psi \int_s \phi Qm' ds$$

4. The motion of the aggregate state is consistent with individual decision rules:

$$[H,\ M',\ y'] = T^X(X) = [h(H_{-1}, M, X), m(H_{-1}, M, X), y']$$

6. Current and future decision rules coincide for all possible states.

3.7 Characterization of Equilibrium

Given that the Savers problem is quite standard, this section focuses the FOCs of the Borrower's problem. The optimal default decision satisfies:

$$\overline{\omega} p h_{-1} = (1 + \lambda Q) m$$

This condition is just equating the current cost of defaulting, which is given by the loss of housing stock of value $\overline{\omega} p h_{-1}$, with the present and future cost of honoring the mortgage obligation, m and $\lambda Q m$ respectively.[38] Notice that the future cost takes into account the option to default in the future through Q. The FOC with respect to the next period's coupon payment, m', reads:

$$u_c[Q + Q_m[m' - \int_{\overline{\omega}}^{\infty} dF(\omega)\lambda m]] = \beta E u_{c'} \int_{\overline{\omega}'}^{\infty} dF(\omega)[1 + \lambda Q']$$

The left-hand side of this equation represents the net marginal benefit of borrowing. By using one extra unit of mortgage debt, Borrowers can increase current consumption by Q units. However, as they face a price schedule, when borrowing more, it decreases the price ($Q_m < 0$) of every unit of additional mortgage debt they take in the current period ($m' - \int_{\overline{\omega}}^{\infty} dF(\omega)\lambda m$), which in turn reduces consumption. The right-hand side of the equation is the marginal cost of borrowing: by taking new debt, Borrowers decrease the expected future consumption, proportionally to the non-defaulted ($\int_{\overline{\omega}}^{\infty} dF(\omega)$) future unit value of total mortgage obligations ($1 + \lambda Q'$).

Notice that for $\lambda > 0$, the government only internalizes as a cost the decline in the prices of new debt taken in the current period. It does not internalizes as a cost, the decrease in the market value of mortgages taken in previous periods. This fact, commonly referred to as debt dilution, is not present in the case of short-term debt ($\lambda = 0$), where the decrease in price applies to the entire stock of debt. For this reason, both debt levels and default rates are higher with long-term fixed-rate contracts. This is also taken into account by banks when setting the price schedule, which results in long-term contracts being *more expensive* in equilibrium.[39]

Finally, the FOC with respect to the current housing consumption is given by:

[38] Notice that, at the family level, the default decision is continuous.

[39] That is, for the same face value of mortgage debt, interest rates are higher in long-term fixed-rate contracts.

$$u_c[p - Q_h[m' - \int_{\overline{\omega}}^{\infty} dF(\omega)\lambda m]] = u_h + \beta E u_{c'} \int_{\overline{\omega}'}^{\infty} \omega dF(\omega) p'$$

The left-hand side of the equation is the net marginal cost of housing consumption. By purchasing one extra unit of housing, Borrowers pay the house price p. However, additional housing also increases the amount of collateral in the mortgage contracts and therefore, increases the price of the mortgage, reducing the total marginal cost. The right-hand side represents the marginal benefit of housing consumption. Additional housing consumption directly increases the Borrowers current utility, as well as future consumption due to price appreciation (p') net of the value of housing seized by banks upon default ($\int_{\overline{\omega}'}^{\infty} \omega dF(\omega)$).

4 Calibration

A summary of the calibration for an annual frequency is shown in Table 1. Details are discussed below.

Income Process. The non-durable good endowment y is assumed to be an AR(1) process of the form:

$$y = (1 - \rho) + \rho y_{-1} + \varepsilon$$

where $E(\varepsilon) = 0$, $E(\varepsilon^2) = \sigma_{\varepsilon}^2$, and ρ is the one-period autocorrelation and the unconditional mean of y is normalized to one. I use two different estimates for ρ and σ_{ε}: one that captures aggregate income fluctuations and a second that accounts for the idiosyncratic ones. For the first one, I use annual HP-filtered data of the US real[40] median household income for the period 1970-2016, which results in $\rho = 0.75$ and $\sigma_{\varepsilon} = 0.025$. On the other hand, recent estimates[41] of the income process for heterogeneous-agent models report $\rho = 0.98$, whereas $\sigma_{\varepsilon} = 0.068$ on average.[42] I choose $\rho = 0.75$ and $\sigma_{\varepsilon} = 0.068$ so that the persistence of the endowment process resembles that of the aggregate median household income, while the volatility is closer to that of idiosyncratic income estimates.

[40] I use the implicit GDP deflator to convert the amounts into real terms. Using data in 2015 dollars delivers similar estimates.

[41] See Storesletten et al. (2004).

[42] The income process in this literature has the form $\log y_t = \widehat{\rho} \log y_{t-1} + (1 - \widehat{\rho}^2)^{0.5} \widehat{\varepsilon}_t$. The estimates are $\widehat{\rho} = 0.98$ and $\sigma_{\widehat{\varepsilon}} = 0.3$, which are equivalent to $\rho = 0.98$ and $\sigma_{\varepsilon} = 0.065$ for the income process used in this document.

In the context of the model, a high value of income volatility also delivers a high house price volatility. In fact, the house price-income ratio has increased its standard deviation from 0.14 during the period 1980-2000 to 0.44 for the period 2001-2017. Even when excluding years of the real estate bubble (2001-2007), the standard deviation increases to 0.25. The resulting house price volatility in the model is 0.29, which between 0.25 and 0.44.

Finally, the AR(1) process of the non-durable good endowment y is approximated with a 5-state Markov chain using Tauchen and Hussey's (1991) algorithm.

Foreclosure Cost. A value a 0.22 is chosen for the foreclosure parameter μ, following the work of Pennington-Cross (2006) studying the liquidation sales revenue from foreclosed houses using national data.

Depreciation Shock. The depreciation shock ω follows a lognormal distribution with mean one and $\sigma = var(\ln \omega)$. Notice that, in the model, both default and foreclosure take place in the same period. In the real world, only a fraction of delinquent mortgages end up being foreclosed two years after the initial date of default on average. Using data collected by ATTOM Data Solutions, the average delinquency rate on single-family residential mortgages during the period 2001-2017 was 5.25 percent, whereas the average foreclosure rate was 1.05 percent.

Motivated by the timing discrepancy between default and foreclosure, the value of σ is chosen to match a slightly higher value than the average foreclosure rates observed in the data. A target of 1.4 percent for the default rate is chosen, which results in a value of 0.13 for σ. This is a relatively conservative target compared to Elenev et al.(2015), who target 2 percent in normal times and 8.5 percent during a foreclosure crisis; and higher than Jeske et al. (2013), who target only a 0.5 percent foreclosure rate.

Preferences. The period utility function has the form

$$u(c, h) = \ln c + \eta \ln h$$

The parameter η is chosen to match the average share of housing in total consumption expenditures from NIPA. The average share is 13.9% for the period 2012-2016, which implies a value of η of 0.161. The discount factor of Savers, $\widetilde{\beta}$, is set at 0.98 to match an equilibrium risk-free rate of 2%. The discount factor of Borrowers, β, is set at 0.97 to match a home equity ratio[43] of 65%. The mass of Borrowers, ψ, is set at 0.47, following Elenev et al.(2015).[44]

Mortgage. The parameter that determines how fast coupon payments decay, λ, is set equal to 0.967 to resemble a 30-year contract. Finally, the administrative cost per unit of mortgage issued, θ, is set at 40 basis points, following Jeske et al. (2013).[45] This cost applies for both one-period and LFR contracts.

Housing stock. The fixed housing stock H_s is set to 1.8 to match a house price-household income ratio of 4.24, which is the averarge ratio for the period 2001-2017.[46]

Table 1: Calibration

Exogenously Calibrated Parameters

Parameter	Description	Value	Source / Target
ρ	Income persistance	0.75	Aggregate household income
σ_ε	Income volatility	0.068	Storsletten et al. (2004)
μ	Foreclosure cost	0.22	Pennington-Cross (2004)
η	Preference for housing	0.161	Housing consump. share NIPA
λ	Coupon decaying factor	0.967	30-year mortgage ($\lambda = \frac{N-1}{N}$)
θ	Mortgage administrative cost	40 BP	Jeske et al. (2013)
ψ	Mass of Borrowers	0.47	SCF asset positions
H_s	Housing Stock	1.8	House-price/Income 4.24
Endogenously Calibrated Parameters			
σ	Volatility of depreciation shock	0.13	Default rate 1.5%
$\widetilde{\beta}$	Discount Factor Savers	0.98	Risk-free rate 2%
β	Discount Factor Borrowers	0.97	Home equity ratio 65%

[43]Home equity is defined as the market (current) value of total housing stock minus the total mortgages obligations attached to the houses. The home-equity ratio is home equity as a fraction of the market value of the total housing stock.

[44]This number is based on calculations a net fixed-income position for households in the Survey of Consume Finance (SCF).

[45]In their paper, banks have to pay 10 basis points for administrative fees and 30 basis points for insurance.

[46]The average of this ratio 3.32 during the period 1980-2000. During 2017, the ratio was 4.33.

5 Main Results

The main results are shown in Table 2. All comparisons are based on steady state numbers and are made with respect to the case with no subsidy, where Borrowers consequently choose the one-period contract. A comparison of the model moments vs those in the data are shown in Table 3 in Appendix 8.1.

The subsidy needed to induce Borrowers to choose a 30-year mortgage contract is, on average, 36 basis points, which is financed by an increase of the income tax of 0.60 percentage points. Compared to the case of no subsidy, where Borrowers choose 1-year contracts, the default rate is 1.38 percentage points higher. Recall that the calibration targets a steady state default rate of 1.5%, which indicates that the model generates a default rate close to zero in the absence of the interest rate subsidy on long-term fixed-rate contracts.

Mortgage debt levels are 37% higher compared to the case of no subsidy, which is the result of both the subsidy and the fact that households use long-term contracts. House prices are only 1.2% higher, which reflects a 4.1% higher housing consumption of Borrowers. Also, the home-equity ratio is 6.4 percentage points lower than the scenario without the subsidy (65% vs 71.4%). Finally, this policy is not Pareto–improving: it implies a welfare gain of 0.31 percent in consumption-equivalent terms for Borrowers, and welfare loss of 1.05 percent for Savers, when compared to the case with no subsidy.

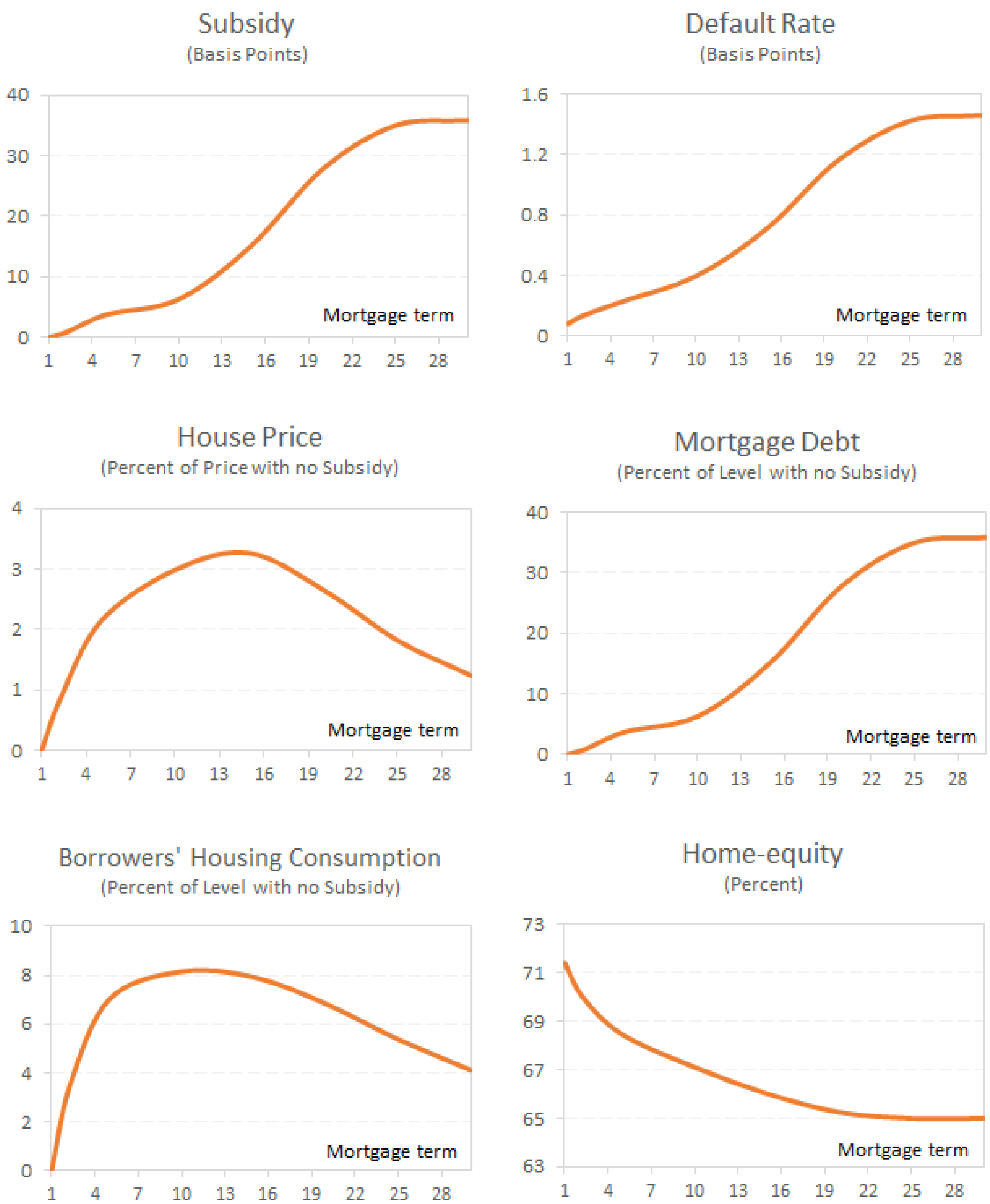
Table 2: Main Results

Comparisons with respect to the case of NO subsidy

Variable	Value
Subsidy (ϕ)	36 BP
Income Tax (τ)	0.60%
Δ default rate	1.38 ptc points
Δ% house price	1.2%
Δ% mortgage debt ($\frac{m'}{1+r_d-\lambda}$)	37%
Δ home equity ratio	-6.4 ptc points
Δ% Borrowers house consumption	4.1%
Borrowers CEV*	0.31%
Savers CEV*	-1.05%

*Consumption-equivalent variation

Figure 3: Results for different Mortgage Terms



Note: All values correspond to the non-stochastic steady state of the model for a given mortgage term

Additionally, I perform the same exercise with shorter terms.[47] The results are shown in Figure 3. The subsidy is increasing on the mortgage term that is being targeted. This result holds because, for this calibration, the incentives provided by one-period contracts dominate the insurance rendered by LFRMs. That is, in the absence of the subsidy, LFRMs are *too expensive* in equilibrium. The longer the mortgage term, the larger Banks will charge initially to compensate for both the interest rate risk and debt dilution; therefore, a higher subsidy is required to make this contracts attractive.

Notice that this last result is consistent with the recent developments in the secondary mortgage market. Since 2007, Fannie Mae and Freddie Mac have increased the guarantee fees they charge banks, an action that is equivalent to reducing the size of the subsidy. In the model, smaller subsidies sustain shorter mortgage terms. In the data, the average term of newly originated mortgages has decreased, as more households take 15-year fixed-rate mortgages as opposed to 30-year fixed-rate ones.

Both the default rate and the level of mortgage debt are increasing on the term of the mortgage. In fact, their levels resemble qualitatively that of the subsidy. On the other hand, the house price and the housing consumption of Borrowers have both increasing and decreasing intervals. This is explained in part by the fact that the subsidy is financed by both Borrowers and Savers.[48] Finally, the home-equity ratio is decreasing in the mortgage term.

6 The Role of Income Volatility

6.1 Optimality of one-year contracts under the Main Calibration

In the main exercise of this paper, the government sets the minimum interest-rate subsidy so that Borrowers choose the long-term mortgage contract over a one-period contract, in equilibrium. In this subsection, I show that, in the absence of subsidy, one-year contracts are optimal under the main calibration. The specifics of how to prove this result (numerically) are as follows:[49]

1. I solve for equilibria in which only N-year contracts are available, $N = 1, 2, ..30$. No subsidies.

[47] Recall that, for an annual frequency, a maturity of N years is equivalent to $\lambda = (N - 1)/N$.

[48] Even though long-term FRMs allow Borrowers to sustain higher levels of debt, they also require increasing levels of the interest-rate subsidy, which Borrowers are also taxed for.

[49] The results of this section are not shown.

2. In the $(N = 1)$-equilibrium—for any point in the ergodic set, I verify that when a Borrower is offered an $\widehat{N}$-year contract with $\widehat{N} > 1$, he/she will choose to stay with the one-year contract.

3. In all $(N > 1)$-equilibria—for any point in the corresponding ergodic sets, I verify that when a Borrower is offered a one-year contract, he/she will choose to switch to it.

4. Therefore, with no subsidy and under the main calibration, one-year contracts are optimal.

Finally, it is important to mention that, the subsidies found in the main exercise, work only on the ergodic set of the equilibrium in which all Borrowers choose N-contracts with $N > 1$. However, initial conditions could be such that there is no subsidy on LFR contracts and, therefore, the economy is in the $(N = 1)$-equilibrium. In that case, the government would need to set a subsidy to make Borrowers switch to longer-term contracts.

When computing this subsidy for every point in the ergodic set of the $(N = 1)$-equilibrium, the resulting subsidy to switch to a 30-year contract is 23 basis points on average. In additi on, this initial subsidy is also lower for shorter mortgage terms.

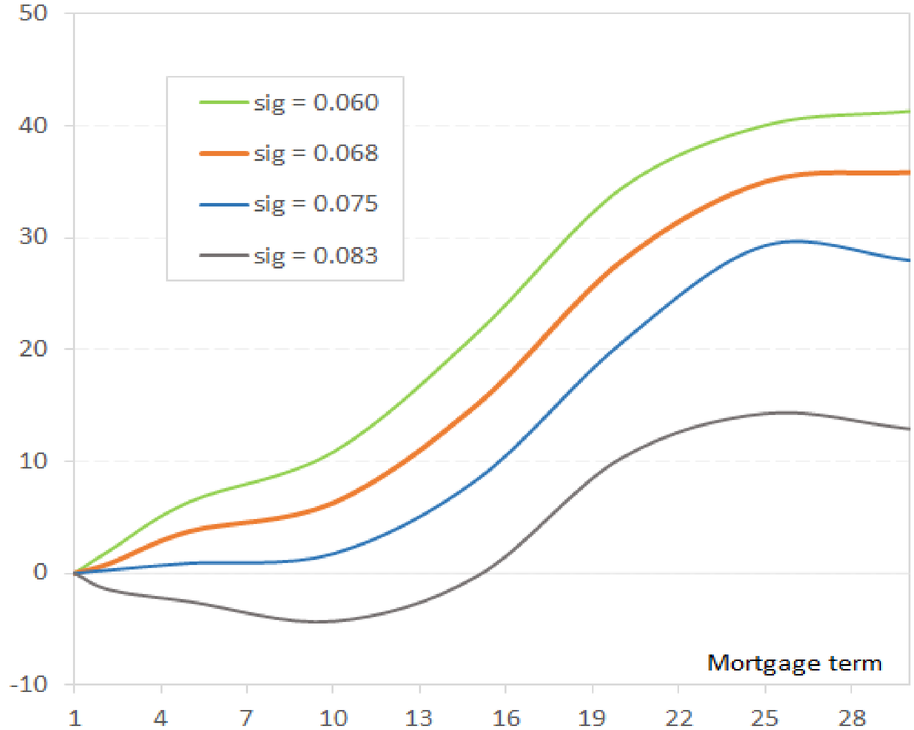
6.2 Different Income Volatilities

This subsection, I show how the results change when varying the value of Income Volatility σ. I focus on this parameter given that it has the largest effects on the equilibrium subsidy. Figure 4 shows the results of this sensitivity analysis for four different values of σ, including the main calibration's value $\sigma = 0.068$.

Notice that, the lower the volatility parameter, the higher the subsidy needed to sustain longer-term contracts; this result holds for every N-contract with $N > 1$. However, if volatility is too low, the equilibrium subsidy will be large enough so that the resulting interest rate on 30-year contracts will be smaller than that on one-year contracts, which is has not been observed in the data. This is the case, for instance, for values of $\sigma \leq 0.064$.

On the other hand, for volatilities above a certain level ($\sigma = 0.075$), the subsidy becomes negative, at least for low terms. This reflects the fact that, when volatility is too high, Borrowers value more the insurance given by longer-term contracts than the additional cost they

Figure 4: Equilibrium Subsidy for different values of Income Volatility σ (basis points)



Note: All values correspond to the non-stochastic steady state of the model for different mortgage terms.

hold. This is shown in Figure 3 for the case of $\sigma = 0.083$. As a consequence, under this alternative calibration, one-period contracts are no longer optimal.

Evidently, there exists a high enough value of σ such that even 30-year contracts do not need to be subsidized. However, quantitatively, it is difficult to justify using such extreme values of income volatility, at least for the U.S. and most advanced economies. It is worth mentioning, though, that, higher levels of income volatility are more common in emerging-market economies.[50]

[50] LFRMs dominate in countries like Argentina, Colombia, Brazil, Peru and Russia, even in the absence of government intervention in the secondary mortgage market. The study of mortgage market in emerging markets is left for future work.

7 Conclusion

The U.S. has an unusually high proportion of long-term fixed-rate mortgages (30 years), a result of government policies lowering the interest rate of these mortgage products. Using a two-agent model (Borrowers and Savers), this paper measures the subsidy-equivalent cost of such policies– that is, the minimum interest rate subsidy so that households choose long-term mortgages over short-term ones as in the data. I find that the subsidy is around 36 basis points on average, and that targeting mortgage terms shorter than 30 years requires lower subsidies. Finally, I find that the size of the subsidy depends mainly on the level of income volatility, but while levels lower than the one in the main calibration deliver a negative spread of fixed and variable rate, higher levels are difficult to justify.

This paper contributes to the strand of literature studying the role of the government sponsored enterprises (GSEs)[51], by providing a methodology to endogenize the size of such policies so that it matches the high market share of long-term fixed-rate mortgages. Evidently, given the simplicity of the model, the numbers resulting from the main exercise are only informative about the forces behind the high market share of LFRMs. For instance, a fixed supply of housing increases the volatility of house prices and, ultimately, make fixed-rate contracts more attractive, which over-estimates the size of the subsidy. On the other hand, the big-family and the no-exclusion-after-default assumptions decrease the cost of default, reduce both one-period and LFR interest rates, but could either increase or decrease the FRM-ARM spread, which will mainly determine the final effect on the needed subsidy.[52]

Finally, the methodology proposed in the paper could be easily applied to models with richer levels of heterogeneity and a life cycle dimension; in particular, one where households facing binding borrowing constraints have to choose ARMs.[53] Additionally, this framework can be used to study mortgage markets in emerging economies, which typically display higher levels of income volatility. Such analyses are deferred for future work.

[51]This strand of the literature usually takes the magnitude of the government intervention as exogenously given.

[52]In other words, this is ultimately a quantitative question.

[53]A life cycle model provides a more suitable framework to analyze policies affecting the downpayment, or to study the possibility that the same policy could benefit or make a household worse off depending on its age.

References

[1] Aguiar, M., Amador, M., Hopenhayn, H., and Werning, I., 2016. "Take the Short Route: Equilibrium Default and Debt Maturity". Working Paper.

[2] Badarinza, C., Campbell, J. Y., and Ramadorai, T., 2015. "What Calls to ARMs? International Evidence on Interest Rates and the Choice of Adjustable-Rate Mortgages". Working Paper.

[3] Campbell, J., 2013. "Mortgage Market Design". Review of Finance. 2013, 17(1), 1-33.

[4] Campbell, J. Y., and Cocco, J., 2003. "Household Risk Managment and Optimal Mortgage Choice". Quaterly Journal of Economics, 118, 1449-1494.

[5] Congretional Budget Office (CBO), 2013, "Federal Subsidies and the Housing GSEs".

[6] Chatterjee, S. and Eyigungor, B., 2011. "A Quantitative Analysis of the U.S. Housing and Mortgage Markets and the Foreclosure crisis". Federal Reserve Bank of Philadelphia Working Paper.

[7] Corbae, D. and Quintin, E., 2014. "Leverage and the Foreclosure Crisis". Working Paper, University of Wisconsin.

[8] Davis, M. and Van Nieuwerburgh, S., 2015. "Housing, Finance, and the Macroeconomy". Handbook of Regional and Urban Economics, 2015, vol. 5, ch. Chapter 12, p. 753-811, Elsevier.

[9] Elenev, V., Landvoigt, T., and Van Nieuwerburgh, S., 2015. "Phasing out the GSEs". NBER Working Paper, Cambrige, MA.

[10] Floetotto, M., Kirker, M., and Strobell, J., 2012. "Government Intervention in the Housing Market: Who Wins, Who Loses". Working Paper Stanford University.

[11] Garriga, C. and Schlagenhauf, D. E., 2009. "Home Equity, Foreclosures and Bailouts". Working Paper.

[12] Gervais, M., 2002. "Housing Taxation and Capital Accumulation". Journal of Monetary Economics, 49(7), 1461-1489.

[13] Jeske, K., Krueger, D., and Mitman, K., 2013. "Housing, Mortgage Bailout Guarantees and the Macro Economy". Journal of Monetary Economics.

[14] Kiyotaki, N. and Moore, J., 1997. "Credit Cycles". Journal of Political Economy, Vol. 105, No. 2, pp. 211-248.

[15] Koijen, R., Hemert, O., and Van Nieuwerburgh, S., 2009. "Mortgage timing". Journal of Financial Economics, 93(2), 292-324.

[16] Krainer, J., 2010. "Mortgage Choice and the Pricing of Fixed-Rate and Adjustable-Rate Mortgages". Federal Reserve Bank of San Francisco Newsletter, February 2010.

[17] Lea, M., 2010. "International Comparison of Mortgage Product Offerings". Research Institute of Housing America, September 2010.

[18] Lucas, D. and McDonald, R., 2010. "Valuing Government Guarantees: Fannie and Freddie revisited", in Lucas, D. (Ed.) *Measuring and Managing Federal Financial Risk*, NBER, Cambrige, MA.

[19] Moech, E., Vickery, J., and Aragon, D., 2010. "Why is the Market Share of Adjustable-Rate Mortgages so Low?", Current Issues in Economics and Finance, 16 (8), 1-11.

[20] Passmore, W., 2005. "The GSE Implicit Subsidy and the Value of Government Ambiguity". Real Estate Economics, 33(3), 465-486.

[21] Peek, J., 1990. "A Call to ARMs: Adjustable Rate Mortgages in the 1980s". New England Economic Review, March/April 1990.

[22] Pennington-Cross, A., 2016, "The durations of foreclosures in the subprime mortgage market: a competing risk model with mixing", Working Papers 2006-2007, Federal Reserve Bank of Saint Louis.

[23] Storesletten, K., Telmer, C., and Yaron, A., 2004, "Consumption and risk sharing over the life cycle", Journal of Monetary Economics, Elsevier, Vol. 51(3), pages 609-633, April.

[24] Tauchen, G. and Hussey, R., 1991, "Quadrature-based methods for obtaining approximate solutions to non-linear asset pricing models", Econometrica, Vol. 59, 371-396.

[25] Van Hemert, O., 2010. "Household Interest Rate Risk Management". Real Estate Economics.

[26] Vickery, J., 2007. "Interest Rates and Consumer Choice in the Residential Mortgage Market". FRB NY Working Paper.

8 Appendix

8.1 Moments of the Model

This section shows how the model perform with respect to the data. This comparison is made in Table 3. All the key variables of the model closely replicate the data, except for the 30 FRM interest rate: while this interest rate had an average spread of 300 bps, the model only generates a spread of 49 bps.[54] The model falls short on this rate mainly because of the chosen discount factor for Borrowers ($\beta = 0.97$), which was calibrated to match an average home-equity ratio of 65%. Additionally, given the main calibration of the rest of the parameters, the model does not have a solution for $\beta < 0.965$.

Table 3: Data vs Model Moments

	Data		Model	
	Mean	Std. Dev.	Mean	Std. Dev.
Risk-free rate	2.00	1.50	2.00	1.70
30 FRM rate*	5.30	2.60	2.49	2.85
House price - income ratio	4.24	0.29	4.24	0.29
Default rate	1.50	0.51	1.50	0.42
Home-equity ratio	0.65	0.04	0.65	0.05

* Risk-free rate of 2% plus average spread

8.2 Solving the Model Linearly

8.2.1 Borrowers Problem

Let F be the cumulative distribution of the idiosyncratic shock ω. Notice that the default rate is given by $F(\overline{\omega})$, and that the repaid fraction is given by:

$$1 - F(\overline{\omega}) = \int_{\overline{\omega}}^{\infty} dF(\omega)$$

Define the function G as the conditional expectation of ω, for values below $\overline{\omega}$:

$$G(\overline{\omega}) = \int_{0}^{\overline{\omega}} \omega dF(\omega)$$

[54]This number already includes the subsidy of 36 bps.

Then, we can state the Borrowers problem in sequential form as follows:

$$V(m_{t-1}, h_{t-1}, X_t) = \max_{c_t, m_t, h_t, \overline{\omega}_t} u(c_t, h_t) + \beta E_X V(m_t, h_t, X_{t+1})$$

$$c_t + p(X)h_t + (1 - F(\overline{\omega}_t))m_{t-1} = y + (1 - G(\overline{\omega}_t))p(X)h_{t-1} + Q(m_t, h_t, X)[m_t - (1 - F(\overline{\omega}_t))\lambda m_{t-1}]$$

The first order conditions are:

$$\begin{aligned} u_{c,t}[Q_t + Q_{m,t}[m_t - (1 - F(\overline{\omega}_t))\lambda m_{t-1}]] &= \beta E_t u_{c,t+1}[1 - F(\overline{\omega}_{t+1})][1 + \lambda Q_{t+1}] \\ u_{c,t}[p_t - Q_{h,t}[m_t - (1 - F(\overline{\omega}_t))\lambda m_{t-1}]] &= u_{h,t} + \beta E_t u_{c,t+1}[1 - G(\overline{\omega}_{t+1})]p_{t+1} \end{aligned}$$

$$\overline{\omega}_t = \frac{(1 + \lambda Q_t)m_{t-1}}{p_t h_{t-1}}$$

The problematic terms are $Q_{m,t}$ and $Q_{h,t}$. Recall that the mortgage price function satisfies:

$$Q_t = \frac{1}{(1 + r_t^d)E_t u_{\tilde{c},t+1}} E_t \left[u_{\tilde{c},t+1}[(1 - \mu)G(\overline{\omega}_{t+1}) \frac{p_t h_{t-1}}{m_{t-1}} + (1 - F(\overline{\omega}_{t+1}))(1 + \lambda Q_t)] \right]$$

If we use the FOC for $\overline{\omega}$ for $t + 1$:

$$Q_t = \frac{1}{(1 + r_t^d)E_t u_{\tilde{c},t+1}} E_t \left[u_{\tilde{c},t+1}[(1 - \mu)\frac{G(\overline{\omega}_{t+1})}{\overline{\omega}_{t+1}} + (1 - F(\overline{\omega}_{t+1}))](1 + \lambda Q_{t+1}) \right]$$

Let' s forget about the SDF for a moment, and simplify the expression to:

$$Q_t = \frac{1}{(1 + r_t^d)} E_t \left[[(1 - \mu)\frac{G(\overline{\omega}_{t+1})}{\overline{\omega}_{t+1}} + (1 - F(\overline{\omega}_{t+1}))](1 + \lambda Q_{t+1}) \right]$$

Now we can find the expressions for the derivatives (recall $\overline{\omega}_{t+1} = g^{\omega}(m, h, X')$)

$$Q_{m,t} = \frac{1}{(1+r_t^d)} E_t \left[\begin{array}{l} \frac{\partial}{\partial m_t}[(1-\mu)\frac{G(\overline{\omega}_{t+1})}{\overline{\omega}_{t+1}} + (1 - F(\overline{\omega}_{t+1}))](1+\lambda Q_{t+1}) + \\ [(1-\mu)\frac{G(\overline{\omega}_{t+1})}{\overline{\omega}_{t+1}} + (1-F(\overline{\omega}_{t+1}))]\lambda \frac{\partial}{\partial m_t} Q_{t+1} \end{array} \right]$$

The first derivative reads:

$$\begin{aligned} FD &= \left[(1-\mu)\frac{G'(\overline{\omega}_{t+1})}{\overline{\omega}_{t+1}} - (1-\mu)\frac{G(\overline{\omega}_{t+1})}{\overline{\omega}_{t+1}^2} - F'(\overline{\omega}_{t+1}) \right] g_m^{\overline{\omega}}(m,h,X') \\ &= \left[(1-\mu)\frac{\overline{\omega}_{t+1} f(\overline{\omega}_{t+1})}{\overline{\omega}_{t+1}} - (1-\mu)\frac{G(\overline{\omega}_{t+1})}{\overline{\omega}_{t+1}^2} - f(\overline{\omega}_{t+1}) \right] g_m^{\overline{\omega}}(m,h,X') \\ &= \left[-\mu f(\overline{\omega}_{t+1}) - (1-\mu)\frac{G(\overline{\omega}_{t+1})}{\overline{\omega}_{t+1}^2} \right] g_m^{\overline{\omega}}(m,h,X') \end{aligned}$$

The second derivative (recall $m_{t+1} = g^m(m,h,X')$ and $h_{t+1} = g^h(m,h,X')$):

$$\frac{\partial}{\partial m_t} Q_{t+1} = Q_{m,t+1}\, g_m^m(m,h,X') + Q_{h,t+1}\, g_m^h(m,h,X')$$

Therefore:

$$\begin{aligned} Q_{m,t} &= \frac{1}{(1+r_t^d)} E_t[\, (-\mu f(\overline{\omega}_{t+1}) - (1-\mu)\tfrac{G(\overline{\omega}_{t+1})}{\overline{\omega}_{t+1}^2}) g_m^{\overline{\omega}}(m,h,X')(1+\lambda Q_{t+1}) + \\ & \quad [(1-\mu)\tfrac{G(\overline{\omega}_{t+1})}{\overline{\omega}_{t+1}} + (1-F(\overline{\omega}_{t+1}))]\lambda \left[Q_{m,t+1}\, g_m^m(m,h,X') + Q_{h,t+1}\, g_m^h(m,h,X') \right] \,] \end{aligned}$$

Similarly:

$$\begin{aligned} Q_{h,t} &= \frac{1}{(1+r_t^d)} E_t[\, (-\mu f(\overline{\omega}_{t+1}) - (1-\mu)\tfrac{G(\overline{\omega}_{t+1})}{\overline{\omega}_{t+1}^2}) g_h^{\overline{\omega}}(m,h,X')(1+\lambda Q_{t+1}) + \\ & \quad [(1-\mu)\tfrac{G(\overline{\omega}_{t+1})}{\overline{\omega}_{t+1}} + (1-F(\overline{\omega}_{t+1}))]\lambda \left[Q_{m,t+1}\, g_h^m(m,h,X') + Q_{h,t+1}\, g_h^h(m,h,X') \right] \,] \end{aligned}$$

8.2.2 Case $\lambda = 0$

In this case, formulas simplify a lot:

$$u_{c,t}[Q_t + Q_{m,t}m_t] = \beta E_t u_{c,t+1}[1 - F(\overline{\omega}_{t+1})]$$

$$u_{c,t}[p_t - Q_{h,t}m_t] = u_{h,t} + \beta E_t u_{c,t+1}[1 - G(\overline{\omega}_{t+1})]p_{t+1}$$

$$\overline{\omega}_t = \frac{m_{t-1}}{p_t h_{t-1}}$$

$$Q_t = \frac{1}{(1+r_t^d)} E_t\left[(1-\mu)\frac{G(\overline{\omega}_{t+1})}{\overline{\omega}_{t+1}} + (1 - F(\overline{\omega}_{t+1}))\right]$$

$$Q_{m,t} = \frac{1}{(1+r_t^d)} E_t\left[\left(-\mu f(\overline{\omega}_{t+1}) - (1-\mu)\tfrac{G(\overline{\omega}_{t+1})}{\overline{\omega}_{t+1}^2}\right) g_m^{\overline{\omega}}(m, h, X')\right]$$

$$Q_{h,t} = \frac{1}{(1+r_t^d)} E_t\left[\left(-\mu f(\overline{\omega}_{t+1}) - (1-\mu)\tfrac{G(\overline{\omega}_{t+1})}{\overline{\omega}_{t+1}^2}\right) g_h^{\overline{\omega}}(m, h, X')\right]$$

Also notice that $g_m^{\overline{\omega}}(m_t, h_t, X') = \frac{1}{p(X')h_t} = \frac{\overline{\omega}_{t+1}}{m_t}$ and $g_h^{\overline{\omega}}(m_t, h_t, X') = -\frac{m_t}{p(X')h_t^2} = -\frac{\overline{\omega}_{t+1}}{h_t}$

$$Q_{m,t} = \frac{1}{(1+r_t^d)} E_t\left[\left(-\mu f(\overline{\omega}_{t+1}) - (1-\mu)\frac{G(\overline{\omega}_{t+1})}{\overline{\omega}_{t+1}^2}\right)\frac{\overline{\omega}_{t+1}}{m_t}\right]$$

$$Q_{h,t} = \frac{1}{(1+r_t^d)} E_t\left[\left(\mu f(\overline{\omega}_{t+1}) + (1-\mu)\frac{G(\overline{\omega}_{t+1})}{\overline{\omega}_{t+1}^2}\right)\frac{\overline{\omega}_{t+1}}{h_t}\right]$$

The case for $\lambda = 0$ can shed some light into how to solve the model when we don't have an analytical expression for the derivatives $g_m^{\overline{\omega}}$ and $g_h^{\overline{\omega}}$. First, assume that both $g_m^{\overline{\omega}}$ and $g_h^{\overline{\omega}}$ are linear. Then we can assume certain values of $g_m^{\overline{\omega}}$ and $g_h^{\overline{\omega}}$, solve the model linearly and later update the guess with the slopes obtained from the linear solution, until we get convergence.

However, notice that to recover $g_m^{\overline{\omega}}(m_{t-1}, h_{t-}, X_t)$ and $g_h^{\overline{\omega}}(m_{t-1}, h_{t-}, X_t)$, we need to make an intermediate step first. Usually, the linear solution will be of the form $T^{\overline{\omega}}(M_{t-1}, H_{t-1}, y_t) = T^{\overline{\omega}}(X_t)$ and $T^{\overline{\omega}}(M_{t-1}, H_{t-1}, y_t) = T^{\overline{\omega}}(X_t)$. That is, it will be expressed in terms of the aggregate state variables. And, generally:

$$
\begin{aligned}
g_m^{\overline{\omega}}(m_{t-1}, h_{t-}, X_t) &\neq T_M^{\overline{\omega}}(M_{t-1}, H_{t-1}, y_t) \\
g_h^{\overline{\omega}}(m_{t-1}, h_{t-}, X_t) &\neq T_H^{\overline{\omega}}(M_{t-1}, H_{t-1}, y_t)
\end{aligned}
$$

Thus, we need solutions that depend separately on both individual and aggregate state variables. One trick to get rid of the dependence on aggregate variables is by solving the model for fixed prices $p(X_{ss})$ and $r^d(X_{ss})$. As with any linear approximation, the solution is accurate in a steady state neighborhood. This way, the decision rule of the model with fixed prices, $\widehat{g}^{\overline{\omega}}(m_{t-1}, h_{t-1})$, is such that:

$$
\begin{aligned}
\widehat{g}_m^{\overline{\omega}}(m_{t-1}, h_{t-1}) &= g_m^{\overline{\omega}}(m_{t-1}, h_{t-}, X_{ss}) \\
\widehat{g}_h^{\overline{\omega}}(m_{t-1}, h_{t-1}) &= g_h^{\overline{\omega}}(m_{t-1}, h_{t-}, X_{ss})
\end{aligned}
$$

which are the derivatives we were looking for.

8.2.3 Case $\lambda > 0$

Recall that

$$
\begin{aligned}
Q_{m,t} = {} & \frac{1}{(1+r_t^d)} E_t[\,(-\mu f(\overline{\omega}_{t+1}) - (1-\mu)\tfrac{G(\overline{\omega}_{t+1})}{\overline{\omega}_{t+1}^2}) g_m^{\overline{\omega}}(m,h,X')(1+\lambda Q_{t+1}) + \\
& [(1-\mu)\tfrac{G(\overline{\omega}_{t+1})}{\overline{\omega}_{t+1}} + (1-F(\overline{\omega}_{t+1}))]\lambda \left[Q_{m,t+1}\; g_m^m(m,h,X') + Q_{h,t+1}\; g_m^h(m,h,X')\right]\,]
\end{aligned}
$$

$$
\begin{aligned}
Q_{h,t} = {} & \frac{1}{(1+r_t^d)} E_t[\,(-\mu f(\overline{\omega}_{t+1}) - (1-\mu)\tfrac{G(\overline{\omega}_{t+1})}{\overline{\omega}_{t+1}^2}) g_h^{\overline{\omega}}(m,h,X')(1+\lambda Q_{t+1}) + \\
& [(1-\mu)\tfrac{G(\overline{\omega}_{t+1})}{\overline{\omega}_{t+1}} + (1-F(\overline{\omega}_{t+1}))]\lambda \left[Q_{m,t+1}\; g_h^m(m,h,X') + Q_{h,t+1}\; g_h^h(m,h,X')\right]\,]
\end{aligned}
$$

Notice that we can get expressions for both $Q_{m,t+1}$ and $Q_{h,t+1}$ using the FOC in $t+1$:

$$Q_{m,t+1} = \frac{\beta E_{t+t} u_{c,t+2}[1 - F(\overline{\omega}_{t+2})][1 + \lambda Q_{t+2}] - u_{c,t+1} Q_{t+1}}{u_{c,t+1}[m_{t+1} - (1 - F(\overline{\omega}_{t+1}))\lambda m_t]}$$

$$Q_{h,t+1} = \frac{u_{c,t+1} p_{t+1} - u_{h,t+1} - \beta E_{t+1} u_{c,t+2}[1 - G(\overline{\omega}_{t+2})] p_{t+2}}{u_{c,t+2}[m_{t+1} - (1 - F(\overline{\omega}_{t+2}))\lambda m_t]}$$

In this case, we not only need to know the values of $g_m^{\overline{\omega}}$ and $g_h^{\overline{\omega}}$, but also of (g_m^m, g_h^m) and (g_m^h, g_h^h). We can solve the model for given values of $g_i^{\overline{\omega}}(m, h, X')$, $g_i^m(m, h, X')$ and $g_i^h(m, h, X')$, and then update the guess—that is, recovering the derivatives by solving the model with fixed prices—until reaching convergence.

8.2.4 Improving the accuracy

We can improve the accuracy of the solution by using the equilibrium price functions and by imposing the equilibrium conditions for aggregate and individual state variables, when recovering the derivatives $g_i^{\overline{\omega}}(m, h, X')$, $g_i^m(m, h, X')$ and $g_i^h(m, h, X')$. Let $p(X_t)$ and $r^d(X_t)$ be the equilibrium price functions for a given initial guess of the derivatives, where $X_t = \{M_{t-1}, H_{t-1}, y_t\}$, and let T^X be the equilibrium transition function for the aggregate state.

Then, we can solve the following problem for Borrowers:

$$V(m_{t-1}, h_{t-1}, X_t) = \max_{c_t, m_t, h_t, \overline{\omega}_t} u(c_t, h_t) + \beta E_{y_t} V(m_t, h_t, X_{t+1})$$

$$\begin{aligned} c_t + p(X_t)h_t + (1 - F(\overline{\omega}_t))m_{t-1} \quad &= \quad y + (1 - G(\overline{\omega}_t))p(X_t)h_{t-1} + \\ &\quad\quad Q(m_t, h_t, X_t)[m_t - (1 - F(\overline{\omega}_t))\lambda m_{t-1}] \end{aligned}$$

$$m_{t-1} = M_{t-1}, h_{t-1} = H_{t-1},$$

$$X_{t+1} = T^X(X_t)$$

Notice that the equilibrium conditions $m_{t-1} = M_{t-1}$ and $h_{t-1} = H_{t-1}$ imply that we solve the model for the cases in which the initial individual state coincides with the initial aggregate state. Notice that aggregate state X_t follows the transition rule T^X, and in general, $h_t \neq H_t$ and $m_t \neq M_t$, where (h_t, m_t) are the Borrowers' optimal choices. However, when convergence is reached in the derivatives $g_i^{\overline{\omega}}(m, h, X')$, $g_i^m(m, h, X')$ and $g_i^h(m, h, X')$, the evolution of aggregate states is consistent with individual optimal decisions.

8.3 Global Solution

I use Coleman (1990)'s algorithm that operates directly on the first-order conditions. Formally, the computation of the competitive equilibrium requires solving for functions $c(X)$, $m(X), h(X), \overline{\omega}(X), \widetilde{c}(X), \widetilde{h}(X), \widetilde{d}(X)$, $p(X), r^d(X)$, $Q(M, H, X)$, where $X = \{M_{-1}, H_{-1}, y\}$ such that:

$$\frac{1}{c(X)}[Q(m(X), h(X), X) + Q_m(m(X), h(X), X)[m(X) - (1 - F(\overline{\omega}(X)))\lambda M_{-1}]]$$

$$= \beta E_y \frac{1}{c(X')}[1 - F(\overline{\omega}(X'))][1 + \lambda Q(m(X'), h(X'), X')]$$

$$\frac{1}{c(X)}[p(X) - Q_h(m(X), h(X), X)[m(X) - (1 - F(\overline{\omega}(X)))\lambda M_{-1}]]$$

$$= \frac{\eta}{h(X)} + \beta E_y \frac{1}{c(X')}[1 - G(\overline{\omega}(X'))]p(X')$$

$$X' = \{M, H, y'\} = \{m(X), h(X), y'\}$$

$$\overline{\omega}(X)p(X)H_{-1} = (1 + \lambda Q(m(X), h(X), X))M_{-1}$$

$$Q(M, H, X) = \frac{E_y\left[\Lambda(X')\,\Gamma(M, H, X')\right]}{(1 + \theta - \phi)}$$

$$\frac{1}{\widetilde{c}(X)} = \widetilde{\beta}(1 + r^d(X))E_y \frac{1}{\widetilde{c}(X)}$$

$$\frac{p(X)}{\widetilde{c}(X)} = \frac{\eta}{\widetilde{h}(X)} + \widetilde{\beta}E_y \frac{p(X')}{\widetilde{c}(X')}$$

and the budget constraint and market clearing conditions. The algorithm follows these steps:

1. Generate a discrete grid for $X = \{M_{-1}, H_{-1}, y\}$.
2. Conjecture $c_k(X), m_k(X), h_k(X), \overline{\omega}_k(X), \widetilde{c}_k(X), \widetilde{h}_k(X), \widetilde{d}_k(X)', p_k(X), r_k^d(X), Q_k(M, H, X)$ at time k for every point in the grid.
3. Solve for values $c_{k-1}(X)$, $m_{k-1}(X)$, $h_{k-1}(X)$, $\overline{\omega}_{k-1}(X)$,$\widetilde{c}_{k-1}(X)$, $\widetilde{h}_{k-1}(X)$, $\widetilde{d}_{k-1}(X)$, $p_{k-1}(X)$, $r_{k-1}^d(X)$, $Q_{k-1}(M, H, X)$, using the equations above and $c_k(X)$, $m_k(X)$, $h_k(X), \overline{\omega}_k(X), \widetilde{c}_k(X), \widetilde{h}_k(X), \widetilde{d}_k(X)', p_k(X), r_k^d(X), Q_k(M, H, X)$. For the derivatives, make:
   - $Q_m(m(X), h(X), X) = Q_{m,k}(m(X), h(X), X)$.
   - $Q_h(m(X), h(X), X) = Q_{h,k}(m(X), h(X), X)$.
4. Evaluate convergence. If $\sup_X \|z_k(X) - z_{k-1}(X)\| < \epsilon$, for $z = c, m, h, \overline{\omega}, \widetilde{c}, \widetilde{h}, \widetilde{d}, p, r^d$, and $\sup_X \|Q_k(M, H, X) - Q_{k-1}(M, H, X)\| < \epsilon$ we have found the competitive equilibrium. Otherwise, set $z_k = z_{k-1}$ and $Q_k = Q_{k-1}$ and go to step 3.