Notes to Understand Migration Policy with International Trade Theoretical Tools

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Abstract: This paper develops a standard model of international trade and makes three contributions. First, it shows that when the welfare function of the recipient country reflects the utility of natives, free-trade and free-migration generate isomorphic results, that is, they increase overall welfare but redistribute income by reducing the returns of the scarce factor. Although this result is frequently evoked in academic circles, this document shows that the equivalence holds for the most relevant measure of welfare from a political economy perspective. Second, this equivalence is extended to the public policy domain: for each level of trade restrictions mutually imposed, it is found an immigration tax that generates the same redistribution and welfare impacts. Third, in the light of these results, the model is enlarged to illustrate a channel through which political economy concerns may influence immigration policy.

Keywords: International Migration, Political Economy, International Trade

JEL Classification: F22, F13, D72, D78

Resumen: Este artículo desarrolla un modelo estándar de comercio internacional y realiza tres contribuciones. Primero, muestra que cuando la función de bienestar del país receptor refleja la utilidad de los nativos, el libre comercio y la libre migración generan resultados isomórficos, es decir, incrementan el bienestar agregado, pero redistribuyen ingreso al reducir los retornos del factor escaso. Si bien este resultado es frecuentemente evocado en círculos académicos, este documento muestra que la equivalencia se sostiene para la medida de bienestar más relevante desde la perspectiva de la economía política. Segundo, la equivalencia es extendida al dominio de la política pública: para cada nivel de restricciones comerciales impuestas mutuamente, se encuentra un impuesto a la inmigración que genera los mismos impactos redistributivos y de bienestar. Tercero, a la luz de estos resultados, el modelo es ampliado para ilustrar un canal a través del cual las consideraciones de economía política podrían influir en la política migratoria.

Palabras Clave: Migración Internacional, Economía Política, Comercio Internacional

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1. Introduction

Immigration policies have always been especially heated, given that they create trade-offs for host economies. Immigration triggers opposing effects on different groups within a country and this opens the door for conflicting debates. Indeed, one could develop arguments both in favor and against immigration. For instance, it could be argued that immigrants stimulate firm creation, boost technological progress and complement natives in the productive process in host economies. In contrast, it could also be argued that immigrants compete with natives, reduce their wages, and impose significant fiscal costs.

These multiple and conflicting arguments emphasize the need for investigating the nature and magnitude of the impacts that could arise from immigration. Hanson (2009) provides an important overview of the state of the literature regarding the measurement of the economic consequences of migration, primarily for the host country which has been the subject of most studies. Special attention is devoted to discussing the impact of migration on the wages of low and high skilled workers, and on the substitutability between native and foreign workers, as the labor market consequences of migration are a key determinant behind the political economy of migration policy. However, much still remains to be done and there still remains a need for accomplishing theoretical and empirical work that can better inform the policy debate. The present note takes a step in this direction by showing how instruments traditionally included in the economists’ toolkit can shed light on relevant but largely controversial issues. In particular, we use a standard model of international trade to show that immigrants with complementary skills may increase scarce factors productivity and, through this channel, enhance overall welfare in host nations. However, some groups within the host country may be affected in their income levels and, thus, may oppose migration.

There are several arguments that either have been or could in principle be used to support immigration. One of these arguments claims that immigrants play an active and indispensable role in promoting dynamism, innovation and scientific progress in host economies. This argument is consistent, for instance, with the finding that 45% of high-tech firms from the Fortune 500 had either a first or a second generation immigrant among its founders.
Moreover, the premise that immigrants largely contribute to scientific progress is consistent with the evidence on Nobel prizes and registered patents provided by Figures 1, 2 and 3. Figure 1 shows that the amount of Nobel prizes obtained by a country is positively associated with the ratio of immigrants-to-natives that have won the prize. At the same time, Figures 2 and 3 show that this ratio is positively correlated with the number of registered patents. The former figure indicates that the correlation holds even after controlling for GDP per capita. At the same time, by controlling for the ratio of R&D expenditure over GDP, Figure 3 suggests that the correlation does not result exclusively from differences in this ratio across countries. Furthermore, beyond the benefits that may arise from firm creation and innovation, Peri (2016) argues that immigrants may enhance the amenity value of the locations to which they migrate by providing greater variety to local services and entertainment.

An additional argument in favor of migration could be that immigrants complement native workers in host labor markets. According to this argument, immigrants’ skills are generally complementary to those possessed by native workers and, therefore, immigration reduces labor shortages in both low and high-skilled occupations. By complementing native workers in production, immigration would create job opportunities and increase their wages. Consistent with this idea, Figure 4 shows that immigrants in the U.S. are relatively concentrated at the top and at the bottom of the skill distribution, while natives are relatively more concentrated in the middle of this distribution. Dustman et al. (2016) provide an overview of these arguments and a discussion as to why empirical studies have had difficulties reaching a consensus regarding the measurement of these effects.

On the opposite side of the debate, several arguments have been or could be used to oppose immigration and favor restrictions to international labor mobility. One of these arguments emphasizes that, in those markets in which natives compete with foreigners, immigration

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1 The report by the Partnership for a New American Economy also finds that by 2010 more than 40% of the Fortune 500 companies were founded by immigrants or by their children, that seven of the ten most valuable brands in the world come from companies founded by first or second generations of immigrants and that the revenue generated by Fortune 500 companies founded by immigrants or children of immigrants is greater than the GDP of every country in the world outside the U.S., except China and Japan.

2 Although the figures provide information only on the correlation between the variables illustrated, they serve to suggest that high-skilled migration entails net benefits for host countries.
triggers competition for the same types of jobs, depressing wages and raising unemployment. Along these lines, De New and Zimmermann (1994) show that immigration increased earnings among German white-collar workers but depressed wages among blue-collar workers in the 1980s.

Another argument against immigration relies on the fiscal costs that foreigners could impose on host countries. It has been argued that, due to their age, skill, fertility and language characteristics, immigrants may consume large amounts of government-funded goods but, on the other hand, increase fiscal revenues only to a small amount (Nowrasteh, 2014).

The fact that immigration can in principle generate several and conflicting effects on welfare, as well as the existence of inconclusive answers to relevant questions, highlights the need for economists to take part of the policy debate that surrounds immigration. Indeed, economists need to steer this debate towards policy prescriptions and cost-benefit analysis that is better informed by economic theory and empirical work. A proper cost-benefit framework and, more generally, a more proactive role by economists, could improve our understanding of relevant trade-offs and increase the reliability of estimates regarding the impact of immigration.

Figure 1. Nobel Prize, 1901-2014 (Top 5 countries)

Source: Author’s own calculations based on data from Nobelprioe.org and the World Atlas.
Figure 2. High-Skill Migration and Patents Registered (As a Percentage of GDP per-capita)

Note: Each observation in the figure is represented as proportion of real GDP per capita.

Figure 3. High-Skill Migration and Patents (As a Percentage of R&D expenditure)

Note: Each observation in the figure is represented as proportion of average R&D expenditures as a percentage of GDP.
In this context, the present note proposes a simple framework to formalize some of the trade-offs that have been associated with immigration. The intuition that the analysis here is based on was first expounded by Mundell (1957), who argued that commodity movements are at least a partial substitute for factor movements in the sense that in a model in which relative factor endowment differences are the motive for trade, trade and migration are substitutes in that the opening of either trade in goods or trade in factors will result in factor price equalization. The result that goods trade and factor trade are substitutes is a special characteristic of models in which the motives for trade are relative factor endowment differences across countries. Markusen and Svensson (1985) consider a model in which the motives for trade are international differences in production technologies. They show that when technology differences are in terms of product-augmenting technology differences, countries will, on average, be net exporters of goods for which they possess a superior

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3 This is true if the initial factor endowments support an equilibrium with factor price equalization. A necessary condition for this is that initial factor endowments are located within the cone of diversification, such that countries do not become completely specialized in the trade equilibrium.
technology. In this case, they show that this correlation implies that when factor movements across countries are allowed, commodity trade and factor trade are complements rather than substitutes in the sense that allowing for factor mobility increases the volume of trade. This result is echoed by the results in Markusen (1986), who shows that in a model where trade is driven by returns to scale and imperfect competition, trade in goods and trade in factors are complements as allowing factor movements increases the volume of trade. However, the standard factor proportions model and its implications for the income distribution across factors of production (i.e. skilled and unskilled workers) is still an important benchmark model that has to a large extent shaped the discussion behind the economic consequences of migration and trade and this is why we focus our attention on this model here.

In this note we study a standard two-good, two-factor, two-country model of trade and explore a topic that has not received sufficient attention in the literature to date. The model explores the importance of assessing the skill composition of migration when performing welfare analysis and investigates how public policies may affect this composition. The analysis is carried out by studying the impacts of international trade and migration under five different scenarios: (i) Autarky Economy with Neither International Trade nor International Labor Mobility; (ii) Free International Trade and no International Labor Mobility; (iii) Mutual Trade Restrictions and no International Labor Mobility; (iv) No International Trade and Free International Labor Mobility and (v) No International Trade and Tax on International Labor Mobility. This analysis generates several interesting and insightful conclusions enabling us to make three contributions:

First, we show that when the welfare function of the recipient country is assumed to reflect the well-being of local inhabitants, either because immigration policy decisions are taken ex-ante or because, just as in any theoretical model of political economy, decision-makers care about voters, the free-trade and free-migration scenarios generate isomorphic results in terms of welfare and redistribution. That is, by complementing the abundant factor, immigrants increase overall welfare, but reduces the scarce factor’s returns in the recipient country, i.e., to construct our measure of overall welfare, we start by assuming the existence of a

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4 These same authors show that for the case of factor-augmenting technology differences across countries, when factor trade is allowed it can either be a complement of or a substitute of goods trade.
representative consumer rather than by aggregating welfare levels corresponding to different
groups, just as in most two-by-two-by-two standard models of international trade.

Although the afore-mentioned equivalence between trade and immigration is frequently
evoked in academic circles, this note contributes by showing that the result holds precisely
for the most relevant measure of welfare from a political point of view (migrants do not vote
and, thus, electoral speeches frequently focus only on natives). Besides, the use of this
measure represents an innovation relative to existing theories that investigate migration
effects in factor proportion models by using different metrics (Dixit and Norman, 1980).

Second, we show that the equivalence between trade and immigration can be extended to
the public policy dimension. Specifically, we compare the outcomes of the mutual trade
restrictions environment analyzed in scenario (iii) with those that arise from the immigration
policy case analyzed in scenario (v). This comparison enables us to show that, for each level
of mutual trade restrictions, there is a migration tax that replicates exactly the same results in
terms of welfare and redistribution from the perspective of the host country.

Third, given the two results we have mentioned above, we propose an extension of the
basic setup. In particular, given that migration enhances overall welfare but generates income
redistribution, just as an imports tariff, it may be the case that migration policies are strongly
influenced by political economy concerns and not necessarily by efficiency considerations,
i.e., protectionist policies on trade and migration have the same redistribution effects and,
thus, may be based on the same kind of political economy considerations. Thus, we propose
an extension of the factor proportion model to illustrate one of the channels through which
this influence may take place.

For this purpose, we combine a standard relative endowment setup with a political
economy model for the determination of immigration policies. In this framework, we show
that, even when a government is benevolent and forward looking, it will propose an
immigration reform that involves a positive tax on immigration. By doing so, the government
will reduce the probability of a strike that could lead to a rejection of its proposal by the
congress and, therefore, an autarky situation. In this sense, our results are consistent with
Galiani and Torrens (2015). In contrast with our setup, their model is able to jointly study the
effects of immigration and international trade because it features Ricardian differences in
productivity but, on the other hand, concentrates on political economy aspects at the international level, i.e., we cannot have trade and migration at the same time since, in our model, the two phenomena result from differences in relative wages. Yet, in the same manner as we do, they conclude that restrictions to international labor mobility may be the result of political economy motivations.

The remainder of the note is structured as follows. Section 2 presents a short literature review. Section 3 develops the model setup and Section 4 points out preliminary considerations that generalize the solution method of the model. Sections 5-9 present and investigate scenarios (i)-(iv). Finally, Section 10 shows the tax equivalence, Section 11 sets the political economy debate and Section 12 concludes.

2. Literature Review

Economists have been long interested in understanding the effects and determinants of immigration, as well as the impacts of different immigration policy measures. This interest has given rise to both theoretical and empirical works, several of which keep a close relationship with the present note. This section provides a brief review of these empirical and theoretical studies.

On the empirical front, economists’ interest has produced a large body of literature performing evaluations in three domains: (i) the determinants of migration decisions; (ii) the impacts of migration on host labor markets and (iii) the effects of migration policies.

As for the factors determining immigration decisions, Borjas (1991) and Chiquiar and Hanson (2005) are two relevant studies. Both studies coincide that educational attainment is a critical determinant of immigration, meaning that differences in the skill returns between source and host countries provide distinct incentives to migrate to workers located in different segments of the skill distribution. Along these lines, the latter of these studies shows that Mexican immigrants to the U.S. are on average more educated than Mexican residents, but less educated than U.S. natives. Continuing with this line of research, Beine et al. (2010), Mayda (2010), and Ortega and Peri (2013) find that immigration decisions are also influenced by three additional determinants: income per capita and unemployment in the
source and destination countries; the stock of people from the source nation residing in the destination country and the restrictiveness of immigration policies.

Regarding the second strand of empirical literature, dealing with the impacts of migration on host labor markets, several of the arguments were proposed by Card and Borjas in the context of the “Mariel boatlift” episode, i.e., 45,000 Cubans arrived to Miami increasing its labor supply by 7% (mostly low-skill labor). On one hand, Card (1990) compared: (a) the labor market outcomes of workers from different ethnicities and workers at different segments of the wage distribution within Miami across time periods; and (b) the outcomes for different workers in Miami with outcomes of similar workers in other American cities. After carefully performing this comparison, he concluded that the surge of labor supply in 1980 had no discernible impact on the labor market outcomes in Miami (i.e., the changes in employment and wages in Miami were comparable to those observed in other American cities over the same time period). On the other hand, Borjas (2016) later revisited the “Mariel boatlift” episode and argued that 60% of the influx of Cuban workers were high-school drop outs and that, as one focused on this specific segment of the labor market, wages in Miami decreased between 10% and 30%.

The third strand of empirical literature explores the effectiveness of immigration policies mainly on two outcome variables, the size and the composition of migration flows. Regarding the former variable, it has been shown that tighter immigration restrictions significantly reduce the size of migration flows, except in the case of asylum migration, which responds more to historical factors than to policy restrictiveness (Thielemann, 2004 and Czaika and de Hass, 2014). In contrast, the literature has found it much harder to assess policy impacts on the composition of migration flows. This is mostly due to difficulties in constructing indexes that can capture the restrictiveness of policies on specific groups of immigrants. Along these lines, Thielemann (2004) and de Haas, Natter and Vezzoli (2014) propose different indexes and show that, using these indexes, immigration policies have affected the size of migration flows but not necessarily their composition. This could be interpreted as evidence that deeper research is required to develop appropriate restrictiveness measures when studying the

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5 Even though both studies measure policy effectiveness in terms of volume, Thielemann (2004) specifically assesses the impact on non-economic migrants. That is, those immigrants whose decisions are driven by political motivations.
composition of migration flows. An additional challenge in this literature is given by the lack of studies evaluating the long-term impact of immigration policies, i.e., most existing studies focus on immediate impacts (Czaika and de Hass, 2013 and 2015).

As a final remark on the empirical literature it should be noted that, while much has been done in terms of assessing relevant impacts, there is further need for empirical work regarding the relationship between the skill profile of migrants and the skill profile of the host country’s labor force. At the same time, it would be important to investigate whether the skill supply of migrants is complementary to or substitute of the skill supply of the native labor force.

On the theoretical front, there are two papers that keep an intimately close relationship with the present note: the studies by Dixit and Norman (1980) and by Galiani and Torrens (2015). Just as we do in this note, Dixit and Norman (1980) set a standard model of international trade and investigate the effects of immigration in this model. In particular, they feature a neoclassical factor proportion framework with two goods, two countries and two goods and further complicate this model later in some of their extensions.

An interesting point of comparison between Dixit and Norman’s (1980) and our work refers to the measure of welfare used in each case. Unlike us, they take as a welfare measure the well-being of both native and migrants workers and, in this context, show that immigration entails two types of effects. First, there is a direct effect that results from changing the size of the host country’s population and holding prices constant, i.e., given our interest in using a more politically relevant measure, this is precisely the effect that our model does not take into account. Besides, there is an indirect effect of migration that results from changes in the terms of trade. In this framework, Dixit and Norman (1980) conclude that there are no clear cut answers for the net effects of migration.

The second theoretical paper that is closely related to ours is Galiani and Torrens (2015). In contrast with our factor proportion approach to international trade, they opt for a Ricardian framework that features differences in technology across sectors and countries. However, just as the present note, their work extends a standard model of international trade to account for political economy motivations. In particular, they combine their Ricardian economy with a simple international political economy model in which governments jointly decide on trade and immigration policies. In their framework, countries specialize in different goods and thus
use different types of technologies. This, in turn, implies that international trade does not reduce real wages in any of the countries. On the other hand, immigration does diminish wages in the technologically advanced-rich nation. In particular, the real wage differences induce workers to migrate there, thereby increasing the labor supply and depressing real labor returns. Hence, while international trade can be supported as a Nash equilibrium of the international political economy game, free international labor mobility cannot. In the same spirit of the present note, they then conclude that restrictions to migration are the result of political economy motivations.

3. Model Setup

Consider a world with two countries, two factors and two goods. The two countries, North and South, are indexed by \( N \) and \( S \); the two factors, skilled and unskilled labor, are denoted by \( H \) and \( L \); and the two goods are a skilled-intensive good and an unskilled-intensive good. The price of the former good is referred to as \( P \), while the price of the latter good is chosen as the numeraire and will, thus, take the value of 1. North is assumed to be the skilled-labor abundant country; hence, \( L_N/H_N < L_S/H_S \).

Technologies are identical across nations. In both countries, production is given by the following Cobb-Douglas constant-returns-to-scale functions:

\[
Y_{js} = \varepsilon_s(H_{js}^\beta L_{js}^{1-\beta}),
\]

\[
Y_{ju} = \varepsilon_u(H_{ju}^\alpha L_{ju}^{1-\alpha}),
\]

where \( Y_{js} \) and \( Y_{ju} \) refer to the production of the skilled- and unskilled-intensive goods in country \( j \) and our skill-intensity assumption implies \( \beta > \alpha \); where \( \beta < 1 \) and \( \alpha < 1 \); \( H_{js} \) and \( L_{js} \) denote the amounts of skilled and unskilled workers used in the production of \( Y_{js} \) and \( \varepsilon_s = \beta^{-\beta} (1 - \beta)^{-(1-\beta)} \) and \( \varepsilon_u = \alpha^{-\alpha} (1 - \alpha)^{-(1-\alpha)} \) are normalizations of the production functions.

Preferences, also identical across counties, are given by the following utility function:

\[
U_j = c_{js}^\gamma c_{ju}^{1-\gamma},
\]
where \( \gamma < 1 \) represents the relative preference for the skilled-intensive good and \( c_{js} \) denotes country \( j \)'s consumption of this product. The indirect utility function associated with Equation (3) can be written as follows (see Appendix 1 for a full derivation):

\[
V_j = \gamma^\gamma (1 - \gamma)^{1-\gamma} I_j P_j^{-\gamma},
\]

(4)

where \( I_j \) and \( P_j \) are the nominal income level and the price of the skilled-intensive good in country \( j \). Equation (4) states a standard microeconomics result: utility is increasing in the real income of country \( j \), i.e., given that the price of \( Y_{ju} \) has been chosen as the numeraire, the price index faced by consumers in this country equals \( (1)^{1-\gamma} P_j^\gamma = P_j^\gamma \).

Labor and product markets are perfectly competitive and, thus, profits must equal to zero in equilibrium. Finally, we assume that relative labor supplies are sufficiently similar across countries that both nations lie within the parallelogram of incomplete specialization in equilibrium.

4. Preliminary Considerations

There are some similarities among the five scenarios that we will take in consideration. In all of these cases, the zero-profit conditions will determine the wages of skilled and unskilled labor as a function of the price for the skilled-intensive good (\( P \)). Moreover, using these wages, the labor market clearing conditions will pin down in all of these cases the supply of goods as a function of \( P \). Finally, depending on the scenario that is being considered, this price will be determined by clearing in the market of the skilled-intensive good and/or by the relevant migration flows.

Indeed, according to the manner in which \( P \) is determined, the five scenarios can be classified in three different groups: (a) in autarky, there is neither migration nor trade and, thus, \( P \) will be determined only by market-clearing conditions and these conditions will be defined at the local level; (b) in scenarios (ii) and (iii) there is international trade and, therefore, \( P \) will be determined only by market-clearing conditions defined at the global level; finally, scenarios (iv) and (v) allows for immigration so that \( P \) will be determined by both market-clearing conditions defined at the country level and by migration flows.
Furthermore, it is possible to illustrate common patterns among the five scenarios in terms of welfare, for which it is necessary to define the measure of well-being that we will use in each case. For scenarios (i)-(iii), we will follow the literature and use the indirect utility function shown in Equation (4) which, as noted above, is increasing in the real income of country \( j \). In cases (iv) and (v), the argument is subtler given that these scenarios consider migration and this, in turn, may threaten the validity of (4) as an appropriate welfare measure. To see this point, note that migration flows increase the population in a given country, generating almost mechanically an increasing effect on utility, as measured by (4), since total income increases with the factor endowment of the country. Nonetheless, by increasing the population, migration flows could also diminish welfare in per capita terms.

To prevent that our measure of welfare from increasing mechanically in response to migration flows, but also to concentrate on political economy aspects of migration policies, we will focus exclusively on the welfare of native inhabitants. That is, in scenarios (iv) and (v), in which immigration will take place, we will use the following measure:

\[
V_j^{(iv)-(v)} = \frac{(1-\gamma)^{1-\gamma}P_j^{(iv)-(v)}}{L_j + H_j},
\]

(5)

where the numerator is given by the indirect utility function shown in Equation (4), and the hats over \( L_j \) and \( H_j \) emphasize that these variables remain fixed for the purpose of welfare calculations. That is, the fact that the population is being held constant at its original size in calculating \( V_j^{(iv)-(v)} \) implies that we can focus only on the numerator of (5). Hence, to evaluate welfare effects in scenarios (iv) and (v), we will use the same expression as in cases (i)-(iii). In this regard, it is important to note that there are common patterns in terms of welfare among the five scenarios only because we have chosen the relevant measure of welfare from a political economy point of view.

Furthermore, note in Equations (4) and (5) that this measure is fully determined by real income \( (L_j P_j^{(iv)-(v)}) \) which is, in turn, equal to the sum of real returns to skilled and unskilled labor. That is, the expression for real income can be written as follows:

\[
I_j P_j^{(iv)-(v)} = L_j w_j P_j^{(iv)-(v)} + H_j q_j P_j^{(iv)-(v)},
\]

(6)
where $w_jP_j^{-\gamma}$ and $q_jP_j^{-\gamma}$ denote the real unskilled and skilled wages, respectively. Given these results, in the remainder of this note we will use real income as our measure of welfare.

Moreover, the fact that in the five scenarios the outcomes derived from the zero-profit conditions can be written as a function of $P_j$ implies that $w_j$ and $q_j$ can also be fully written in terms of this price. In particular, using the zero-profit conditions, it is possible to write the real unskilled and skilled wages in all scenarios as follows (see Appendix 2 for a full derivation): \(^6\)

\[
q_j = P_j^{1-a} \beta^{-a}, \quad (7)
\]

\[
w_j = P_j^{-a}. \quad (8)
\]

Intuitively, simple algebra on Equations (7) and (8) shows that an increase in the price of the skilled-intensive good ($P_j$) leads to a rise in the skilled wage ($q_j$) and a reduction in the unskilled wage ($w_j$). This simply reflects the Stolper Samuelson Theorem. Moreover, note that the skilled wage increases more than proportionally with $P_j$; this is the so-called Jones Magnification Effect (Jones, 1965).

Substituting the definitions of $q_j$ and $w_j$ in Equation (6), we can write real income as follows:

\[
I_jP_j^{-\gamma} = \bar{L}_jP_j^{-a} + \bar{H}_jP_j^{-a} = P_j^{-a} \left( \bar{L}_j + \bar{H}_jP_j^{-a} \right). \quad (9)
\]

Equation (9) states that, as the “politically relevant” measure of welfare is taken into account, it is possible to assess the impact of immigration flows on welfare just by knowing how they affect final goods prices, i.e., it can be argued that in the present setup $P_j$ is a sufficient statistic for welfare. In addition, this equation states that an increase in $P_j$ has two opposing effects on real income: on the one hand, the increase reduces the real returns to unskilled workers but, on the other hand, it raises the real returns to skilled employees. Note

\(^6\) As understood from the equations shown in upcoming sections, for the case of mutual trade restriction, the price of $P_j$ in (7) and (8) must be interpreted as the effective relative price of the skilled-intensive good.
that the strength of the former negative effect is increasing in \( \hat{L}_j \) and \( \alpha \), while the strength of the latter positive negative effect is increasing in \( \hat{H}_j \) and \( \beta \).

Using these results, Appendix 2 shows that the expression in Equation (9) has a single critical point, i.e., a minimum, within the interval \( P_j \in (0, \infty) \). Thus, under the proper assumptions on the concavity on \( I_j P_j^{-\gamma} \), Figure 5 plots real income as a function of \( P_j \). In this figure, \( W^M \) indicates that real income is our measure of welfare and the minimum of the functions is reached at \( P_j = P_j^* = \left( \frac{\hat{L}_j}{\hat{H}_j} \right)^{\beta - \alpha} \left( \frac{\alpha(1-\gamma) + \beta \gamma}{1 - \alpha(1-\gamma) - \beta \gamma} \right)^{\beta - \alpha} \). Intuitively, the larger \( \hat{L}_j \), the higher the value of \( P_j^* \) is: a great value of \( \hat{L}_j \) raises the negative impact of increases in \( P_j \) on real income, so that this negative impact is only offset at a higher value of \( P_j \). The same result holds true for \( \beta \).

![Figure 5. Real Income (Our Measure of Welfare) as a Function of Pj](image)

Even though Figure 5 would seem to suggest that the welfare function attains a minimum at \( P_j^* \) for given endowment levels, it is not the case that we are minimizing welfare through the allocation of resources. What the figure illustrates is that the most efficient attainable equilibrium is reached under autarky for a constant supply of production factors, i.e., Section 5 shows that \( P_j^* \) is in fact the equilibrium price in autarky. Any other point on the curve
depicted in Figure 5 which yields a higher level of welfare requires relaxing some of the constraints defining this function. Indeed, as we will see below, reaching a higher welfare level will only be possible by separating consumption and production decisions through the implementation of prices that result from international trade or by enhancing the labor supply of the economy through immigration.

5. Analysis of the First Scenario: Autarky Economy with Neither International Trade nor International Labor Mobility

This section studies the equilibrium properties and welfare characteristics of two autarkic economies, North and South. The absence of international trade or migration implies that the two economies will have different prices of the skilled-intensive good and that real skilled and unskilled wages will not equalize across countries.

Indeed, skilled and unskilled wages are determined by each country’s zero-profit conditions. In a perfectly competitive environment, these conditions are fulfilled when the effective price of each goods equals its unitary cost. When technology is given by the constant-returns-to-scale production functions shown in Equations (1) and (2), unitary costs equal marginal costs and, therefore, the zero-profit conditions in North can be written as follows (see Appendix 3 for a general derivation of marginal costs and zero-profit conditions):

\[
q_{N}^{\text{aut}} \beta w_{N}^{\text{aut}} (1-\beta) = p_{N}^{\text{aut}}, \tag{10}
\]

\[
q_{N}^{\text{aut}} \alpha w_{N}^{\text{aut}} (1-\alpha) = 1, \tag{11}
\]

where \(q_{N}\) and \(w_{N}\) refer to the skilled and unskilled wages in North, \(p_{N}^{\text{aut}}\) is the price of the skilled-intensive good under the autarkic regime in this country and the price of the unskilled-intensive good has again been chosen as the numeraire. Combining Equations (10) and (11) one can write \(q_{N}\) and \(w_{N}\) in terms of \(p_{N}^{\text{aut}}\) as follows:

\[
q_{N}^{\text{aut}} = p_{N}^{\text{aut}} \frac{1-\alpha}{\beta-\alpha}, \tag{12}
\]

\[
w_{N}^{\text{aut}} = p_{N}^{\text{aut}} \frac{-\alpha}{\beta-\alpha}. \tag{13}
\]
By dividing (12) and (13) by the price index, i.e., which equals $P_{N\text{aut}}^\gamma$ because the price of the unskilled-intensive is equal to 1, one can obtain the expressions for real skilled and unskilled wages. In turn, these wages can be used to derive the expression for real income, i.e., our measure of welfare. Hence, using Equations (12) and (13), real wages and income can be written as follows:

\[ \frac{q_{N\text{aut}}^\gamma}{P_{N\text{aut}}^\gamma} = P_{N\text{aut}}^\gamma \frac{1-a(1-\gamma) - \beta \gamma}{\beta - \alpha}, \]  

(14)

\[ \frac{w_{N\text{aut}}^\gamma}{P_{N\text{aut}}^\gamma} = P_{N\text{aut}}^\gamma \frac{-a(1-\gamma) - \beta \gamma}{\beta - \alpha}, \]  

(15)

\[ I_{N\text{aut}}^\gamma P_{N\text{aut}}^{-\gamma} = (H_N^\gamma q_{N\text{aut}}^\gamma + L_N^\gamma w_{N\text{aut}}^\gamma) = P_{N\text{aut}}^\gamma \frac{-a(1-\gamma) - \beta \gamma}{\beta - \alpha} (L_N^\gamma + H_N^\gamma P_{N\text{aut}}^\gamma). \]  

(16)

Note that Equation (16) shows exactly the same definition of real income that appears in Equation (9), with the exception of the superscripts indicating that we are dealing with the autarky case. This fact illustrates that Equation (9) provides a general definition that can be applied to the any of five scenarios under consideration in the present note.

Furthermore, using Equations (12) and (13), one can write the skill-premium in North as $P_{N\text{aut}}^\gamma \frac{1}{\beta - \alpha}$. In turn, this premium determines the total demands for skilled and unskilled labor as a function of $P_{N\text{aut}}^\gamma$. Equating these demands to $H_N$ and $L_N$, the Northern supplies of skilled and unskilled labor, the supply of each good can also be written as a function of $P_{N\text{aut}}^\gamma$ (see Appendix 4 for a general derivation of labor market-clearing conditions). The following equations summarize the results:

\[ \gamma_{Ns}^\text{aut} = P_{N\text{aut}}^\gamma \frac{-\beta}{\beta - \alpha} (H_N P_{N\text{aut}}^\gamma)^\frac{1}{\beta - \alpha} (1 - \alpha) - L_N^\gamma \alpha) / (\beta - \alpha), \]  

(17)

\[ \gamma_{Nu}^\text{aut} = P_{N\text{aut}}^\gamma \frac{-\alpha}{\beta - \alpha} (L_N^\gamma \beta - H_N P_{N\text{aut}}^\gamma)^\frac{1}{\beta - \alpha} (1 - \beta) / (\beta - \alpha). \]  

(18)

Equations (17) and (18) have left the most relevant endogenous variables as a function of $P_{N\text{aut}}^\gamma$. To find the equilibrium value of this price, one needs to equate the supply shown in Equation (17) to the demand for the skilled-intensive good. Hence, this equilibrium price can be written as follows (see Appendix 5 for a full derivation of this price under general product market-clearing conditions):
\[ p_{N}^{\text{aut}} = \left( \frac{L_N}{H_N} \right)^{\beta - \alpha} \left( \frac{a(1-\gamma) + \beta \gamma}{1 - a(1-\gamma) - \beta \gamma} \right)^{\beta - \alpha}. \] \hfill (19)

Intuitively, Equation (19) states that the autarky price of the skilled-intensive good in equilibrium is increasing in the unskilled-to-skilled labor endowment ratio, i.e., the skilled intensive good in autarky will be relatively more expensive when the relative supply of the factor used intensively to produce it is scarcer.

Using the same logic we have followed above, one can obtain real wages and the autarky price in South:

\[ \frac{q_S^{\text{aut}}}{p_S^{\text{aut}}} = p_S^{\text{aut}} \frac{1 - a(1-\gamma) - \beta \gamma}{\beta - \alpha}, \] \hfill (20)

\[ \frac{w_S^{\text{aut}}}{p_S^{\text{aut}}} = p_S^{\text{aut}} \frac{-a(1-\gamma) - \beta \gamma}{\beta - \alpha}, \] \hfill (21)

\[ p_S^{\text{aut}} = \left( \frac{L_S}{H_S} \right)^{\beta - \alpha} \left( \frac{a(1-\gamma) + \beta \gamma}{1 - a(1-\gamma) - \beta \gamma} \right)^{\beta - \alpha}. \] \hfill (22)

A comparison between (19) and (22), along with the fact that \( L_N/H_N < L_S/H_S \), reveals that \( p_N^{\text{aut}} < p_S^{\text{aut}} \): the autarky price of the skilled-intensive good is smaller in the skilled-abundant country (North). This is a standard result in the literature of factor-proportion models.

Furthermore, close inspection of Equations (19) and (22) shows that the autarky prices are precisely the values of \( P_j \) at which real income reaches its minimum in each country, i.e., using notation from Figure 5, we have that \( p_N^{\text{aut}} = P_N^* \) and that \( p_S^{\text{aut}} = P_S^* \). The outcome is depicted in Figure 6. In this regard, note that this does not imply that the autarky equilibrium is inefficient. Indeed, it associates with the most efficient situation among all the choices for the amount of resources held by the economy. Nonetheless, Figure 6 is not constrained by feasibility or resource availability, it is simply a description of real income behavior as a function of \( P_j \), i.e., as it will be shown below, international trade leads economies to different points of the curves by allowing them to consume a different bundle from the one they produce, just as if these economies actually had more resources.
Figure 6. Real Income as a Function of $P_j$ in Autarky

Figure 7. Equilibrium in Autarky

The allocation properties of the equilibrium are depicted in Figure 7 for a given level of initial resources i.e., skilled and unskilled labor, in North and South. This figure, as well as all subsequent figures dealing with the allocation characteristics of the equilibrium are not based on the explicit functional forms used in the model. Instead, the goal of these figures is to provide intuition on the results by making conceptual points. Having said that, note that in
Figure 7 differences in the supplies of relative endowments across countries are represented by distinct degrees of skewness in each of the panels. Note also that the production and consumption bundles coincide in each country and are given by the point at which the utility curve is tangent to the budget constraint. Differences in the slope of the tangency line reflect differences in the equilibrium relative price across nations.

6. Analysis of the Second Scenario: Free International Trade and no International Labor Mobility

This section considers a scenario in which there is an international exchange of goods but there is no migration flows. While international trade equalizes the price of the skilled-intensive good across countries, the absence of migration implies that real wages will be determined only by this price, and, in particular, will not be determined by migration flows.

Just as in the previous section, we begin with the zero-profit conditions. Marginal costs are given exclusively by the technologies used in production and are, therefore, the same that appear in the left-hand sides in Equations (10) and (11). For the purpose of solving the model, the only difference between the relevant conditions in this section and Equations (10) and (11) is that, in this case, the price of the skilled-intensive good is the same across countries and should not carry, as a result, a $j$ sub-index, i.e., thus, the derivation of the zero-profit conditions are, in this case, also consistent with the general derivation shown in Appendix 3. Hence, it follows that the skilled and unskilled wages are also the same across countries and that their expressions can be found by using Equations (10) and (11). In particular, these wages can be written as follows:

$$q^{FT} = p^{FT} \frac{1-a}{\bar{p} - a},$$

(23)

$$w^{FT} = p^{FT} \frac{-a}{\bar{p} - a}.$$  

(24)

As for real wages, they can also be obtained in this case by dividing nominal wages through the price index, i.e., which is equal to $p^{FT} \gamma$ given that the price of the unskilled-intensive good is now also equal to 1. Thus, real wages can be written as follows:
\[
\begin{align*}
\frac{q^{FT}}{p^{FT}} &= P^{FT} \frac{1-a(1-\gamma)-\beta\gamma}{\beta-a} \\
\frac{w^{FT}}{p^{FT}} &= P^{FT} \frac{-a(1-\gamma)-\beta\gamma}{\beta-a}
\end{align*}
\]

The difference with the autarky case in terms of the solution method lies in the products market equilibrium, which is in this section defined at the global level. The supply of goods differs across countries due to differences in factor proportions. Using the same logic as in the previous section, one can obtain labor demands in each country by using the skill-premium implied by Equations (23) and (24). The difference is that now the sum of demands must be equated to the sum of supplies, leading to the following equilibrium values:

\[
\begin{align*}
Y_{Ns}^{FT} &= P^{FT} \frac{1}{\beta-a} (H_N P^{FT} \frac{1}{\beta-a}(1 - \alpha) - L_N \alpha) / (\beta - \alpha) \\
Y_{Nu}^{FT} &= P^{FT} \frac{1}{\beta-a} (L_N \beta - H_N P^{FT} \frac{1}{\beta-a}(1 - \beta)) / (\beta - \alpha) \\
Y_{Ss}^{FT} &= P^{FT} \frac{1}{\beta-a} (H_S P^{FT} \frac{1}{\beta-a}(1 - \alpha) - L_S \alpha) / (\beta - \alpha) \\
Y_{Su}^{FT} &= P^{FT} \frac{1}{\beta-a} (L_S \beta - H_S P^{FT} \frac{1}{\beta-a}(1 - \beta)) / (\beta - \alpha)
\end{align*}
\]

Regarding the equilibrium value of \( P^{FT} \), it is possible to find it by solving the product market-clearing conditions. Nonetheless, unlike in the autarky case, these conditions are now defined at the global level. In particular, \( P_N^{FT} \) is the price that equates the global demand for the skilled-intensive good and the sum of supplies shown in Equations (27) and (29). By equating global supply and demand, we obtain the following equilibrium price (see Appendix 5 for a proof):

\[
p^{FT} = \left( \frac{L_W}{H_W} \right) \beta - \alpha \left( \frac{\alpha(1-\gamma)+\beta\gamma}{1-\alpha(1-\gamma)-\beta\gamma} \right) \beta - \alpha
\]

where \( H_W = H_N + H_S \) and \( L_W = L_N + L_S \) refer to the world’s supplies of skilled and unskilled labor. Note in Equation (31) that the expression for \( P^{FT} \) is similar to the expression defining the equilibrium price in the autarky case; the difference is that, while in the autarky regime there is a relevant unskilled-to-skilled labor ratio for each country, in the free trade equilibrium the single relevant ratio refers to the entire world. Indeed, the latter result implies that, as well-known, the free-trade
equilibrium replicates the allocations of the integrated economy, i.e., the one we would observe in a world with no boundaries.

Equation (31) can also be used to evaluate changes in \( P_j \) that result from trade liberalization, i.e., the transition from the autarky case to the free trade equilibrium. Recall that \( P_j \) provides enough information to judge changes in welfare and redistribution. Note in this regard that, given that \( L_N/H_N < L_S/H_S \), we know that \( P_N^{aut} < P_{FT} < P_S^{aut} \): the equilibrium price under free trade lies within the range of prices determined by the autarky prices and, therefore, this price increases in North but diminishes in South, relative to autarky. This result also takes part of the standard set of outcomes in the literature of factor-proportion models of international trade.

As for the welfare implications of the transition from trade liberalization, we can explore them by using the results obtained in Section 4. In particular, using Figure 5, it is possible to build a three-step argument showing that free trade enhances welfare in both countries; the three steps are as follows: (i) \( I_j P_j^{-\gamma} \), and thus welfare, is decreasing in \( P_j \) for any \( P_j \not\in (0, P_j^*) \) and increasing for any \( P_j \in (P_j^*, \infty) \); (ii) \( P_N^{aut} \) is \( P_N^* \) and \( P_S^{aut} \) is \( P_S^* \) and (iii): as noted in the previous paragraph, \( P_N^{aut} < P_{FT} < P_S^{aut} \) (see Figure 8). International trade leads both countries to an equilibrium price that is not feasible in the autarky regime. This result is consistent with the idea that free trade enhances welfare by allowing countries to separate consumption from production bundles and, in particular, to reach a consumption bundle that lies in the set of unfeasible allocations for the autarky case.

The fact that we know how \( P_j \) changes in the transition from the autarky to the free trade equilibrium implies that we can also find out the corresponding redistributive effects. These redistributive impacts can be summarized as follows:

\[
P_{FT} > P_N^{aut} \quad \text{yields} \quad \frac{q_N^{aut}}{p_N^{aut}} < \frac{q_N^{FT}}{p_{FT}} \quad ; \quad \frac{w_N^{aut}}{p_N^{aut}} > \frac{w_N^{FT}}{p_{FT}} \quad (32)
\]

\[
P_{FT} < P_S^{aut} \quad \text{yields} \quad \frac{q_S^{aut}}{p_S^{aut}} > \frac{q_S^{FT}}{p_{FT}} \quad ; \quad \frac{w_S^{aut}}{p_S^{aut}} < \frac{w_S^{FT}}{p_{FT}} \quad (33)
\]

Trade liberalization increases the real skilled wage and diminishes the real unskilled wage in the skilled-abundant country. By contrast, the former wage falls and the latter wage rises.
in the unskilled labor abundant country. This result is also standard in international trade theory and frequently referred to in the context of the Stolper-Samuelson Theorem.

Figure 8. Real Income as a Function of $P_j$ under Free Trade

![Figure 8. Real Income as a Function of $P_j$ under Free Trade](image)

Figure 9. Free Trade Equilibrium

![Figure 9. Free Trade Equilibrium](image)

The allocation properties of the equilibrium are depicted in Figure 9, which shows the free trade equilibria for given levels of skilled and unskilled labor in South and North, i.e., recall that, just as Figure 7, Figure 9 simply makes a conceptual point and is, thus, not based on the
functional forms assumed in the model. This latter figure states that, unlike in the autarky case, consumption and production in each country are now different and this, in turn, constitutes a source of welfare gains. At the same time, the changes in the slopes of the tangency lines relative to Figure 7 figure shows the change in relative prices that generate the redistributive implications noted above.

7. Analysis of the Third Scenario: Mutual Trade Restrictions and no International Labor Mobility

This section considers a scenario in which migration is still not allowed. However, starting from a free trade environment, countries mutually impose an imports tariff. The mutual trade restrictions regime lies between the autarky and the free trade scenarios and thus, as is well known, a mutual imports tariff harms abundant factors, benefits scarce factors and reduces welfare in aggregate terms in each country, as compared to the free trade equilibrium.

In this context, the mutual trade restriction scenario sets a benchmark for comparison with Section 9, in which immigration is restricted by a choice of migration policy, i.e., just as international trade is restricted by the import tariff in the present section. As more clearly noted below, the mutual trade restriction equilibrium can be replicated by the appropriate choice of a migration tax.

In the present section, the imports tariff is assumed to be identical across countries and to take the iceberg form so that, in order for one unit of a product to arrive in the other country, \( \tau > 1 \) must be shipped, i.e., the rest melts away in transit. By creating a wedge between domestic and international prices, the tariff will prevent the effective price of the skilled-intensive good from equalizing across countries. In the absence of migration, this will in turn also prevent real wage equalization.

Regarding the zero-profit conditions, it should be noted that the tariff increases the effective price of imported products, i.e., the skilled-intensive good in South and the unskilled-intensive good in North. In contrast, because marginal costs depend exclusively on the technology used for production, they remain unaltered relative to previous sections. Hence, the zero-profits conditions can be written as follows:
\[ q_N^\beta w_N^{1-\beta} = P^{\text{MTR}} \quad (34) \]
\[ q_N^\alpha w_N^{1-\alpha} = \tau \quad (35) \]
\[ q_S^\beta w_S^{1-\beta} = \tau P^{\text{MTR}} \quad (36) \]
\[ q_S^\alpha w_S^{1-\alpha} = 1 \quad (37) \]

where \( P^{\text{MTR}} \) is the price of the skilled-intensive good in the mutual trade restriction scenario.

Solving the systems formed by equations (34)-(35) and by equations (36)-(37), it is possible to write wages as follows:

\[ q_N^{\text{MTR}} = P^{\text{MTR}} \frac{1-\alpha}{\beta-a} \frac{\tau}{\tau-\alpha} \quad (38) \]
\[ w_N^{\text{MTR}} = P^{\text{MTR}} \frac{-\alpha}{\beta-a} \frac{\beta}{\tau-\alpha} \quad (39) \]
\[ q_S^{\text{MTR}} = P^{\text{MTR}} \frac{1-\alpha}{\beta-a} \frac{1-\alpha}{\tau-\alpha} \quad (40) \]
\[ w_S^{\text{MTR}} = P^{\text{MTR}} \frac{-\alpha}{\beta-a} \frac{-\alpha}{\tau-\alpha} \quad (41) \]

Equations (38)-(41) can be used to derive the real skilled and unskilled wages in each country. When calculating real wages, it is important to note that the imports tariff modifies the price index in both nations. In particular, while the price index in North is now given by \((P^{\text{MTR}})^\gamma (\tau)^{1-\gamma}\), in South this index is given by \((P^{\text{MTR}} \tau)^\gamma\). Taking this into account, we write real wages as follows:

\[ \frac{q_N^{\text{MTR}}}{(P^{\text{MTR}})^\gamma (\tau)^{1-\gamma}} = \left( \frac{P^{\text{MTR}}}{\tau} \right)^{\frac{1-\alpha(1-\gamma)-\beta\gamma}{\beta-a}} , \quad (42) \]
\[ \frac{w_N^{\text{MTR}}}{(P^{\text{MTR}})^\gamma (\tau)^{1-\gamma}} = \left( \frac{P^{\text{MTR}}}{\tau} \right)^{\frac{-a(1-\gamma)-\beta\gamma}{\beta-a}} , \quad (43) \]
\[ \frac{q_S^{\text{MTR}}}{(P^{\text{MTR}} \tau)^\gamma} = \left( \frac{P^{\text{MTR}} \tau}{\tau} \right)^{\frac{1-\alpha(1-\gamma)-\beta\gamma}{\beta-a}} , \quad (44) \]
\[ \frac{w_S^{\text{MTR}}}{(P^{\text{MTR}} \tau)^\gamma} = \left( \frac{P^{\text{MTR}} \tau}{\tau} \right)^{\frac{-a(1-\gamma)-\beta\gamma}{\beta-a}} , \quad (45) \]

A comparison of Equations (42)-(43) with (14)-(15) and (25)-(26) reveals that Northern real wages have symmetric expressions in the mutual trade restriction case and in the
remaining scenarios; however, in the former case, the price of the skilled-intensive good is divided by the imports tariff \((P_{MTR}^{\tau}/\tau)\). Note that the same result holds for South; nonetheless, in this case, the price of the skilled-intensive good must be multiplied by the imports tariff. Indeed, trade restriction creates a wedge between relative prices across countries.

Moreover, inspection of (38)-(41) shows that a similar result can be formulated in terms of skill-premia. While the Northern skill-premia in the autarky and free trade scenarios equal \((P_{N}^{aut})^{\frac{1}{\beta - \alpha}}\) and \((P_{FT}^{FT})^{\frac{1}{\beta - \alpha}}\), respectively, the skill-premium implied by (38)-(39) is obtained by dividing the price of the skilled-intensive good through the imports tariff, i.e., and equals \((P_{MTR}^{\tau})^{\frac{1}{\beta - \alpha}}\). By the same token, to obtain the Southern skill-premium implied by (40)-(41) one must multiply the price by the tariff \((P_{MTR}^{\tau})^{\frac{1}{\beta - \alpha}}\). Just by multiplying or by dividing through the imports tariff, one can go back and forth from the mutual trade restriction and the remaining scenarios.

Just as in previous sections, the skill-premia can be used to derive the demand for skilled and unskilled labor in each country and, subsequently, to obtain the supply of goods. Indeed, the fact that technologies are the same implies that labor demands as a function of skill-premia are also exactly the same. Thus, the only difference in setting up the labor market-clearing conditions lies in the divergence of skill-premia among the different scenarios. Taken this difference in consideration, we can solve for the labor market clearing conditions and derive the following output supplies (Appendix 6 proves this result):

\[
Y_{NS}^{MTR} = (P_{MTR}^{\tau})^{\frac{1}{\beta - \alpha}}(H_{N}(P_{MTR}^{\tau})^{\frac{1}{\beta - \alpha}}(1 - \alpha) - L_{N}\alpha)/(\beta - \alpha) \tag{46}
\]

\[
Y_{Nu}^{MTR} = (P_{MTR}^{\tau})^{\frac{1}{\beta - \alpha}}(L_{N}\beta - H_{N}(P_{MTR}^{\tau})^{\frac{1}{\beta - \alpha}}(1 - \beta)))/(\beta - \alpha) \tag{47}
\]

\[
Y_{SS}^{MTR} = (P_{MTR}^{\tau})^{\frac{1}{\beta - \alpha}}(H_{S}(P_{MTR}^{\tau})^{\frac{1}{\beta - \alpha}}(1 - \alpha) - L_{S}\alpha)/(\beta - \alpha) \tag{48}
\]

\[
Y_{Su}^{MTR} = (P_{MTR}^{\tau})^{\frac{1}{\beta - \alpha}}(L_{S}\beta - H_{S}(P_{MTR}^{\tau})^{\frac{1}{\beta - \alpha}}(1 - \beta)))/(\beta - \alpha) \tag{49}
\]

Now, note that the Northern supply of goods represented in Equations (46) and (47) allows calculating the income level in North as a function of \(P_{MTR}^{\tau}\) and, therefore, solving for real
income in this country. Obtaining the expression for real income in North will be critical to set a benchmark for comparison with the study of a migration tax. In particular, in Section 10, we will investigate whether a migration policy can replicate the same real income as the imports tariff considered in this section. Taking this in consideration, we use expressions (46) and (47) to write income and real income in North as follows:

\[ I_N^{MTR} = Y_{Ns}^{MTR} P^{MTR} + Y_{Nu}^{MTR} \tau = P^{MTR} \frac{\beta}{\beta - \alpha} \frac{\alpha}{\beta - \alpha} (L_N + H_N P^{MTR} C (1 - \alpha)) (50) \]

\[ I_N^{MTR} ((P^{MTR})^\gamma (\tau)^{1 - \gamma})^{-1} = (P^{MTR} / \tau) \frac{1}{\beta - \alpha} \frac{1}{(P^{MTR} / \tau)^{1 - \alpha} H_N + L_N} \] (51)

Not surprisingly, a simple comparison between (9) and (51) shows that we have again found the same pattern: the expression for real income, and thus welfare, in the mutual trade restriction case is symmetric to the expression for the remaining scenarios, but in the former case the price of the skilled-intensive good must be divided by the import tariffs. Putting together Equations (42)–(43) and (51) we know that as long as a migration tax is able to generate an equilibrium price equal to \( P^{MTR} / \tau \), it will generate the welfare level and redistributive implications as the imports tariff. In Section 10, it will be shown that such a migration tax actually exists.

Given what we have just said, it is critical to find the equilibrium value of \( P^{MTR} \). For this purpose, we will use the supplies of goods shown in Equations (46)-(49) to set product market equilibrium.

Nonetheless, in contrast with the product market equilibrium conditions from previous sections, in this section the conditions must consider that a country can only satisfy a certain demand for imports by producing that demand plus the quantity lost in transit. Taking this into account, we solve for product market clearing and find the following price (see Appendix 6 for a full proof):

\[ P^{MTR} = \frac{(\alpha(1 - \gamma) + \beta \gamma) (L_N + \tau L_S)}{(1 - \alpha(1 - \gamma) - \beta \gamma) (H_N \tau^{\alpha - \beta - \alpha} + \tau^{\alpha - \beta} H_S)} \] (52)
Figure 10. Equilibrium in Mutual Trade Restrictions for North

Figure 11. Equilibrium in Mutual Trade Restrictions for South

Just as in previous sections, we depict the allocation properties of the equilibrium for the same levels of initial resources as in Figures 10 and 11, i.e., for North and South, respectively. Note in these figures that the mutual trade restrictions equilibrium lies between the free trade and the autarky cases. In other words, the imposition of mutual imports tariffs reverts the process triggered by trade so that it reduces overall welfare and favors scarce factors (the reversion in the income redistribution process is given by the change in the slope of the tangency lines and the fall in utility is represented by the fact that \( U^{FT} > U^{MTR} > U^{TAR} \))
\(U^\text{aut}\). In particular, note that the tariff raises the unskilled wage and reduces the skilled wage, as compared to the free trade regime in North (where the abundant factor is skilled labor). In this context, it is natural to expect that Northern skilled workers will support trade liberalization, while Northern unskilled workers will oppose it.

8. Analysis of the Fourth Scenario: No International Trade and Free International Labor Mobility

This section investigates the equilibrium characteristics and welfare implications of allowing for migration. For this purpose, it considers a scenario in which workers are allowed to migrate freely to a different country and there is no international trade. In order to construct this scenario, we take as a point of departure the autarky regime presented in Section 5. In particular, starting from this point, we will find the ensuing incentives for migration and resulting equilibrium prices.

As clearly stated in Section 5, the autarky price of the skilled-intensive good is smaller in North \(P_N^\text{aut} < P_S^\text{aut}\). In turn, it is easy to see that this implies \(\frac{q_N^\text{aut}}{r_N^\text{aut}} > \frac{q_S^\text{aut}}{r_S^\text{aut}}\) and \(\frac{w_N^\text{aut}}{p_N^\text{aut}} > \frac{w_S^\text{aut}}{p_S^\text{aut}}\). In a scenario with no international trade and migration barriers, skilled workers have incentives to migrate South and unskilled workers have incentives to migrate North. Moreover, these incentives are only exhausted once migration flows lead real skilled and unskilled wages to be equal across countries. Indeed, this will be our main equilibrium condition in the present section.

Using the expressions for real wages shown in Section 5, we can write real wages in a situation with no international trade as follows:

\[
\frac{q_N^\text{FM}}{p_N^\text{FM}} = p_N^\text{FM} \frac{1 - \alpha(1 - \gamma) - \beta y}{\beta - \alpha} \tag{53}
\]

\[
\frac{w_N^\text{FM}}{p_N^\text{FM}} = p_N^\text{FM} \frac{-\alpha(1 - \gamma) - \beta y}{\beta - \alpha} \tag{54}
\]

\[
\frac{q_S^\text{FM}}{p_S^\text{FM}} = p_S^\text{FM} \frac{1 - \alpha(1 - \gamma) - \beta y}{\beta - \alpha} \tag{55}
\]
where the superscript $FM$ denotes that we are dealing with the free migration case and $P_{N}^{FM}$ and $P_{S}^{FM}$ are the prices of the skilled-intensive good in North and South, respectively. Following Section 5, we know that the equilibrium values of these prices can be written as follows:

\[ P_{N}^{FM} = \left( \frac{L_{N}}{H_{N}} \right)^{\beta-a} \left( \frac{\alpha(1-\gamma)+\beta \gamma}{1-\alpha(1-\gamma)-\beta \gamma} \right)^{\beta-a} \quad (57) \]

\[ P_{S}^{FM} = \left( \frac{L_{S}}{H_{S}} \right)^{\beta-a} \left( \frac{\alpha(1-\gamma)+\beta \gamma}{1-\alpha(1-\gamma)-\beta \gamma} \right)^{\beta-a} \quad (58) \]

Note, however, that unlike in Section 5, the number of unskilled workers in North in this section is given not only by the original population but also by the number of unskilled migrants, i.e., recall that we have left these migrants aside from welfare calculation but not from price calculation. In other words, we can write: $L_{N} = \overline{L}_{N} + M_{LN}$, where $M_{LN}$ refers to the number of (net) unskilled immigrants to North. By the same token, it is known that $H_{N} = \overline{H}_{N} + M_{HN}$ and that $L_{S} = \overline{L}_{S} + M_{LS}$ and $H_{S} = \overline{H}_{S} + M_{HS}$.

As noted above, in the absence of migration barriers skilled workers have incentives to migrate South and unskilled have incentives to migrate North until real wages equalize. Using Equations (53)-(59), it is easy to see that this equilibrium condition can be written as follows:

\[ \frac{q_{N}^{FM}}{q_{S}^{FM}} = \frac{q_{S}^{FM}}{q_{N}^{FM}} ; \quad \frac{w_{S}^{FM}}{w_{N}^{FM}} \quad \text{yields} \quad P_{N}^{FM} = P_{S}^{FM} \quad \text{yields} \quad \frac{L_{S}}{H_{S}} = \frac{L_{N}}{H_{N}} \quad (59) \]

Equation (59) states that in equilibrium good prices and therefore unskilled-to-skilled labor ratios must be the same in both countries; it is only in this way that real wages are identical across regions. Importantly, note that because real wages do not depend on the supply of skilled and unskilled workers in absolute terms, the equilibrium condition only pins down the corresponding unskilled-to-skilled labor ratios.

In order to find the precise value of the unskilled-to-skilled labor ratio that fulfills (59), let us use the following full-employment definition:
\[ L_S + L_N = L_W \]  
\[ (60) \]

where \( L_W \) denotes the world’s unskilled labor supply. Dividing Equation (60) through \( H_W \) (the world’s labor supply) we can write:

\[
\left( \frac{H_S}{H_W} \right) L_S + \left( \frac{H_N}{H_W} \right) L_N = \frac{L_W}{H_W} \]  
\[ (60') \]

Let us now substitute \( \frac{H_N}{H_W} \) with the expression \( 1 - \frac{H_S}{H_W} \) in Equation (60’) and write:

\[
\left( \frac{H_S}{H_W} \right) L_S + \left( 1 - \frac{H_S}{H_W} \right) L_N = \frac{L_W}{H_W} \]  
\[ (60'') \]

Equation (60’’) states the full employment condition for the world. Note that this condition is only fulfilled when:

\[
\frac{L_S}{H_S} = \frac{L_N}{H_N} = \frac{L_W}{H_W} \]  
\[ (61) \]

The equilibrium value of the unskilled-to-skilled labor ratio in the free migration regime must be the ratio prevailing in the entire world. This has a relevant implication for the price of the skilled-intensive good because this price depends directly on the unskilled-to-skilled labor ratio. In particular, putting together Equations (57)- (58) and (61) yields the following result:

\[
P_N^{FM} = P_S^{FM} = P^{FT} = P^{INT} \]  
\[ (62) \]

Equation (62) states that, just as international trade, migration leads to price and wage convergence and that the underlying prices are the same as in the integrated economy. This makes it easy to find out the welfare implications of migration. As shown above, and is largely known, free trade is welfare-improving relative to the autarky equilibrium, in which there neither international trade nor international labor mobility. More generally, it is known that the prices under free trade reproduce any of the allocations within the continuum set of Pareto efficient allocations i.e., this can be proved simply by applying the First Fundamental Welfare Theorem and by noting that our framework does not exhibit market failures. Driven by these results, we know that, because free migration implements the same price vector as
free trade, this migration is not only welfare-improving but also optimal from a Pareto-point of view.\textsuperscript{7}

Furthermore, an important feature of the free migration equilibrium is that it does not determine the absolute number of worker types in each country, i.e., it only determines the unskilled-skilled labor ratio. The fact that the absolute number of workers in each country is not determined confronts us with the need of choosing the focus of our study. In response to this need, we proceed by focusing exclusively on migration going on a single direction, from South to North. Besides being consistent with one of the equilibria, a situation in which migrants only go North constitutes the most interesting one and, importantly, is also the only equilibrium configuration in a more realistic scenario in which this country has a technological advantage over South.

Let us present the redistributive effects of free migration through the following results:

\[ P_N^{FM} = P_N^{FT} > P_N^{aut} \quad \text{yields} \quad \frac{q_N^{aut}}{p_N^{aut} \gamma} < \frac{q_N^{FM}}{p_N^{FM} \gamma} ; \quad \frac{w_N^{aut}}{P_N^{FM}} > \frac{w_N^{FM}}{P_N^{FM} \gamma} \quad (63) \]

\[ P_S^{FM} = P_S^{FT} < P_S^{aut} \quad \text{yields} \quad \frac{q_S^{aut}}{p_S^{aut} \gamma} > \frac{q_S^{FM}}{p_S^{FM} \gamma} ; \quad \frac{w_S^{aut}}{P_S^{FM}} < \frac{w_S^{FM}}{P_S^{FM} \gamma} \quad (64) \]

In the spirit of the Stolper-Samuelson Theorem for the case of international trade, Equations (63) and (64) state that migration benefits the abundant factor and harms the scarce factor in each country, relative to the autarky regime. In particular, free migration increases the real return of skilled workers and reduces the real return of unskilled workers in North.

Using the same methodology as in previous sections, Figure 12 shows two relevant results: (i) immigration improves production capacity and, through this channel, affects consumption and increases welfare, i.e. the production possibility frontier shifts in each country and (ii) the resulting equilibrium relative price is exactly the same as in the free trade equilibrium so that trade and immigration has precisely the same redistributive implications.

\textsuperscript{7} Furthermore, we know that in any of these Pareto optimal allocations, the skilled-to-unskilled labor ratio is the world’s ratio in both countries.
9. Analysis of the Fifth Scenario: No International Trade and Tax on International Labor Mobility

9.1 General Welfare Implications of Immigration

Even though we have been able to show that fully removing migration barriers is optimal from a Pareto point of view, this section takes a more general approach and investigates its welfare consequences, regardless of whether crossing border is fully or only partially free. The goal of this exercise is to simplify the welfare analysis that we will undertake in the upcoming section.

As a first step, let us focus on North and consider the expression for real income, our measure of welfare: \( I_N P_N^{-\gamma} = P_N^{\frac{-a(1-\gamma)-\beta\gamma}{b-a}} \left( L_N + H_N P_N^{\frac{1}{b-a}} \right). \) As noted above, we will hold the number of residents constant for welfare calculation purposes and, therefore, in our framework migration will only affect well-being through its impact on prices (see further details above). Following this point, and as a first glance to the welfare effects of migration, let us take the partial derivative of real income with respect to \( P_N \):
\[
\frac{\partial I_N^P}{\partial P_N} = P_N^{\frac{\beta}{\beta-\alpha}} \left( \frac{1}{(H_N P_N)^{\frac{1}{\beta-\alpha}}} - L_N (a(1-\gamma) + \beta y) \right) / \beta - \alpha
\]  

(65)

Simple algebra on (65) shows that \( \frac{\partial I_N^P}{\partial P_N} = 0 \) precisely at the value \( P_N \) at which this derivative is evaluated in a non-international trade regime; that is, the partial derivative equal 0 exactly at \( P_N^* = \left( \frac{L_N}{H_N} \right)^{\beta-\alpha} \left( \frac{a(1-\gamma) + \beta y}{1-a(1-\gamma) - \beta y} \right)^{\beta-\alpha} \), which is always the equilibrium price in a non-trade regime.

Figure 13. Welfare, Immigration and the Envelope Theorem

In the light of this result, one may be tempted to argue that migration has no implications for welfare. This statement is partially true: a marginal change in \( P \) has no effects on welfare and this marginal change could, in turn, be originated by migration flows and the ensuing change in the number of unskilled workers.\(^8\) Indeed, an increase in the supply of unskilled labor generating a marginal change in \( P \) would at first affect real wages; nonetheless, this initial impact would be fully offset by agents’ re-optimization. That is, the fact that marginal

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\(^8\) Note that the manuscript does not claim that marginal changes in the number of unskilled workers have no welfare impacts; instead, what the manuscript claims is that marginal changes in prices have no such effects.
changes in prices have no welfare implications can be interpreted simply as an application of The Envelope Theorem. Figure 13 shows the result.

Figure 14. Discrete Changes in Prices

![Figure 14. Discrete Changes in Prices](image)

Although migration flows associated with marginal changes on prices have no welfare impacts, migration inducing discrete (non-marginal) changes do affect welfare in a positive way. To see this, use the expression for $P_N^*$ to take the partial derivative with respect the amount of unskilled workers and note:

$$\frac{\partial P_N^*}{\partial L_N} > 0$$  \hspace{1cm} (66)

Equation (66) states that migration indeed shifts upwards the curves presented in previous graphs. Thus, we can conclude migration flows that induce discrete changes on prices have positive impacts on welfare. This result is summarized in Figure 14.

Finally, note that to investigate the welfare implications of a migration tax, it is sufficient to use the graphical method developed above. As noted previously, our measure of welfare is restricted exclusively to native residents and, therefore, migration affects welfare through its effect on prices.
9.2 General Welfare Implications of Immigration

This section investigates the effects of a tax on migration. For this purpose, it takes as a point of departure from the autarky regimes presented in Section 4 and focuses on migration of unskilled workers from South to North, just as Section 8.

Consider a migration tax that takes the iceberg form: it is assumed that a worker migrating to North receives only $1 - \phi$ of her wage and the rest melts away in “her transit.” This tax must be thought of as sufficient statics of migration policy and, in particular, as an indicator of policy-induced restrictions to labor mobility across countries.

Just as in Section 8, it is assumed that the expression for wages are initially the same as in the autarky regime. Because we concentrate on migration by unskilled workers from South to North, we hereby refer only to real unskilled wages

$$\frac{w_{TM}^N}{p_{TM}^N} = \frac{b_{TM}^N}{\beta - \alpha}$$  \hspace{1cm} (67)

$$\frac{w_{TM}^S}{p_{TM}^S} = \frac{b_{TM}^S}{\beta - \alpha}$$  \hspace{1cm} (68)

where $b_{TM}^N$ and $b_{TM}^S$ are the prices of the skilled-intensive good in the tax migration case in North and South, respectively. Following Section 4, it is known that the equilibrium values of these prices can be written as follows:

$$p_{TM}^N = \frac{l_{N}}{h_{N}} \beta - \alpha \left( \frac{\alpha(1-\gamma) + \beta y}{1 - \alpha(1-\gamma) - \beta y} \right) \beta - \alpha$$  \hspace{1cm} (69)

$$p_{TM}^S = \frac{l_{S}}{h_{S}} \beta - \alpha \left( \frac{\alpha(1-\gamma) + \beta y}{1 - \alpha(1-\gamma) - \beta y} \right) \beta - \alpha$$  \hspace{1cm} (70)

where the definitions given in the free migration case also apply here: $L_N = \tilde{L}_{N} + M_{LN}$; $H_N = \tilde{H}_{N} + M_{HN}$; $L_S = \tilde{L}_{S} + M_{LS}$ and $H_S = \tilde{H}_{S} + M_{HS}$. In contrast with the free migration case, however, in this section the incentives for migration are not exhausted when real wages equalize across countries. In particular, the presence of a tax diminishes the benefit from migration, inducing unskilled workers from South to migrate only until

$$\frac{w_{TM}^N}{p_{TM}^N} (1 - \phi) = \frac{w_{TM}^S}{p_{TM}^S},$$

i.e., only until the real wage net of iceberg costs in North is equal to the real wage in
South. In other words, the equilibrium condition in this section is different from the equilibrium condition considered in the free migration case.

The combination of this new equilibrium condition with Equations (69)-(70) yields the following result:

\[
\frac{P_{TM}^{N}}{P_{TM}^{S}} \frac{\alpha(1-\gamma)-\beta\gamma}{1-\phi} = \frac{P_{TM}^{S}}{P_{TM}^{N}} \frac{\alpha(1-\gamma)-\beta\gamma}{1-\phi} \Rightarrow \frac{L_{N}}{H_{N}} = \frac{L_{S}}{H_{S}}(1-\phi) \]  

(71)

The migration tax introduces a wedge between real wages across countries and, therefore, between unskilled-to-skilled labor ratios. In particular, by discouraging unskilled migration, the tax reduces this ratio in North while increasing it in South. Along these lines, substituting Equation (71) in the world’s full-employment conditions generates the following results:

\[
\frac{L_{S}}{H_{S}} = \frac{1}{1-\phi(1-\frac{H_{S}}{H_{W}})} \left( \frac{L_{W}}{H_{W}} \right) \]  

(72)

\[
\frac{L_{N}}{H_{N}} = \frac{1-\phi}{1-\phi(1-\frac{H_{S}}{H_{W}})} \left( \frac{L_{W}}{H_{W}} \right) \]  

(73)

From these expressions, it is easy to see that \( P_{TM}^{S} < P_{TM}^{N} = P^{FT} < P_{TM}^{N} \); just as an imports tax, a tax on migration introduces a wedge between prices across countries, reducing this price in North with respect to the free migration case.

Once the change in the price of the skilled-intensive good is known, we can infer the welfare and redistributive impacts of the migration policy. Because the tax diminishes the price of the skilled-intensive good in North, it triggers welfare-reducing effects, i.e., it is easy to see this by using the graphical tools shown above and, in particular, the analysis presented in Figure 15.

Furthermore, the fall in \( P \) arising from the immigration tax generates redistributive effects with respect to the free international labor mobility equilibrium. In particular, the fact that there are fewer unskilled immigrants implies that the supply of unskilled labor increases to a lesser extent in North. This, in turn, makes the real unskilled wages greater than in the free migration regime and implies that the real skilled wage must fall in order for the zero-profit conditions to be satisfied. At the same time, these factor price changes must be compensated for with a fall in the price of the skilled-intensive good. In sum, the migration tax increases
the real return to unskilled workers and reduces the real return to skilled workers with respect to a free migration regime and therefore, in this sense, it can be argued that it has some effects as an imports tax in the context of international trade. This result is crucial to understand why special interests groups may want to constrain migration and, therefore, to motivate the political economy analysis of Section 11. The allocation properties of the equilibrium are shown in Figure 15.

Figure 15. Equilibrium with Migration Tax in North

10. Tax Equivalence

The previous analysis has shown that a tax on migration harms the abundant factor, it benefits the scarce factor and it triggers welfare-reducing effects with respect to a case in which free international labour mobility is allowed. Interestingly, note that these impacts are isomorphic to the impacts of a mutual imports tax. In this context, the present section derives a migration policy that replicates the same equilibrium as the imports tariffs presented in Section 6.

As largely discussed in that section, for that purpose, it suffices to find for each $\tau$ a value of $1 - \phi$ that implements exactly the same relative price $P_{\text{MTC}} / \tau$ and is, therefore, associated with the prices, real wages and welfare of the import tax regime. To find this value, we divide the expression for $P_{\text{MTC}}$ in Equation (52) by $\tau$ and equate the resulting expression to the price
that arises from plugging (73) in the definition given in (69). This process yields the following “equivalent” migration tax (see Appendix 8 for a full derivation of this result):

$$\phi_{Eq} = \frac{\tau^\beta (\tau^{a-\beta} H_N + \tau H_S) L_W - \tau^{a+1} \alpha^{1} H_W (L_M^{MTR} + \tau L_S^{MTR}) H_W}{\tau^\beta ((\tau^{a-\beta} H_N + \tau H_S) L_W + \tau^{a+1} \alpha^{1} H_N (1-\tau) L_M^{MTR} + \tau L_S^{MTR}) (H_S - H_W)} \quad (74)$$

where $L_N^{MTR}$ and $L_S^{MTR}$ are the supplies of unskilled labor in North and South under the mutual trade restriction scenario, respectively, and the supplies of skilled labor do not carry out not a superscript given that they are the same under the two scenarios considered.

In order to draw intuition on Equation (74), it will be useful to note at all times the sum of the labor supplies in North and in South must equal the world labor supply for both skilled and unskilled workers. That is, it will be useful to impose in the definition of $\phi_{Eq}$ the following feasibility constraints: $L_S^{MTR} = L_W - L_N^{MTR}$ and $H_S = H_W - H_N$ and write the resulting equation as follows

$$\phi_{Eq} = \frac{\tau^\beta (\tau^{a-\beta} H_N + \tau H_S) L_W - \tau^{a+1} \alpha^{1} H_W ((1-\tau) L_N^{MTR} + L_W)}{\tau^\beta ((\tau^{a-\beta} H_N + \tau H_S) L_W + \tau^{a+1} \alpha^{1} H_N (1-\tau) L_N^{MTR} + L_W)} \quad (74')$$

Using Equation (74') it is possible to assess the effects of an increase in the supply of unskilled labor in North, for a given supply of this labor around the world, i.e., a redistribution of $L_W$ in favor of North. Note that, given that North is the skilled labor-abundant country, an exogenous redistribution of unskilled labor to its favor makes the countries more similar in terms of relative factor endowments and, therefore, diminishes the welfare gains from international trade. In a context in which gains form trade are exogenously diminished, one would expect that the extent to which mutual trade restrictions reduce welfare is smaller; consequently, the equivalent migration tax that appears in Equations (74) and (74') should also be lower, so that it generates a harm of a smaller magnitude. Indeed, Appendix 8 demonstrates that $\partial \phi_{Eq} / \partial L_N^{MTR} < 0$: a more equal distribution of unskilled labor is associated with a smaller equivalent migration tax.
11. Political Economy Analysis

The previous analysis has shown that immigration increases overall welfare but generates income redistribution effects. In this context, restricting migration may favor special interest groups. For the particular case analyzed in the present note, a migration policy that restricts unskilled migration benefits skilled workers in North at the costs of smaller welfare aggregate levels. More generally, this suggests that political economy concerns may have an influence on the design and implementation of migration policies in advanced economies.

Indeed, the premise that public policies can be influenced by special interest groups has a long tradition in both economics theory and empirical work. For the case of regulatory measures, Djankov et al. (2002) suggest that entry regulation generates rents that accrue to bureaucrats and administrative employees. Yet, bureaucrats, politicians and administrative employees may be tempted to implement regulation not only to obtain profits directly, but also to collect bribes and contributions from the relevant interest groups (McChesney, 1987; De Soto, 1990; Shleifer and Vishny, 1993 and Tobal, 2017 for the consequences on international trade).

In the domain of international trade policy, probably the most influential work has been written by Grossman and Helpman (1994). In their seminal paper, they show that lobbying groups have incentives for having an influence on the design of import tariffs. Along these lines, the present technical note has shown that there is some equivalence between international trade and migration policies in terms of welfare and redistribution effects. This, again, suggests that, just as international trade policies, immigration policy may be influenced by interest groups.

In this section, we develop an extension of the factor proportion model presented previously with the goal of illustrating one among the several channels through which special interest groups may affect immigration policy. In particular, we will illustrate a case in which those who are damaged by free international labor mobility, i.e., unskilled workers, exert political pressure to restrict migration flows. Indeed, there are several channels through which this pressure could be exerted and, therefore, several modelling options to be considered. For instance, in line with Grossman and Helpman's seminal work (1994) for the
case of international trade policy, one could think of a scenario in which a group of unskilled workers attempts to influence politicians through lobbying and contributions to campaigns.

However, in contrast with several of the channels through which political economy concerns shape public policy in the literature, the extension we present does not rely on the existence of a government attempting to extract private rents or to maximize political support. Instead, our extension shows that, even when a government is forward-looking and benevolent, it may have incentives to deviate from the migration policy associated with the first-best equilibrium. We do not claim that our modelling choice is unique or even the best, our claim is that, by depicting a channel that has not been considered in the literature yet, we make a significant contribution.

As noted above, we consider the economy described by the factor proportion model presented in previous sections and set as our point of the departure the autarky equilibrium of Section 3. To simplify, it is assumed that there is a single interest group representing unskilled workers, e.g., unions. This group can affect congress’ decisions, possibly because their actions influence media coverage and, through this channel, have an impact on public opinion, i.e., as many unskilled workers go on a strike, large media coverage frequently exerts pressure on the congress. For instance, if unskilled workers go on a strike, the pressure of the media forces the congress to reject the migratory reform proposed by the government, i.e., represented by the $\phi$ parameter.

More formally, assume that there is a probability that unskilled workers do not go on a strike $f(\phi)$ and that this probability fulfills the traditional Inada conditions: (i) $f(\phi = 0) = 0$: when free migration is proposed, unskilled workers always go on a strike; (ii) $f(\phi)$ is continuously differentiable; (iii) $f(\phi)$ is strictly increasing in $\phi$: the probability of going to strike falls with the severity of the policy proposed (i.e. the higher the tax on migration, the lower the probability of going on strike); (iii) the second derivative is negative; (iv) the limit of the first derivative of $f(\phi)$ is infinite when $\phi$ tends to 0; (v) the limit of the first derivative of $f(\phi)$ is 0 when $\phi$ tends to infinite.

In this environment, the government is interested in maximizing expected welfare. In the context of our extension, this welfare can be written as follows: $EW(\phi) = f(\phi) W_N(\phi) +$
At the same time, it is known from the factor proportion model that \( W_N(\phi) > W_N^{aut} \) and that \( \frac{\partial W_N(\phi)}{\partial \phi} < 0 \).

Under these conditions, it is easy to show that the benevolent and forward-looking government never chooses a migration tax equal to zero (Appendix 9 provides a formal proof of this outcome). The intuition for this result goes as follows. Even though the government knows that choosing a zero tax would be optimal in the absence of political economy conflicts, it is also aware that doing so would lead the union to a strike and, consequently, the congress to reject the proposal. Under these conditions, the economy would remain in the autarky regime and reach the lowest possible welfare level. Hence, to avoid this situation, the benevolent and forward-looking government opts for proposing a positive migration tax and improve the probability that the reform gets accepted.

12. Conclusions

In the present technical note, we have provided some theoretical tools that illustrate useful and interesting insights into the economic effects of migration, as well as on political factors that may affect the design of migration policy. To illustrate these points, we have set a standard factor proportion model of international trade and have used it to investigate the impacts of free trade, free migration, an imports tariff and a tax on migration.

The analysis generates several interesting conclusions. First, free-trade and free-migration generate isomorphic results, precisely when the most relevant measure of welfare from a political economy perspective is taken into account. In particular, both trade and migration increase aggregate welfare but have redistributive effects. Second, along these lines, it is shown that the welfare outcomes arising from an imports tax can be replicated by implementing the proper migration policy. In the light of these results, we then conclude that it may be the case that migration policies are influenced by political economy concerns. Thus, we develop an extension of our standard model to illustrate one of the several channels through this may actually take place.

In sum, migration constitutes an important source of potential gains in terms of welfare. Nonetheless, it generates redistribution effects and may cause losses to specific groups in the
society. From a public policy perspective, this implies that policy-makers are confronted with the need of developing policies that mitigate potential adverse effects on sectors of the population that do not directly benefit from international labor mobility. This could be accomplished through the implementation of compensatory non-distortionary taxes that increase government’s revenues and, through this channel, allows raising progressive public spending.

From a more general point of view, the conclusion that one should take from our analysis is that there is large room for using economists’ tools to contribute to the understanding of topics that generally create controversy, such as immigration policies. In particular, just as we already have a toolkit to analyze trade policy and its welfare, redistributive and political economy dimensions, we can easily extend this literature to analyze migration policy in these same dimensions.
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Appendix Section

Appendix 1. Indirect Utility Function: Derivation of General Results

This appendix demonstrates that the indirect utility function shown in (4) can be derived from the utility function shown in Equation (3). Using Equation (3), we know that the indirect utility function is given by \( U_j = c_{js}^*(I_j, P_j) c_{ju}^{1-\gamma}(I_j, P_j) \), where \( c_{js}(I_j, P_j) \) and \( c_{ju}(I_j, P_j) \) are the consumption levels of the skilled- and unskilled-intensive goods that result from the following maximization problem:

\[
Max_{c_{js}, c_{ju}} U_j = c_{js}^\gamma c_{ju}^{1-\gamma},
\]  

(A.1)

subject to:

\[
I_j = P_j c_{js} + c_{ju},
\]

where \( I_j \) and \( P_j \) are the income level and price of the skilled-intensive good in country \( j \), respectively. Solving for the first order conditions of this problem we obtain the following Marshallian demands:

\[
c_{js}^*(I_j, P_j) = \frac{\gamma I_j}{P_j},
\]

(A.2)

\[
c_{ju}^*(I_j, P_j) = (1 - \gamma) I_j.
\]

(A.3)

Replacing these solutions in \( U_j = c_{js}^*(I_j, P_j) c_{ju}^{1-\gamma}(I_j, P_j) \) one obtains the following indirect utility function:

\[
V_j = \gamma^\gamma (1 - \gamma)^{1-\gamma} I_j P_j^{-\gamma}.
\]

(A.4)

Note that this is the expression for the indirect utility function shown in Equation (4).

Appendix 2. Real Income as a Function of \( P_j \): Derivation of General Results

This appendix demonstrates that the function shown in (9) has a single critical point and that, abusing on the concavity properties of this function, we can illustrate it by using Figure 5. To show this, let us constrain our analysis to prices of the skilled-intensive good contained
within the interval \((0, \infty)\), i.e., \(P_j \in (0, \infty)\), and take the first derivative of (9) with respect to \(P_j\). This yields the following expression

\[
\frac{\partial (I_j P_j^{-\gamma})}{\partial P_j} = \frac{P_j^{\frac{\beta}{\beta - \alpha} - \gamma} (P_j^{\frac{1}{\beta - \alpha} (1 - \alpha(1 - \gamma) - \beta \gamma) - \gamma_j (\alpha(1 - \gamma) + \beta \gamma))}{\beta - \alpha}.
\]

(A.5)

Simple algebra on this equation shows that \(P_j^* = (\frac{\gamma_j}{\beta})^{\beta - \alpha} \left(\frac{\alpha(1 - \gamma) + \beta \gamma}{1 - \alpha(1 - \gamma) - \beta \gamma}\right)^{\beta - \alpha}\) is the only critical point of \(I_j P_j^{-\gamma}\), i.e., the only value of \(P_j\) at which \(\partial (I_j P_j^{-\gamma}) / \partial P_j = 0\). This is precisely the value of \(P_j\) referred to Section 4.

Furthermore, it is easy to see in (A.5) that \(\partial (I_j P_j^{-\gamma}) / \partial P_j > 0\) for any \(P_j \in (P_j^*, \infty)\) and that \(\partial (I_j P_j^{-\gamma}) / \partial P_j < 0\) for any \(P_j \in (0, P_j^*)\). That is, within the set of prices considered real income, and thus welfare, is decreasing in \(P_j\) for any \(P_j < P_j^*\) and is increasing in \(P_j\) for any \(P_j < P_j^*\). This property is crucial to derive all of the welfare results of the model. This proves that, under the appropriate assumptions on concavity, Figure 5 properly represents the indirect utility function shown in (9).

Appendix 3. Marginal Costs and Zero-Profit Conditions: Derivation of General Results

This appendix shows the derivation of marginal costs that are subsequently used in the setup of the zero-profit conditions. Let us begin with the zero-profit condition of the skilled-intensive good shown in (10). To derive the marginal costs associated with the skilled-intensive good, we will use the production function shown in Equation (1). In particular, marginal costs are obtained from the cost function that results from solving the following optimization problem:

\[
\text{Min}_{H_{js} L_{js}} C_{js} = q_j H_{js} + w_j L_{js},
\]

(A.6)

subject to:

\[
\bar{Y}_{js} = \varepsilon_s \left(\frac{H_{js}^{\beta} L_{js}^{1-\beta}}{\beta}\right),
\]

Solving for the first order conditions of this problem we obtain the following output-constrained demands:
\[ H_{js}^*(\overline{Y}_{js}, q_j, w_j) = \beta \left( \frac{w_j}{q_j} \right)^{1-\beta} \overline{Y}_{js}, \]  
(A.7)

\[ L_{js}^*(\overline{Y}_{js}, q_j, w_j) = (1 - \beta) \left( \frac{w_j}{q_j} \right)^{-\beta} \overline{Y}_{js}. \]  
(A.8)

Substituting these solutions in the production function yields the following costs function:

\[ C_{js}(\overline{Y}_{js}, q_j, w_j) = q_j^\beta w_j^{1-\beta} \overline{Y}_{js}, \]  
(A.9)

The marginal cost associated with the skilled-intensive good is given by the partial derivative of this function with respect to \( \overline{Y}_{js} \) and is, therefore equal to:

\[ MC_{js}(\overline{Y}_{js}, q_j, w_j) = q_j^\beta w_j^{1-\beta}, \]  
(A.10)

As for the zero-profits condition of the unskilled-intensive good, note that by analogy to (A.10) we can use Equation (2) and write:

\[ MC_{ju}(\overline{Y}_{ju}, q_j, w_j) = q_j^\alpha w_j^{1-\alpha}, \]  
(A.11)

Using the marginal costs displayed in Equations (A.10) and (A.11), it is possible to write the zero-profit-conditions of country \( j \) as follows:

\[ q_j^\beta w_j^{1-\beta} = P_j, \]  
(A.12)

\[ q_j^\alpha w_j^{1-\alpha} = 1, \]  
(A.13)

Solving these system of two equations and two unknowns, we can write the unskilled and skilled wages as follows:

\[ q_j = P_j^{\frac{1-\alpha}{\beta-\alpha}}, \]  
(A.14)

\[ w_j = P_j^{\frac{-\alpha}{\beta-\alpha}}. \]  
(A.15)

These are precisely the same expressions that appear in Equations (7) and (8).

Appendix 4. Labor Market-Clearing Conditions: Derivation of General Results

The demands for skilled and unskilled labor depends only the technology used for production. As noted in Appendix 3, with the Cobb-Douglas functions shown in Equations (1) and (2), these labor demands for skilled labor are written as follows:
\[ H_{js}^*(\overline{Y}_{js}, q_j, w_j) = \beta \left( \frac{w_j}{q_j} \right)^{1-\beta} \overline{Y}_{js}, \]  
(A.7')

\[ H_{ju}^*(\overline{Y}_{ju}, q_j, w_j) = \alpha \left( \frac{w_j}{q_j} \right)^{1-\alpha} \overline{Y}_{ju}, \]  
(A.16)

Using these demands, we can write the market-clearing condition for the skilled-intensive good as follows:

\[ \beta \left( \frac{w_j}{q_j} \right)^{1-\beta} \overline{Y}_{js} + \alpha \left( \frac{w_j}{q_j} \right)^{1-\alpha} \overline{Y}_{ju} = H_j, \]  
(A.17)

By the same token, the demands for unskilled labor are summarized by the following equations

\[ L_{js}^*(\overline{Y}_{js}, q_j, w_j) = (1 - \beta) \left( \frac{w_j}{q_j} \right)^{-\beta} \overline{Y}_{js}, \]  
(A.8')

\[ L_{ju}^*(\overline{Y}_{ju}, q_j, w_j) = (1 - \alpha) \left( \frac{w_j}{q_j} \right)^{-\alpha} \overline{Y}_{ju}. \]  
(A.18)

Thus, the market-clearing condition for the unskilled-intensive good is given by the following equation:

\[ (1 - \beta) \left( \frac{w_j}{q_j} \right)^{-\beta} \overline{Y}_{js} + (1 - \alpha) \left( \frac{w_j}{q_j} \right)^{-\alpha} \overline{Y}_{ju} = L_j, \]  
(A.19)

Equations (A.13) and (A.15) form a system of two equations with a higher number of unknowns. Nonetheless, this number boils down to two as we impose the skill-premium to be given by \( P_j^{\frac{1}{\beta-\alpha}} \), i.e., which is indeed the premium arising from the zero-profit conditions.

As one imposes this conditions in (A.13) and (A.15), one is left with a system with two equations and two unknowns that solve for the following supplies of goods:

\[ \overline{Y}_{js} = P_j^{\frac{1}{\beta-\alpha}} (H_j P_j^{\frac{1}{\beta-\alpha}} (1 - \alpha) - L_j \alpha)/(\beta - \alpha), \]  
(A.20)

\[ \overline{Y}_{ju} = P_j^{\frac{1}{\beta-\alpha}} (L_j \beta - H_j P_j^{\frac{1}{\beta-\alpha}} (1 - \beta))/(\beta - \alpha). \]  
(A.21)

Appendix 5. Product Market-Clearing Condition: Derivation of General Results

As noted in the main body text, the equilibrium value of \( P_j \) is determined by the product market-clearing conditions. Nonetheless, depending on the particular scenario that is being
taken under consideration, these conditions are set in a different manner (see Section 3 for a thorough discussion in this regard). Let us consider to this end the three groups referred to in Section 3:

✓ In autarky, there is neither migration nor international trade and, thus, $P$ is determined only by market-clearing conditions and these conditions are defined at the local level. Thus, in autarky, the relevant supplies are given in Equations (17) and (18) for North. By using sub-indexes $j$ rather than $N$ to generalize, we can write:

$$Y_{js}^{aut} = P_j^{aut} \frac{1}{\beta - \alpha} (H_j P_j^{aut} (1 - \alpha) - L_j \alpha) / (\beta - \alpha), \quad (A.22)$$

$$Y_{ju}^{aut} = P_j^{aut} \frac{1}{\beta - \alpha} (L_j \beta - H_j P_j^{aut} (1 - \beta)) / (\beta - \alpha). \quad (A.23)$$

Given that we consider the autarky case, the relevant demand for the good is defined at the local level. Maximization of the utility function shown in (3) yields a demand for the skilled-intensive good that can be obtained by appropriately interpreting Equation (A.2) in Appendix 1. In particular, the demands for the skilled-intensive good can be written as:

$$c_j^{s, aut} (I_j^{aut}, P_j^{aut}) = Y_{js}^{aut} \frac{1}{P_j^{aut}}, \quad (A.24)$$

where $I_j^{aut}$ is defined as $Y_{js}^{aut} P_j^{aut} + Y_{ju}^{aut}$. Using (A.22)-(A.24) one can equate the demand for the skilled-intensive good to its supply. This yields the following price:

$$P_j^{aut} = \left( \frac{L_j}{H_j} \right)^{\beta - \alpha} \left( \frac{\alpha (1 - \gamma) + \beta \gamma}{1 - \alpha (1 - \gamma) - \beta \gamma} \right)^{\beta - \alpha}. \quad (A.25)$$

This expression represents a generalization of the cases shown in Equations (19) and (22).

✓ In the free trade equilibrium, $P$ is determined only by the product market-clearing conditions and these conditions are defined at the global level. Thus, the relevant supplies are given by Equations (27)-(30):

$$Y_{NS}^{FT} = P_j^{FT} \frac{1}{\beta - \alpha} (H_N P_j^{FT} (1 - \alpha) - L_N \alpha) / (\beta - \alpha), \quad (27)$$

$$Y_{Nu}^{FT} = P_j^{FT} \frac{1}{\beta - \alpha} (L_N \beta - H_N P_j^{FT} (1 - \beta)) / (\beta - \alpha), \quad (28)$$
\[ Y_{SS}^{FT} = \frac{P^{FT}}{\beta^a} \frac{1}{\beta^a} (H_s P^{FT} (1-\alpha) - L_s \alpha) / (\beta - \alpha), \quad (29) \]
\[ Y_{Su}^{FT} = \frac{P^{FT}}{\beta^a} (L_s \beta - H_s P^{FT}) (1 - \beta) / (\beta - \alpha). \quad (30) \]

The global supply of each good is obtained as the sum of supplies by Northern and Southern producers. Thus, the supply of the skilled-intensive good is given by the following expression:
\[ Y_{world}^{FT} = Y_{Ns}^{FT} + Y_{SS}^{FT}. \quad (A.24) \]

On the demand-side, the demand for each product is obtained as the sum of the demands for the good from Northern and Southern consumers. In the case of the skilled-intensive good, we can turn again to Equation (A.2) and write:
\[ c_{world}^{FT} (I_{world}^{FT}, P_{world}^{FT}) = \gamma I_{Ns}^{FT} + \gamma I_{SS}^{FT} = \gamma I_{world}^{FT}. \quad (A.25) \]

Equating the demand that appears in (A.24) and (A.25) yields the following equilibrium price:
\[ P^{FT} = \frac{L_w H_w}{\alpha \gamma (1 - \gamma) + \beta \gamma} \frac{1}{1 - \alpha (1 - \gamma) - \beta \gamma} \beta^a. \quad (A.26) \]

This is precisely the same expression that appears in Equation (31).

Appendix 6. Labor Market-Clearing Conditions: Case of Mutual Trade Restrictions

This appendix derives the labor market-clearing conditions for the scenario presented in Section 7. Note first that the demands for skilled and unskilled labor depends only the technology used for production and, therefore, the demands derived in Appendix 3 and subsequently used in Appendix 4 are still valid in the present appendix. In particular, the labor demands are given by the following expressions:
\[ H_{js}^{*} (Y_{js}, q_j, w_j) = \beta (\frac{w_j}{q_j})^{1-\beta} Y_{js}, \quad (A.7') \]
\[ H_{ju}^{*} (Y_{ju}, q_j, w_j) = \alpha (\frac{w_j}{q_j})^{1-\alpha} Y_{ju}, \quad (A.16) \]
\[ L_{js}^{*} (Y_{js}, q_j, w_j) = (1 - \beta) (\frac{w_j}{q_j})^{-\beta} Y_{js}, \quad (A.8') \]
\[ L_{ju}^*(\bar{Y}_{ju}, q_j, w_j) = (1 - \alpha) \left( \frac{w_j}{q_j} \right)^{-\alpha} \bar{Y}_{ju}. \]  
(A.18)

Given that in section 7 there is no migration, the supplies of skilled and unskilled labor remains unchanged relative to Appendix 4. Hence, we can still use the same expressions for the labor market clearing conditions:

\[
\beta \left( \frac{w_j}{q_j} \right)^{1-\beta} \bar{Y}_{js} + \alpha \left( \frac{w_j}{q_j} \right)^{1-\alpha} \bar{Y}_{ju} = H_j, \quad \text{(A.17)}
\]

\[
(1 - \beta) \left( \frac{w_j}{q_j} \right)^{-\beta} \bar{Y}_{js} + (1 - \alpha) \left( \frac{w_j}{q_j} \right)^{-\alpha} \bar{Y}_{ju} = L_j, \quad \text{(A.19)}
\]

All of the equations so far presented reveal that labor market clearing requires the same conditions as in the case of free trade. Nonetheless, unlike in that case, the solution of the regime presented in Section 7 must consider the skill-premia that arise from Equations (38)-(41). As noted in Section 7, these skill-premia are equal to \( \left( \frac{P_{MTTR}}{\tau} \right)^{\frac{1}{\beta - \alpha}} \) in North and equal to \( \left( \frac{P_{MTTR}}{\tau} \right)^{\frac{1}{\beta - \alpha}} \) in South. Using this information to solve for the system formed by (A.17) and (A.19) for North on the one hand and for South on the other hand, we obtain the following result:

\[
Y_{Ns}^{MTTR} = \left( \frac{P_{MTTR}}{\tau} \right)^{-\beta} \left( H_N \left( \frac{P_{MTTR}}{\tau} \right) \right)^{\frac{1}{\beta - \alpha}} (1 - \alpha) - L_N \alpha) / (\beta - \alpha) \quad \text{(A.27)}
\]

\[
Y_{Nu}^{MTTR} = \left( \frac{P_{MTTR}}{\tau} \right)^{-\alpha} \left( L_N \beta - H_N \left( \frac{P_{MTTR}}{\tau} \right) \right)^{\frac{1}{\beta - \alpha}} (1 - \beta) / (\beta - \alpha) \quad \text{(A.28)}
\]

\[
Y_{Ss}^{MTTR} = \left( \frac{P_{MTTR}}{\tau} \right)^{-\beta} \left( H_S \left( \frac{P_{MTTR}}{\tau} \right) \right)^{\frac{1}{\beta - \alpha}} (1 - \alpha) - L_S \alpha) / (\beta - \alpha) \quad \text{(A.29)}
\]

\[
Y_{Su}^{MTTR} = \left( \frac{P_{MTTR}}{\tau} \right)^{-\alpha} \left( L_S \beta - H_S \left( \frac{P_{MTTR}}{\tau} \right) \right)^{\frac{1}{\beta - \alpha}} (1 - \beta) / (\beta - \alpha) \quad \text{(A.30)}
\]

These are precisely the same expressions we have presented in Equation (46)-(49).

Appendix 7. Product Market-Clearing Condition: Case of Mutual Trade Restrictions

This appendix derives the equilibrium price of the skilled-intensive good in the mutual trade restrictions regime by solving the product market equilibrium conditions. The existence of an imports tariff of the iceberg form makes the setup of the product market clearing condition subtle. This setup must take into account that a fraction of an exported good is lost in transit.
and, therefore, consumption a good in the recipient country is smaller than its exports supply.
Hence, for the particular case of the skilled-intensive good, which is exported by North, market clearing requires:

\[ \frac{\gamma_{NS} MTR}{\gamma_{NS} P MTR} - \frac{\gamma_{SS} MTR}{\gamma_{SS} P MTR} = \tau \left( \frac{\gamma_{S} MTR}{\gamma_{S} P MTR} \right) \]  

(A.31)

Equation (A.31) states that the net supply of the skilled-intensive good by North must equal the net demand by South times the iceberg costs tariff. As for the different components in (A.31), both \( \gamma_{NS} MTR \) and \( \gamma_{SS} MTR \) have been calculated in Appendix 6 and are shown in (A.27) and (A.29). Thus, we only need to calculate the corresponding demands for the goods. Using the solution to the optimization problem shown in Appendix 1, it is known that the demand in North can be written as follows:

\[ \frac{\gamma_{NS} MTR}{\gamma_{NS} P MTR} = \frac{\gamma_{NS} MTR}{\gamma_{NS} P MTR} \]  

(A.32)

where the income level is given by the following expression:

\[ \frac{\gamma_{NS} MTR}{\gamma_{NS} P MTR} = \frac{\gamma_{NS} MTR}{\gamma_{NS} P MTR} + \gamma_{Nu} MTR = P^{MTR} \frac{1}{\beta-a} \frac{\beta}{\tau^B-\beta} \left( L_N + H_N P^{MTR} \frac{1}{\beta-a} \frac{1}{\tau^B-\beta} \right) \]  

(A.33)

where \( \gamma_{NS} MTR \) and \( \gamma_{Nu} MTR \) have been defined in Appendix 6. By the same token, the demand for the skilled-intensive in South is given by:

\[ \frac{\gamma_{SS} MTR}{\gamma_{SS} P MTR} = \frac{\gamma_{SS} MTR}{\gamma_{SS} P MTR} \]  

(A.34)

where the income level is given by the following expression:

\[ \frac{\gamma_{SS} MTR}{\gamma_{SS} P MTR} = \frac{\gamma_{SS} MTR}{\gamma_{SS} P MTR} + \gamma_{Su} MTR = P^{MTR} \frac{1}{\beta-a} \frac{\beta}{\tau^B-\beta} \left( L_S + H_S P^{MTR} \frac{1}{\beta-a} \frac{1}{\tau^B-\beta} \right) \]  

(A.35)

where \( \gamma_{SS} MTR \) and \( \gamma_{Su} MTR \) have been defined in Appendix 6.

Substituting for \( \frac{\gamma_{NS} MTR}{\gamma_{NS} P MTR} \) and \( \frac{\gamma_{SS} MTR}{\gamma_{SS} P MTR} \) in (A.31) with (A.32)-(A.35) we obtain the following result:

\[ P^{MTR} = \frac{(\alpha(1-\gamma) + \beta)(L_{N} + \tau L_{S})}{(1-\alpha(1-\gamma) - \beta)(H_{N} \tau^B - \beta + \tau^B H_{S})} \]  

(A.36)

This is precisely the expression shown in Equation (52).
Appendix 8. Tax Equivalence
This appendix shows the equivalence between the imports tax presented in Section 7 and the migration policy. As noted above, for that purpose, we need to find for each \( \tau \) a value of \( 1 - \phi \) that implements exactly the same relative price \( p_{MTR}^{MTR}/\tau \) and is, therefore, associated with the prices, real wages and welfare as in the import tax regime.

Let us first divide the expression for \( p_{MTR}^{MTR} \) in Equation (52) by \( \tau \) and write

\[
p_{MTR}^{MTR}/\tau = \left( \frac{(a(1-\gamma)+\beta y)(l_N+\tau l_S)\tau^{-\beta+a}}{(1-a(1-\gamma)-\beta y)(1+\tau l_S)} \right)^{\beta-a}
\]  

(A.37)

Let’s now look at the price in tax migration case by plugging (73) in the definition given in (69):

\[
p_{TM} = \left( \frac{(a(1-\gamma)+\beta y)}{(1-a(1-\gamma)-\beta y)} \right) \left( \frac{(1-\phi)}{(1-\phi_H/H_S)} \right) \left( \frac{l_W}{H_W} \right)^{\beta-a}
\]  

(A.38)

From these expressions, it is easy to see that to make \( p_{TM} \) equal to \( p_{MTR} \), it suffices to equate \( \frac{(l_N+\tau l_S)\tau^{-\beta+a}}{(1-a(1-\gamma)-\beta y)(1+\tau l_S)} \) and \( \frac{1}{(1-\phi_H/H_S)} \frac{l_W}{H_W} \). Solving for the value of \( \phi \) that produces this result, we obtain:

\[
\phi^{Eq} = \frac{\tau^\beta(2\tau^{-\beta}H_N+\tau H_S)l_W - \alpha H_W l_N^{MTR}}{\tau^\beta(2\tau^{-\beta}H_N+\tau H_S)l_W + \alpha H_S l_N^{MTR} + \tau L_{S}^{MTR}}
\]  

(A.39)

This is precisely the expression for the index of migration that has been stated in the main body text of Section 11. By imposing in this expression the feasibility constraints, we can write:

\[
\phi^{Eq} = \frac{\tau^\beta((2\tau^{-\beta}-\tau)H_N+\tau H_W)l_W - \alpha H_W l_N^{MTR}}{\tau^\beta((2\tau^{-\beta}-\tau)H_N+\tau H_W)l_W + \alpha H_S l_N^{MTR} + \tau L_{S}^{MTR}}
\]  

(A.39’)

This is precisely the expression for \( \phi^{Eq} \) that appears in Equation (74’). Taking the derivative of this expression with respect to \( L_N^{MTR} \), we find that:

\[
\frac{\partial \phi^{Eq}}{\partial L_N^{MTR}} = -\frac{\frac{2}{2} (1+\tau)^{\beta} \frac{\alpha + H_W l_N^{MTR}}{\alpha - H_N + \tau H_W l_L^{MTR}}}{(\tau^\beta((2\tau^{-\beta}-\tau)H_N+\tau H_W)l_W + \alpha H_S l_N^{MTR} + \tau L_{S}^{MTR})^2}
\]  

(A.40)
Note that the term \((-\tau + \frac{2}{\tau^u - \beta})H_N + \tau H_W\) is greater than zero given that \(H_W > H_N\) and \(\tau > -\tau + \frac{2}{\tau^u - \beta}\); this, in turn, implies that \(\frac{\partial \phi^E}{\partial L_N^{MT}} < 0\) and completes the proof.

Appendix 9. Political Economy Analysis
This appendix shows that the forward-looking and benevolent agent chooses a positive migration tax. Note first that the expected payoff of the benevolent government is equal to:

\[
\max_{\phi} EW(\phi) = f(\phi) W_N(\phi) + (1 - f(\phi)) W_N^{aut}
\]

The desirable outcome for the government is associated with the proposed policy \(\phi = 0\) while for the union the desirable outcome is realized when \(\phi > 0\). Recall the probability of not going to strike is \(f(\phi)\). The government determines the optimal level of \(\phi\) that maximizes the expected welfare:

Having this mind, we implement a proof that goes in two steps:

It is easy to prove this in two steps:

(a) Consider the first order conditions associated with the optimization problem:

\[
\frac{\partial EW(\phi=0)}{\partial \phi} = \frac{\partial f(\phi=0)}{\partial \phi} W_N(\phi = 0) + \frac{\partial W_N(\phi=0)}{\partial \phi} f(\phi = 0) - \frac{\partial f(\phi=0)}{\partial \phi} W_N^{aut}
\]

Note that \(f(\phi = 0), \frac{\partial f(\phi=0)}{\partial \phi}\) is infinite and \(W_N(\phi = 0) = W_N^{FT} > W_N^{aut}\); hence \(\frac{\partial EW(\phi=0)}{\partial \phi}\) always tends to infinite.

(b) Infinite is greater than \(W_N^{aut}\);

Put differently, a benevolent and forward-looking government always proposes a positive migration tax so that the probability that it gets accepted is positive and expected welfare is maximized.