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Abstract: We study the impact of adaptive learning for the design of a robust monetary policy using a small open-economy New Keynesian model. We find that slightly departing from rational expectations substantially changes the way the central bank deals with model misspecification. Learning induces an intertemporal trade-off for the central bank, i.e., stabilizing inflation (output gap) today or stabilizing it tomorrow. The central bank should optimally anchoring private agents expectations in the short term in exchange of easier future intratemporal trade-offs. Compared to the rational expectations equilibrium, the possibility to conduct robust monetary policy is limited in a small open economy under learning for any exchange rate pass-through level and any degree of trade openness. The misspecification that can be introduced into all equations of the model is lower in a small open economy, and approaches zero at high speed as the learning gain rises.

Keywords: Robust control, model uncertainty, adaptive learning, small open economy.
JEL Classification: C62, D83, D84, E52, E58

Resumen: Se analiza el impacto del aprendizaje adaptativo en el diseño de una política monetaria robusta con un modelo Nuevo Keynesiano de economía pequeña y abierta. Se encuentra que alejarse ligeramente de las expectativas racionales podría cambiar sustancialmente la forma en que el banco central se ajusta ante un error de especificación del modelo. El aprendizaje induce una disyuntiva inter-temporal para el banco central, es decir, estabilizar la inflación (brecha del producto) en el periodo actual o en el siguiente. El banco central debe anclar de manera óptima las expectativas privadas a corto plazo a cambio de disyuntivas intra-temporales futuras más sencillas. En comparación con el equilibrio de expectativas racionales, la posibilidad de llevar a cabo una política monetaria robusta es limitada bajo aprendizaje en una economía pequeña y abierta para cualquier nivel de traspaso cambiario y cualquier grado de apertura comercial. El error de especificación del modelo que se introduce es menor para una economía pequeña y abierta, y se aproxima a cero rápidamente a medida que aumenta la ganancia del aprendizaje.

Palabras Clave: Política monetaria robusta, economía pequeña y abierta, aprendizaje adaptativo, incertidumbre.

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1   Introduction

The intensive research on monetary policy using New Keynesian models since late 1970s has resulted in a so-called "science of monetary policy" (Clarida, Galí and Gertler 1999). It identifies a set of principles and prescriptions central to the design and implementation of monetary policy under the assumption of perfect information and full rationality. In particular, by adopting inflation targeting monetary policy is conducted with transparency and credibility, and is correctly understood by the public, giving the central bank (CB) the ability to manage private expectations.

Since the Global Financial Crisis, many question the efficacy of monetary policy based on these principles and prescriptions, given that the CB has to conduct policy in a rapidly evolving macroeconomic environment affected by many national and international disturbances. As Bernanke (2007) stresses, the traditional rational expectations (RE) model is less helpful for thinking about an economy whose structure is constantly evolving in ways that may take time to be understood by by both the public and policymakers. Consequently, the CB should include the latter in its own decision making, along with the fact that private agents may not fully grasp how the complex economy works as a whole and what is the objective function of policy makers. In this paper, we study the role of limited rationality on the conduct of robust monetary policy under discretion in a small open economy under two calibrations for an advanced economy and an emerging market economy.

Due to such imperfect knowledge, the monetary policy designed under the assumption that all agents perfectly know the structure of the economy might perform sub-optimally in practice. When designing optimal policy, the CB should take into account the issues arising from imperfect knowledge, such as model uncertainty and the learning behavior of private agents. As highlighted by Schmidt-Hebbel and Walsh (2009), a key lesson learned from recent research on monetary policy is that neither uncertainty nor learning can be ignored.

With regard to model uncertainty, the robust control approach introduced by Hansen and Sargent (2001, 2003, 2007) gives the tools to design policy that would be robust to plausible deviations from the benchmark New Keynesian model. Without a complete description of reality, central bankers are more inclined to base policy on principles that remain valid even if the model's assumptions are demanding. The central bank cannot formulate a probability distribution over the full set of realistic models, and thus design a robust policy that would respond to the worst possible outcome within a pre-specified set of models (Leitemo and Söderström 2008b). Said differently, the worst-case outcome corresponds to a situation where a malevolent agent (Nature) chooses model misspecification to be as damaging

as possible, while the CB's policy rule and private agents' expectations reflect this misspecification. Generally, optimal interest rate policy is more aggressive to deviations from the inflation target under the robust control approach in both closed and open economy (Leitemo and Söderström 2008a, b). This approach is related to (but different from) the seminal contribution of Brainard (1967) who considers the impact of parameter uncertainty and advocates that the CB should be cautious by using less intensively each policy instrument following an attenuation principle.[1]

Models of learning dynamics are particularly well suited for assessing whether CBs can influence expectations with as much control as previously thought (Eusepi and Preston 2018a). Replacing the RE hypothesis by the assumption that private agents use adaptive learning algorithms, among others, better reflects the idea that they have imperfect knowledge of the economy and are subject to limited rationality. Moreover, model uncertainty makes it even more difficult for private agents to forecast how economic variables evolve, providing thus another strong rationale for learning behavior. Such an alternative approach to expectations would lead to results that challenge conventional wisdoms in the theory of monetary policy obtained under the RE hypothesis (Woodford 2013). Notably, when private agents are learning, the optimal policy designed with the RE hypothesis could perform poorly in steering inflation towards the target (Milani 2008, and Orphanides and Williams 2008). A number of studies demonstrate that beliefs based on learning are not only relevant for explaining patterns in standard macroeconomic time series, but are also consistent with expectations survey data characterized by low-frequency drift.[2] In particular, it is acknowledged that DSGE models with adaptive learning outperform those with the RE hypothesis in terms of fitting the data (Slobodyan and Wouters 2012, Ormeño and Molnár 2015).

Under learning, beliefs become state variables and represent a new constraint on what can be achieved by monetary policy. Therefore, optimal policy response to private expectations can improve economic outcomes by inducing agents to revise their beliefs, and this interaction can generate further movements in output and inflation (Molnár and Santoro 2014,

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[1]The attenuation principle is also named the "conservatism principle" by Blinder (1998). According to this principle, the CB has to be cautious based on the fact that the choice of a policy instrument can have more severe consequences than in the absence of parameter uncertainty. Researchers who study robust monetary policy have reversed the meaning of "cautious" so that "being cautious or precautionary" signifies "to do more". In other words, the CB struggles to avoid worst outcomes by responding more aggressively to shocks (Söderström 2002, Gianonni 2007).

[2]Empirical evidence is provided by, among others, Branch and Evans (2006), Milani (2007, 2011), Malmendier and Nagel (2016), and Eusepi and Preston (2011, 2018b). According to this literature, consumers and firms react sluggishly to persistent shifts in inflation (Trehan 2011, Trehan and Lynch 2013) while financial operators are slow to react to news (Altavilla, Giannone and Modugno 2017), suggesting that they slowly adapt their forecast.

Eusepi and Preston 2018a). In response to the intertemporal trade-off generated by this kind of interactions, the CB's optimal policy should be more aggressive in fighting inflation than what is standard under RE. Monetary policy decisions taking account of learning substantially reduce social loss compared to the ones made by assuming RE even when agents are learning, giving thus a rationale for the CB to monitor private expectations (Molnár and Santoro 2014). To do that, the CB acts more aggressively in the present period, through an optimal path-dependent policy, than it would under standard RE. However, learning may generate inflation persistence, which is suboptimal, during the transition to the steady-state equilibrium compared to the equilibrium under discretion and RE. According to André and Dai (2017), to deal with this, the CB should be less conservative in the sense of Rogoff (1985) than society, under discretionary monetary policy. André and Dai (2018) examine in a closed economy the design of optimal robust monetary policy when private agents are learning, and find that learning significantly limits the possibility for the CB to conduct robust policy compared to RE.

This paper studies optimal robust discretionary monetary policy in a small open-economy New Keynesian model with private agents using adaptive learning algorithm to form expectations. The aim of the paper is to study how adaptive learning affects optimal robust monetary policy in the worst-case small open economy model. The robust control approach focuses on the worst-case scenario within a set of admissible models as economic agents are not able to attach probabilities to all plausible outcomes, which is translated into misspecification in the Phillips curve in the closed economy. In the open economy, the CB is aware of possible misspecification not only in the Phillips curve but also in the IS equation and the uncovered interest rate parity (UIP) because all shocks affect the real exchange rate and hence, via its presence in the Phillips curve, the intratemporal inflation-output gap trade-off. Consequently, the equilibrium values of inflation, the output gap, the exchange rate and the interest rate depend on all shocks and model misspecification in all equations. The extent to which the CB is aware of possible misspecification in each equation could be significantly affected by a slight departure from RE. The intertemporal trade-off due to learning incentivizes the CB to curtail the intratemporal trade-off between inflation and the output gap in the current period with the aim of better anchoring private expectations and improving future intratemporal trade-offs. Learning or the robust control considered alone lead the CB to conduct a more aggressive interest rate policy in the short run towards inflation compared to the case under RE. The direct effect of considering a robust control policy under learning is that the inertia in endogenous variables, introduced by private agents' learning, calls for anchoring private expectations and imposing an additional constraint on the design of optimal robust monetary policy.

Our main results are: 1) Compared to the RE equilibrium, the possibility to conduct robust monetary policy faces additional constraints in the open economy when private agents are learning, which greatly limit the set of worst-case scenarios as the learning gain increases. 2) Adaptive learning makes robust monetary policy more (less) aggressive compared to RE in its response to cost-push and demand (exchange-rate) shocks for an advanced open-economy calibration. 3) The misspecification that can be introduced into all equations of the model is very small and approaches zero at high speed as the learning gain rises. 4) A change in the CB's focus in favor for robustness, against model misspecification, has no significant impact on the feedback coefficients in the laws of motion of the economy once this focus is bounded to ensure the determinacy of the equilibrium. The results are valid for a representative advanced economy following the calibrations of Leitemo and Söderström (2008b). We perform robustness checks for an emerging economy, represented by Mexico, confirming our results.

The small open-economy New Keynesian model used in this paper is based on Galí and Monacelli (2005) and Leitemo and Söderström (2008b). Regarding the assumptions used and the main results obtained, our paper is closely related to two strands of literature. The first is the literature on optimal robust monetary policy under RE and the second studies the implications of adaptive learning for the design of monetary policy.

The robust control approach adopted in this paper considers additive model misspecification. Taking worst-case scenarios for the economy into consideration, the CB's response tends to amplify, rather than attenuate, when facing shocks in a closed economy (e.g., Giannoni and Woodford 2002, Onatski and Stock 2002, Giordani and Söderlind 2004, Leitemo and Söderström 2008a, Gonzalez and Rodriguez 2013). By being more aggressive against inflation, robust monetary policy is able to avoid particularly sub-optimal outcomes but at the cost of inducing some inflation persistence under optimal robust control under RE (Qin, Sidiropoulos and Spyromitros 2013). With misspecification of the Phillips curve, (Dai and Spyromitros 2010), and being aware of possible misspecification in the true degree of shock persistence or the potential output, the CB should be more hawkish (Tillmann 2009, 2014).

An alternative approach to robustness is to consider multiplicative Knightian uncertainty implying that the uncertainty is located in one or more specific parameters of the model, and the true values of these parameters are bounded between minimum and maximum plausible values (Giannoni 2002, 2007, Onatski and Stock 2002, Tetlow and von zur Muehlen 2004, Tetlow 2019). Numerical simulations show that under parameter uncertainty, the robust interest rate rule generally responds more strongly to changes in inflation and the output gap, with greater inertia than in the absence of such uncertainty, invalidating thus the Brainard attenuation principle. The CB is less cautious in the sense of Brainard (1967) but actually

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more cautious in the sense commonly used in the robust control literature by conducting a policy that is more aggressive towards inflation, as it is more averse to worst-case scenarios.

Our paper contributes to a large and growing learning literature, of which most has focused on evaluating the robustness of RE policy prescriptions to learning dynamics.[3] It shares with a number of studies in the monetary policy literature the assumption of adaptive learning. When policy is conducted through exogenous Taylor rules, adaptive learning helps selecting among all possible equilibria obtained under RE, and in this sense it can be viewed as a process that converges towards RE equilibrium (Bullard and Mitra 2002, Evans and Honkapohja 2003, 2006, Machado 2013, Airaudo, Nisticò and Zanna 2015). Furthermore, adaptive learning solves the disinflationary-booms anomaly in the New Keynesian model under RE (Moore 2016).

In particular, our paper is closely linked to a few recent studies that have paid attention to the design of optimal policy conditional on belief structures based on learning (Gaspar, Smets and Vestin 2006, 2010, Molnár and Santoro 2014, André and Dai 2017, 2018, Eusepi, Giannoni, and Preston 2018, Mele, Molnár and Santoro 2019). These studies explore how learning affects the design of optimal policy compared to RE and provide insights on the constraints that non-rational belief structures place on optimal monetary policy. Such constraints induce a fundamental change: since beliefs based on learning are state variables, there is no distinction between commitment and discretion under learning. As Molnár and Santoro (2014), Eusepi and Preston (2018a), and Eusepi, Giannoni, and Preston (2018) point out, monetary policy anchoring private agents' beliefs that evolve with learning improves equilibrium outcomes. Nevertheless, internalizing the effects of such beliefs on policymaker's actions induces drifts into such beliefs through a feedback mechanism and can thus amplify fluctuations in output and inflation during the transition to the steady state. This might reduce the space for dynamic stability when monetary policy should be more aggressive due to the adoption of the robust control technique (André and Dai 2018).

The remainder of the paper is structured as follows. Section 2 outlines the model. Section 3 derives equilibrium solutions under RE. Section 4 explores the effects of constant-gain learning on robust monetary policy and the equilibrium. Section 5 develops a robustness check for a different type of economy, i.e. an emerging economy. Section 6 concludes.

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[3]See, for surveys, Evans and Honkapohja (2009), Woodford (2013) and Eusepi and Preston (2018a).

2 The Model

We use a New Keynesian model of a small open economy similar to the one derived by Galí and Monacelli (2005) and Leitemo and Söderström (2008b) as a baseline.[4] Following Leitemo and Söderström, we add a time-varying premium on foreign bond holdings and consider the robust control problem of the CB that conducts policy under discretion and is aware of possible misspecification, in the terminology of Hansen and Sargent (2001), in all structural model equations. Notice that the time-varying risk premium represents an important source of uncertainty in open economies and its introduction enables analyzing misspecification (specification errors) in the UIP condition. The implications of private agents' adaptive learning for the design of discretionary monetary policy are derived using the approach of Molnár and Santoro (2014).

2.1 The structural equations

The small domestic country freely trades with the rest of the world (foreign country), constituted of a continuum of foreign economies. We assume that foreign and domestic countries share preferences and technology. Domestic and foreign firms produce tradeable consumption goods, using labor as the sole input. Households derive their utility from consuming both domestic and foreign goods, and have a marginal decreasing disutility in labor supply to firms.

Denote by e_t the log-linearized real exchange rate (units of domestic currency against one unit of foreign currency). We have by definition

$$e_t = s_t + p_t^f - p_t, \tag{1}$$

with s_t being the nominal exchange rate, p_t^f the price level of the goods produced in the foreign country and p_t the price level of domestically produced goods.

The real exchange rate is directly related to the inflation rate in the domestic goods sector, π_t, via the New Keynesian Phillips curve:[5]

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[4]By assuming complete international financial markets, Galí and Monacelli make their small open economy New Keynesian model isomorphic to a closed economy New Keynesian model with assets in zero net supply (e.g. no government bonds), so that the open-economy Phillips curve is independent of the real exchange rate. This isomorphism is not present in Leitemo and Söderström.

[5]To the difference of Galí and Monacelli (2005), Leitemo and Söderström (2008b) derive a Phillips curve including the real exchange rate. For the microfoundations of the model, see Leitemo and Söderström (2008b). Notice that π_t is different from the inflation rate of the consumer price index that also takes into account the inflation of foreign goods consumed by residents. In the closed economy, π_t represents both producer and

$$\pi_t = \beta E_t^* \pi_{t+1} + \kappa x_t + \phi e_t + h_t^\pi + \varepsilon_t^\pi, \tag{2}$$

where x_t denotes the output gap representing the log deviation of the flexible-price equilibrium level of domestic output from the steady-state output, $0 < \beta < 1$ the discount factor, and E_t^* the expectation operator with the asterisk signaling that private agents may use learning algorithm. The composite parameter κ is the output-gap elasticity of inflation and encompasses the effect of the output gap on inflation via real marginal costs. It is defined as $\kappa = \hat{\kappa}(\eta + \sigma)$, where $\hat{\kappa} \equiv \frac{(1-\vartheta)(1-\vartheta\beta)}{\vartheta}$, η represents the steady-state Frisch elasticity of labor supply, and $\sigma \equiv \frac{\hat{\sigma}}{1-\omega}$ with $\hat{\sigma}$ denoting the inverse of the elasticity of intertemporal substitution and $0 \le \omega \le 1$ the share of foreign goods in domestic consumption. The composite parameter κ crucially depends on ϑ, the share of firms that do not optimally adjust but simply update in period t their previous price by the steady-state inflation rate. The real exchange rate affects the Phillips curve with a composite coefficient $\phi = -\omega\hat{\kappa}[(2-\omega)\zeta\sigma - 1]$, where ζ stands for the elasticity of substitution across domestic and foreign goods.[6] Indeed, when households choose labor supply, they care about the purchasing power of their wage deflated by the consumer price index that also includes prices of imported goods, implying that the equilibrium wage and hence the real marginal costs depend on the real exchange rate. The noise $\varepsilon_t^\pi \sim \mathcal{N}(0, \sigma_\pi^2)$ is an *i.i.d.* cost-push shock. The term h_t^π represents the misspecification in the Phillips curve being defined below by (9).

The New Keynesian IS equation is given by

$$x_t = E_t^* x_{t+1} - \sigma^{-1}(r_t - E_t^* \pi_{t+1}) - \delta\left(E_t^* e_{t+1} - e_t\right) + h_t^x + \varepsilon_t^x, \tag{3}$$

where r_t is the nominal short-term interest rate, and δ a composite parameter defined by $\delta \equiv \frac{1}{\sigma}\left[\frac{\Omega}{(1-\omega)} - 1\right]$ with $\Omega \equiv (1-\omega)[(1-\omega) + (2-\omega)\omega\zeta\sigma]$. The composite parameter δ is the elasticity of the output gap with respect to the expected change in the real exchange rate, reflecting the substitution effect induced by such a change on the demand of domestically produced goods.[7] We introduce an *i.i.d.* demand shock $\varepsilon_t^x \sim \mathcal{N}(0, \sigma_x^2)$ and a term h_t^x denoting the misspecification in the IS equation, below defined by (9).

The real UIP condition relates the real interest rate differential with the expected rate of

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consumer price inflation rates.

[6]The composite parameter ϕ is negative as long as $(2-\omega)\zeta\sigma > 1$. The latter is generally true according to Leitemo and Söderström (2008b).

[7]Note that Ω and δ are positive for $(2-\omega)\zeta\sigma > 1$.

real depreciation:

$$r_t - E_t^* \pi_{t+1} = E_t^* e_{t+1} - e_t + h_t^e + \varepsilon_t^e, \tag{4}$$

where foreign variables are set to zero; h_t^e denotes the misspecification in the UIP equation below defined by (9), and $\varepsilon_t^e \sim \mathcal{N}(0, \sigma_e^2)$ an *i.i.d.* real exchange rate disturbance. A positive ε_t^e means that investors require a positive risk premium on domestic bonds compared to foreign bonds.

Here, we consider the worst-case model where the CB sets the interest rate to minimize its loss function while a fictitious malevolent agent in the sense of Hansen and Sargent (2007) selects the level of specification errors to maximize the CB's loss.[8] Such an agent represents the policy maker's biggest challenge about model misspecification. The worst-case scenario is the outcome that the CB is most averse to, and against which the CB conducts robust policy. The model misspecification cannot arise independently of random noises that affect model equations and are positively dependent on the variance of such noises (Giordani and Söderlind 2004). This is because if the variance of the disturbance in one equation was null, then the misspecification would be detected at once. Therefore, the larger the variance of the disturbance is, the larger the specification error that cannot be detected.

The presence of the real exchange rate in the Phillips curve is the key factor that differentiates open from closed economies and hence their transmission mechanism of monetary policy. In the present model, movements in the exchange rate negatively affect inflation, for a given output gap and for an advanced economy, according to Leitemo and Söderström (2008b).

On the one hand, an increase in the exchange rate (depreciation of domestic currency) raises domestic consumer prices and reduces the real wage, for a given nominal wage. Given households' marginal rate of substitution between leisure and consumption, this incites households to supply less labor and enjoy more leisure. Firms must increase the real wage to offset the reduction in the households' income, leading to higher marginal cost and inflation.

On the other hand, the depreciation increases the relative price of foreign goods in terms of domestic goods and the resulting substitution effect leads to a decrease in the demand for imports and, therefore, a reduction in aggregate consumption due to imperfect substitution between domestic and foreign goods, given the level of output. The marginal rate of substitution between leisure and consumption then falls, leading to lower real wages and marginal cost. This induces domestic firms to hire more labor to increase production while decreasing

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[8]An alternative approach is to consider the 'approximating model' (Hansen and Sargent 2007) postulating that while the policy rule and agents' expectations reflect the CB's focus on robustness, there is no model misspecification in the reference model that turns out to be correct.

product prices.

Under the condition $(2 - \omega)\zeta\sigma > 1$, which is typically verified according to Leitemo and Söderström (2008b) for an advanced economy, the second effect dominates so that the total effect of an increase in the real exchange rate on inflation is negative, and aggregate demand decreases.[9]

The fact that the monetary transmission mechanism in the small open economy is qualitatively different from that of a closed economy does not always hold. In Galí and Monacelli (2005), the open-economy model with perfect international risk sharing and a Phillips curve that does not incorporate the real exchange rate is isomorphic to the closed economy, so that all closed-economy results are qualitatively similar to those in the open economy. Following Leitemo and Söderström (2008b), our framework keeps the real exchange rate in the Phillips curve and assumes imperfect access to international capital markets. These features imply that both, demand shocks in the IS equation and the risk premium shocks on foreign bond holdings, become new sources of macroeconomic volatility that are absent in the closed economy.[10] Said features break the isomorphism result and justify the study of robustness against misspecification in both the IS equation and the UIP condition besides the misspecification in the Phillips curve.

2.2 Learning rules of private agents

Given the complexity and the uncertainty that characterize the economy, it is hard for private agents to know the actual law of motion (ALM) for inflation, the output gap, and the exchange rate such that they learn their evolution using an algorithm.[11] Consequently, they recursively estimate a Perceived Law of Motion (PLM), i.e., a noisy steady state in the terminology of Evans and Honkapohja (2012), which is consistent with the law of motion followed by the CB under RE. By believing that the steady-state levels of endogenous variables only depend on exogenous shocks, private agents perceive their expected levels as constant and act as if the conditional and unconditional expectations of these variables were identical. This justifies why private agents estimate these variables via sample means.

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[9]For a discussion on the effects of the sign for the exchange rate pass-through to inflation, see section 5.

[10]In the closed economy, demand shocks do not contribute to macroeconomic volatility when the CB conducts optimal policy since their effect on the aggregate demand is fully offset by an adequate change in the policy interest rate. This is impossible in the present open-economy model since such shocks affect the trade-off between inflation and the output gap through the presence of the exchange rate in the Phillips curve.

[11]The modern literature on learning algorithms was pioneered by Marcet and Sargent (1989) who study the convergence to RE equilibrium when private agents form expectations using least-squares learning. For a survey of the literature, see Evans and Honkapohja (2012).

Private agents form their expectations using the following learning algorithms (Marcet and Nicolini 2003, Molnár and Santoro 2014):

$$E_t^* \pi_{t+1} \quad \equiv \quad a_t = a_{t-1} + \gamma_t(\pi_{t-1} - a_{t-1}), \tag{5}$$

$$E_t^* x_{t+1} \quad \equiv \quad b_t = b_{t-1} + \gamma_t(x_{t-1} - b_{t-1}), \tag{6}$$

$$E_t^* e_{t+1} \quad \equiv \quad z_t = z_{t-1} + \gamma_t(e_{t-1} - z_{t-1}), \tag{7}$$

where $\gamma_t \in (0,1)$ is the learning gain that is assumed to be constant henceforth, i.e. $\gamma_{t+1} = \gamma_t = \gamma$.[12] It corresponds to the speed at which new data are integrated into current expectations. Equations (5)-(7) are the correct PLMs for private agents if the CB has no credibility. They are consistent with the CB's optimal policy under RE and discretion. These learning algorithms establish a positive relationship between a variable's expectation and its last period value. Given that the last period value of a variable depends on past shocks, these algorithms make the expectations of endogenous variables dependent on all past shocks.

We apply learning algorithms (5)-(7) to the linearized structural equations obtained under RE. This Euler-equation approach, attractive for its analytical tractability, is used notably by Molnár and Santoro (2014). Under this approach, spending and pricing decisions are only determined by one-period-ahead expectations. Nevertheless, the Euler-equation approach ignores the complex issues raised by the fact that under learning, optimizing private agents are required to take into account possible future revisions to beliefs in their current decisions process. The learning literature addresses this problem by taking the "anticipated utility" approach to individual optimization (Kreps 1998, Preston 2005, and Woodford 2013).[13]

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[12] An alternative approach is to assume that γ_t is decreasing over time. Compared to decreasing-gain learning, constant-gain learning generally buys tractability of the model.

[13] According to Molnár and Santoro (2014), optimal policy obtained with the Euler-equation approach achieves divine coincidence, i.e., the complete stabilization of disturbances to the IS equation in a closed economy. This result is a special case in the anticipated-utility framework (Eusepi and Preston 2018).

2.3 Monetary policy objectives

The CB shares the preferences for inflation and output-gap stabilization with society, whose expected loss function is given by:[14]

$$L_t^s = \frac{1}{2} E_t \sum_{i=0}^{+\infty} \beta^i \left(\pi_{t+i}^2 + \alpha x_{t+i}^2 \right), \tag{8}$$

where $\alpha > 0$ denotes the relative weight assigned to the objective of stabilizing the output gap. For simplicity, we assume that inflation and output-gap targets are both zero. Without the overly ambitious output-gap target that is common in the Barro-Gordon framework, the discretionary monetary policy set with the aim of minimizing social loss (8) would avoid an average inflation bias when private agents form RE.

Optimal monetary policy results from a sequential Nash game between the CB conducting robust policy to minimize the social loss and the malevolent agent (or nature) who sets the level of model misspecification to maximize the social loss.[15]

Given the model misspecification set by the malevolent agent, the CB designs the robust discretionary policy for the worst possible model within a given set of plausible models. The CB allocates a budget χ_j^2, $j = \pi, x, e$, to the malevolent agent, for the misspecification to be created in the Phillips curve, the IS equation and the UIP condition, respectively. This budget means that the specification errors, h_t^j, are finite.

The specification errors, h_t^j, with $j = \pi, x, e$, monitored by the malevolent agent are subject to the following budget constraints:

$$E_t \sum_{i=0}^{+\infty} \beta^t \left(h_{t+i}^j \right)^2 \le \chi_j^2, \; j = \pi, x, e. \tag{9}$$

In the absence of robust control, $\chi_j = 0$ for all j.

Under discretion, the CB designs a robust policy that takes into account not only different shocks affecting the economy but also model misspecification. The optimal robust monetary

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[14]This type of objective function is commonly used to characterize inflation-targeting policy in small open economies. Woodford (2003) demonstrate that a welfare loss function based on a second-order approximation of the representative consumer's utility loss has a similar form.

[15]Alternatively, the CB and the malevolent agent can play a Stackelberg game with the first acting as a Stackelberg leader. Notice that if the malevolent agent is the Stackelberg leader, the CB could adjust its policy according to the scenario designed by the malevolent agent (Hansen and Sargent, 2003). It results that the approach in terms of model misspecification would lose its interest.

policy is obtained by solving the min-max problem:

$$\min_{\pi_t, x_t, e_t, r_t} \max_{h_t^j} L_t^{CB} = \frac{1}{2} E_t \sum_{i=0}^{+\infty} \beta^i \left(\pi_{t+i}^2 + \alpha x_{t+i}^2 - \theta^\pi h_{t+i}^{\pi}{}^2 - \theta^x h_{t+i}^{x}{}^2 - \theta^e h_{t+i}^{e}{}^2 \right), \qquad (10)$$

subject to the misspecified structural equations (2)-(4), and the malevolent agent's budget constraints (9). The penalty parameter $\theta^j > 0$, with $j = \pi, x, e$, controls the CB's focus in favor for robustness. The higher θ^j are, the lower the focus in favor for model robustness. The specification errors h_t^j, with $j = \pi, x, e$, are inversely proportional to θ^j. The absence of concern regarding robustness corresponds to the case where $\theta^j \rightarrow \infty$, implying that $h_t^j \rightarrow 0$. In the following, we assume for simplicity that the malevolent agent's budget constraints (9) are not binding.

The CB is assumed to form RE and knows the reference model of the economy. This assumption makes it possible on the one hand, to highlight the degree to which imperfect knowledge on the part of households and firms constrains what can be achieved by the CB, and on the other hand, to avoid a more difficult control problem for the CB (Eusepi and Preston 2018). Moreover, assuming that the CB forms RE is supported by the fact that CBs in real world dedicate significant effort and human capital investment to understand the economic environment and hence are better informed than private agents (Molnár and Santoro 2014).[16]

3 Benchmark Equilibrium with Rational Expectations

We shortly summarize, with some slight changes in notation, the equilibrium solution of Leitemo and Söderström (2008b) in the worst-case scenario. This benchmark allows examining the effects of adaptive learning on the equilibrium.

3.1 Optimal monetary policy decisions

Solving the min-max problem of the CB (10), subject to the constraints represented by (2)-(4), yields the first-order conditions that are arranged to obtain the intratemporal trade-off

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[16]To check the robustness of their results, Molnár and Santoro (2014) also consider the case where the CB is uncertain about the nature of agents' beliefs in the spirit of Hansen and Sargent's (2007) min-max approach but does not care about misspecification in the structural equations of the model.

condition (optimal targeting rule) and equations that relate all specification errors to inflation:

$$x_t = -\Gamma \pi_t, \tag{11}$$

$$h_t^\pi = \frac{1}{\theta^\pi} \pi_t, \tag{12}$$

$$h_t^x = \frac{\sigma \phi}{\theta^x (1 + \sigma \delta)} \pi_t, \tag{13}$$

$$h_t^e = -\frac{\phi}{\theta^e (1 + \sigma \delta)} \pi_t. \tag{14}$$

where $\Gamma \equiv \frac{\kappa}{\alpha} + \frac{\sigma \phi}{\alpha (1 + \sigma \delta)} > 0$, so that the openness of the economy does not change the sign of the Phillips curve's slope but only makes it less steep (Leitemo and Söderström 2008b).

The open economy feature implies that there is an "exchange rate channel" that attenuates the effects of monetary policy through the demand channel. It creates for the CB a more difficult intratemporal trade-off compared to the closed economy. The optimal targeting rule (11) gives rise to a "leaning against the wind" policy, meaning that the CB should aim to contract demand below capacity to disinflate the economy when the output gap is above its target. Such a rule is independent of the CB's focus in favor for robustness.

However, monetary policy robust against model misspecification is obtained at the cost of higher output-gap volatility. As shown by (12)-(14), the absolute value of specification errors increases with π_t. These specification errors, in turn, tend to push inflation even further away and thus force the CB to accept greater variations in the output gap. The feedback loop between the policy interest rate and the exchange rate, which enters the Phillips curve, prevents the CB from entirely offsetting demand and exchange-rate disturbances. This makes the CB being averse to misspecification in both IS and UIP equations, additionally to its aversion to misspecification in the Phillips curve. Thus, in the open economy, the CB cannot aim to set h_t^x (hence h_t^e) to zero under optimal robust monetary policy as in the closed economy. Notice that with an increase in the interest-rate elasticity of the demand for domestic goods (smaller σ), the CB can more easily conduct policy to offset specification errors in the IS equation and hence worries less about h_t^x, regardless of the consequence of raising misspecification in the UIP equation. In contrast, a stronger effect of the exchange rate on inflation (larger ϕ) or a smaller exchange-rate elasticity of the demand for domestic goods (smaller δ) raises the costs of specification errors in both IS and UIP equations. The latter implies that the interest rate movements aiming at offsetting these errors, have a stronger direct effect on inflation and hence the CB worries more about misspecification in these equations.

The robust monetary policy with the above characteristics could be implemented through

an optimal interest rate obtained using (4), (11) and (13)-(14) to eliminate x_t and h_t^x in (3)

$$r_t = \sigma[\Gamma + \frac{\sigma\phi}{\theta^x(1+\sigma\delta)}]\pi_t + (1-\sigma\Gamma)E_t\pi_{t+1} - \sigma\delta(E_t e_{t+1} - e_t) + \sigma\varepsilon_t^x. \tag{15}$$

Equation (15) is an optimal implicit instrument rule in the terminology of Giannoni and Woodford (2002). An increase in the CB's focus on robustness in the IS equation (a decrease in θ^x) implies a more aggressive response to inflation. The CB's focus on robustness in the UIP equation affects the policy interest rate through the term $(E_t e_{t+1} - e_t)$, which negatively (positively) depends on h_t^e (θ^e).

3.2 The equilibrium of the worst-case model

The equilibrium of the worst-case model could be obtained by solving the system of equations (2), (4), and (11)-(15). The state variables are the shocks ε_t^π, ε_t^x and ε_t^e. The solution of the worst-case model through the method of undetermined coefficients (McCallum1983) is assumed to be a function of state variables:[17]

$$
\begin{bmatrix} \pi_t \\ x_t \\ e_t \\ r_t \\ h_t^\pi \\ h_t^x \\ h_t^e \end{bmatrix}
=
\begin{bmatrix} d_\pi^{RE} & d_x^{RE} & d_e^{RE} \\ k_\pi^{RE} & k_x^{RE} & k_e^{RE} \\ j_\pi^{RE} & j_x^{RE} & j_e^{RE} \\ m_\pi^{RE} & m_x^{RE} & m_e^{RE} \\ \hat{d}_\pi^{RE} & \hat{d}_x^{RE} & \hat{d}_e^{RE} \\ \widehat{k}_\pi^{RE} & \hat{k}_x^{RE} & \widehat{k}_e^{RE} \\ \hat{j}_\pi^{RE} & \hat{j}_x^{RE} & \hat{j}_e^{RE} \end{bmatrix}
\begin{bmatrix} \varepsilon_t^\pi \\ \varepsilon_t^x \\ \varepsilon_t^e \end{bmatrix}. \tag{16}
$$

The main results derived by Leitemo and Söderström (2008b) under RE and discretion for the worst-case model can be summarized as follows. In the worst-case model, a stronger focus on robustness (against misspecification in any equation) increases the sensitivity of inflation, the output gap, and the exchange rate to all shocks as evidenced in Appendix A.1 by equations (A.13)-(A.15).[18] This result arises from the CB's being more averse to misspecification and to more volatile inflation, output gap, and exchange rate than in the reference model, i.e. the model without any misspecification. This result stands in contrast to the one in the closed-economy model.

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[17]For the detailed solution, see Appendix A.1.

[18]Differentiating (A.13)-(A.15) with respect to θ^j, with $j = \pi, x, e$ leads immediately to this result. See Proposition 1 of Leitemo and Söderström (2008b).

Using (13)-(14) it is straightforward to deduce that, for $\phi = 0$, one has $h_t^x = h_t^e = 0$ but $h_t^\pi \neq 0$ for $\pi_t \neq 0$. In the closed economy (equivalent to $\phi = 0$), the CB that desires to conduct robust policy worries that only inflation is more volatile than in the reference model, but not that the output gap is more volatile. This is because demand shocks do not alter the trade-off for the CB in the closed economy, since the policy interest rate can be used as intensively as it is necessary to fully offset the effects of such shocks without affecting the CB's loss.

In the open economy, the presence of the real exchange rate in the Phillips curve implies that all shocks worsen the trade-off for monetary policy, that is stabilizing inflation or the output gap in the same time period, inducing the CB to worry that all variables become more volatile compared to the reference model. Therefore, in the open economy, the CB should be more cautious than in the closed economy, and hence more aggressive towards inflation when setting the policy interest rate according to the solution of r_t, given by (16).[19]

4   The ALMs under Learning

This section studies how constant-gain learning interacts with the conduct of robust monetary policy and affects the equilibrium compared to the misspecified benchmark model where private agents form RE. It highlights the consequences of the intertemporal trade-off that adaptive learning introduces into the CB's decision process in addition to the intratemporal one already existing under the RE hypothesis. Notice that under RE, assuming *i.i.d.* shocks implies there is no inefficiency of discretionary monetary policy compared to commitment to a policy rule, and hence the absence of stabilization bias when there is no overambitious output target (Woodford 1999). Learning reintroduces inflation persistence as serially correlated cost-push shocks do under RE, and a stabilization bias compared to the RE equilibrium. Unlike the worst-case closed economy model is extensively studied under learning by André and Dai (2018), this section focuses on the difference introduced by opening the economy.

4.1   The min-max problem and influencing private expectations

Private agents' learning behavior gives rise to an intertemporal trade-off, allowing the CB to embed expectations interactions in policy decisions. Deriving the CB's optimal robust policy

---

[19]See the coefficients definition of the policy interest rate's solution (16) in Appendix A.1.

under discretion amounts to solving the Lagrangian of the following min-max problem:

$$
\begin{aligned}
\min_{\Psi} \max_{h_t^j} \mathscr{L}_t^{CB} \;=\; & E_t \sum_{i=0}^{+\infty} \beta^i \{ \tfrac{1}{2} \left[\pi_{t+i}^2 + \alpha x_{t+i}^2 - \theta^\pi h_{t+i}^{\pi}{}^2 - \theta^x h_{t+i}^{x}{}^2 - \theta^e h_{t+i}^{e}{}^2 \right] \\
& - \lambda_{1,t+i} \left[\pi_{t+i} - \beta a_{t+i} - \kappa x_{t+i} + \phi e_{t+i} - h_{t+i}^\pi - \varepsilon_{t+i}^\pi \right] \\
& - \lambda_{2,t+i} \left[x_{t+i} - b_{t+i} + \sigma^{-1}(r_{t+i} - a_{t+i}) + \delta(z_{t+i} - e_t) - h_{t+i}^x - \varepsilon_{t+i}^x \right] \\
& - \lambda_{3,t+i} \left[e_{t+i} - z_{t+i} + (r_{t+i} - a_{t+i}) - h_{t+i}^e - \varepsilon_{t+i}^e \right] \\
& - \lambda_{4,t+i} \left[a_{t+1+i} - a_{t+i} - \gamma_{t+i}(\pi_{t+i} - a_{t+i}) \right] \\
& - \lambda_{5,t+i} \left[b_{t+1+i} - b_{t+i} - \gamma_{t+i}(x_{t+i} - b_{t+i}) \right] \\
& - \lambda_{6,t+i} \left[z_{t+1+i} - z_{t+i} - \gamma_{t+i}(e_{t+i} - z_{t+i}) \right] \}.
\end{aligned}
\tag{17}
$$

where $\Psi \equiv \{ r_t, \pi_t, x_t, e_t, a_{t+1}, b_{t+1}, z_{t+1} \}$, $j = \pi, x, e$, and $\lambda_{n,t}$, with $n = 1, 2, \dots 6$ are Lagrange multipliers that are respectively associated with (2)-(4) in which we substitute $E_t^* \pi_{t+1} = a_t$, $E_t^* x_{t+1} = b_t$ and $E_t^* e_{t+1} = z_t$, and (5)-(7). Compared to the decision problem in the benchmark, there are additional first-order conditions with respect to private expectations because as expectations deviate from full rationality, they become state variables but also new control variables for the CB as long as the latter desires to influence private beliefs.

We now express the intertemporal trade-off condition implied by first-order conditions from the Lagrangian (17):

$$
\pi_t + \frac{\alpha(\delta + \sigma^{-1})}{\left[\kappa(\delta + \sigma^{-1}) + \phi \right]} x_t = \beta(1 - \gamma) E_t \pi_{t+1} + \frac{\alpha\beta(\delta + \sigma^{-1})\left[1 - \gamma(1 - \beta) \right]}{\left[\kappa(\delta + \sigma^{-1}) + \phi \right]} E_t x_{t+1}.
\tag{18}
$$

For $\gamma = 0$, the optimality condition $\lambda_{4,t} = -\frac{1}{\gamma} \left\{ \pi_t + \frac{\alpha(1+\delta\sigma)}{[\kappa(1+\delta\sigma)+\sigma\phi]} x_t \right\}$ is verified only when $\pi_t + \frac{\alpha(1+\delta\sigma)}{[\kappa(1+\delta\sigma)+\sigma\phi]} x_t = 0$. The rule (18) is thus reduced to the targeting rule under RE and discretion (11) and only embeds the well-known intratemporal trade-off between inflation and the output gap, which implies a "leaning against the wind" policy.

For $\gamma > 0$, the intertemporal trade-off is reflected by the terms associated with $E_t \pi_{t+1}$ and $E_t x_{t+1}$ at the right-hand side of (18). The rule (18) shows the optimal way to exploit the intertemporal trade-off and how to balance the stance of "leaning against the wind" policy in the present and in the future. The optimal policy decision depends on the current inflation and output-gap expectations. Given that inflation and output-gap targets are both zero, the right-hand side of (18) could be either positive or negative with its value depending on γ. When the right-hand side of (18) is positive, it is optimal to contract more sharply the output

gap in the present compared to the policy under RE and discretion and *vice versa*.

After numerous rearrangements exposed in Appendix A.3, we obtain the IS condition depending on output gap and exchange rate expectations, the exchange rate, output misspecifications and shocks.

$$x_t = b_t - (\delta + \sigma^{-1})(z_t - e_t) + \left(1 + \frac{\theta^x}{\sigma^2 \theta^e}\right) h_t^x - \sigma^{-1} \varepsilon_t^e + \varepsilon_t^x. \tag{19}$$

We then find a Taylor-rule-like equation where the nominal interest rate moves with expected inflation and exchange rate, as well as the exchange rate, the output misspecification as defined by (22) and an exchange rate shock (see Appendix A.1. for more details).

$$r_t = a_t + z_t - e_t - \frac{\theta^x}{\sigma \theta^e} h_t^x + \varepsilon_t^e. \tag{20}$$

After tedious calculation, we obtain the misspecifications equations depending on the intertemporal trade-off condition (see A.3.):

$$h_t^\pi = -\frac{\alpha(1 + \sigma\delta)}{\theta^\pi [\kappa(1 + \sigma\delta) + \sigma\phi]} x_t, \tag{21}$$

$$h_t^x = \frac{\alpha\sigma\phi}{\theta^x [\kappa(1 + \sigma\delta) + \sigma\phi]} [\beta(1 - \gamma) E_t x_{t+1} - x_t] + \beta E_t h_{t+1}^x, \tag{22}$$

$$h_t^e = -\frac{\theta^x}{\sigma \theta^e} h_t^x. \tag{23}$$

The equilibrium solutions can be solved using a reduced but still complex system of ten equations, i.e., the ten first-order conditions given in A.2. (see equations (A.21)-(A.30)). To obtain the equilibrium solutions of h_t^π and h_t^e, further calculations are exposed in A.3.

The dynamic nature of the above-mentioned system and the complex interactions between endogenous variables induced by learning and the openness of the economy imply that it is impossible to reduce this system to smaller and tractable subsystems that allow obtaining reasonably simple analytical solutions. Consequently, we numerically simulate the model using calibrations proposed by Galí and Monacelli (2005) and Leitemo and Söderström (2008b) for the baseline small open economy model.

Notice that in the closed economy, the equilibrium with RE is identical to the equilibrium with a learning gain γ equal to zero (André and Dai 2018). This is because, when $\gamma = 0$,

the CB cannot influence anymore private expectations and the expected values of π_{t+1} and x_{t+1} are exogenous and must be equal to their past values as well as the steady-state values (identical to those under RE so that $a_t = b_t = E_t\pi_{t+1} = E_t x_{t+1} = 0$) for the model to converge to the steady state equilibrium. Similarly in the open economy, setting $\gamma = 0$ and $a_t = b_t = E_t\pi_{t+1} = E_t x_{t+1} = 0$, we find that the intertemporal optimal trade-off condition (18) is identical to (11), implying the equilibrium solution obtained is the same as the one with RE.

4.2   The degree of model robustness ensuring determinacy

As we cannot derive the explicit condition to be imposed on the degree of model robustness to ensure determinacy, we simulate the model by taking values for the structural parameters from Galí and Monacelli (2005): $\hat{\sigma} = \zeta = 1$, $\eta = 3$, $\vartheta = 0.75$, $\beta = 0.99$, and $\omega = 0.4$. This implies that $\kappa = 0.401$, $\phi = 0.057$, $\delta = 0.4$, and $\sigma = 1.667$ according to Leitemo and Söderström (2008). We set the relative weight on output stabilization in the CB's loss function to $\alpha = 0.25$.[20]

We use the system of equations (5)-(7) and (18)-(23) to simulate the lower bound to be imposed on θ^π, θ^x and θ^e for the robust monetary policy not to induce indeterminacy under RE and learning for different values of γ in the interval $(0, 1)$. The results about the determinacy condition obtained in the open economy do not allow a direct comparison with the those in the closed economy obtained by André and Dai (2018) since several parameters take the standard parameter values (i.e., $\kappa = 0.024$, $\sigma = 0.157$, and $\alpha = 0.048$) in the literature of closed-economy New-Keynesian models are very different from the calibration used here (i.e., $\kappa = 0.401$, $\sigma = 1.667$ and $\alpha = 0.25$).

Assuming that $\theta^\pi = \theta^x = \theta^e$, we have simulated their value under which the model describing the economy does not respect the Blanchard and Kahn conditions. Table 1 shows that the thresholds of θ^π, θ^x, θ^e ensuring the determinacy of the equilibrium, denoted as $\underline{\theta}^\pi$, $\underline{\theta}^x$, $\underline{\theta}^e$ respectively, exponentially increase with the learning gain. This is to compare with a threshold for θ^π equal to 12.80 when $\gamma = 0.99$ in the closed-economy, using the calibration for the open-economy model's parameters. In the small open economy, such a low value for θ^π can be obtained for γ very close to zero. Notice that as discussed previously, the "learning" equilibrium with $\gamma = 0$ is identical to the equilibrium with RE.

---

[20]Here, we adopt $\alpha = 0.25$ following Leitemo and Söderström. This is higher than the value set by Galí and Monacelli who consider an objective function derived as a second-order approximation to the household's utility, which is characterized by $\alpha = (1-\vartheta)(1-\beta\vartheta)(1+\eta)/(\epsilon\vartheta)$, where ϵ is the elasticity of substitution across the differentiated domestic goods. Using $\epsilon = 6$, Galí and Monacelli obtain $\alpha = 0.0572$.

[21]This table is obtained with Dynare, upon authors' own calculation, using the above calibration.

γ^{21}	0	0.01	0.05	0.10	0.20	0.50	0.99
$\underline{\theta}^\pi, \underline{\theta}^x, \underline{\theta}^e$	1.8139	331.393	5528.009	20304.969	77582.851	471216.826	1829645.999

Table 1: The thresholds for the CB's focus in favor for robustness for $\phi = -0.057$ and $\omega = 0.4$.

For further insight, we numerically analyze the threshold for the $\underline{\theta}^\pi, \underline{\theta}^x, \underline{\theta}^e$ for a same level of learning gain but for different levels of ω, and hence δ, with ω being the share of foreign goods in domestic consumption and δ the elasticity of the output gap with respect to the expected change in the real exchange rate in the IS equation, and then two different levels of ϕ, the feedback coefficient from the exchange rate to inflation in the Phillips curve.

This analysis shows that for a fixed learning gain, the thresholds for robustness focus are higher when ω (δ) is smaller, meaning that not opening the economy reduces the possibility for the CB to conduct a robust policy other things being equal. This contrasts with the effect of ϕ, another indicator of the openness of the economy. Notably, it follows from the simulations that the thresholds for robustness focus increase with ϕ of which a negative value means that a depreciation leads to lower inflation. As the degree of isomorphism of the open economy with respect to the closed economy depends entirely on how close is the parameter value to zero, the effects of the openness of the economy on the thresholds for the CB's focus in favor for robustness is mainly determined by the values of ϕ.

Summarizing the results reported in Table 1 and the discussion in the above leads to the following result.

Result 1: *The degree of openness of the economy puts an additional constraint on the conduct of robust monetary policy. The more open the economy is to the rest of the world, the smaller the set of worst-case scenarios against which the monetary policy should be robust for the range of parameters used in the simulation. Moreover, the set of worst-case scenarios monotonically decreases with the learning gain for gains superior to 0.1.*

This result demonstrates that the hypothesis of constant-gain learning in an open economy substantially reduces the scope for the CB to conduct robust policy. For $\gamma = 0.01$, which is a very low learning gain, the threshold for the CB's focus on robustness that ensures determinacy is $\theta^\pi = \theta^x = \theta^e = 331.393$ (see Table 1). The misspecification that is allowed in the Phillips curve, the IS equation and the UIP condition is related to cost-push, demand and exchange-rate shocks, and is given respectively as follows: $h_t^x = 0.000117\varepsilon_t^\pi$, $h_t^e = -0.000070\varepsilon_t^\pi$, $h_t^\pi = 0.002043\varepsilon_t^\pi$; $h_t^x = -0.000004\varepsilon_t^e$, $h_t^e = 0.000002\varepsilon_t^e$, $h_t^\pi = -0.000069\varepsilon_t^e$; $h_t^x = 0.000006\varepsilon_t^x$, $h_t^e = -0.000004\varepsilon_t^x$, $h_t^\pi = 0.000114\varepsilon_t^x$. This means that under learning, the misspecification is negligible for all shocks and all equations of the model, and even

more so as γ rises to a value between 0.2 and 0.5, i.e. the interval commonly used in the learning literature. For $\gamma = 0$, which is a proxy for the RE equilibrium, we have approximately $\underline{\theta}^\pi, \underline{\theta}^x, \underline{\theta}^e = 1.8139$ for a negative pass-through of the exchange rate to the inflation, $\phi = -0.057$.

This comparison suggests that the CB cannot take into account many model misspecifications under learning compared to those under RE. The result is due to the fact that opening the economy increases the volatility of inflation and the output gap, since cost-push shocks and also demand and exchange-rate shocks affect the economy, meaning that adding too much model misspecification may feed too much volatility into the economy. The mechanism underlying our main finding is that the presence of the real exchange rate in the Phillips curve through the wage-setting process makes the intertemporal trade-off for the CB due to learning more difficult, i.e., the social cost of inflation volatility is higher when choosing between stabilizing inflation today or tomorrow. This radically changes the conduct of monetary policy and the interactions between endogenous variables and shocks compared to the closed economy.

A second explanation is that learning gain reduces the space where the robust policy can be defined. Learning is one kind of model misspecification that already introduces inflation persistence through the intertemporal trade-off so that to conduct an optimal policy, the CB has to take into account the intertemporal trade-off first to better anchor inflation expectations, before acknowledging the misspecifications present in the model.

The key factor making the thresholds of $\underline{\theta}^\pi, \underline{\theta}^x, \underline{\theta}^e$ so sensitive to an increase in γ is the openness of the economy δ, and the sign of ϕ. Some simulation exercises allow to show that the threshold for $\underline{\theta}^\pi, \underline{\theta}^x, \underline{\theta}^e$ increases significantly as ϕ rises, and to a lesser extent as δ decreases. This is because a rise in ϕ deteriorates the intertemporal trade-off and a decrease in δ makes it less effective for the CB to use the interest rate policy to respond to an exchange-rate shock, making the economy more volatile and dissuading thus the CB to introduce very bad worst-case scenarios.

Notice that in the closed economy, for a set of different parameter values, André and Dai (2018) find that the threshold of θ^π is 83.33 for $\gamma < 1$, compared to a threshold of $\theta^\pi = 45.45$ under RE. André and Dai assume that the misspecification in the IS equation is set to zero. This assumption is justified since Leitemo and Söderström (2008a) find that the CB would optimally set $h^x = 0$ and Dai and Spyromitros (2012) confirm this result even when asset prices are included into the closed-economy model.

4.3 Learning effects on the equilibrium

The ALMs for inflation, the output gap, the exchange rate and the interest rate are a function of inflation, output-gap, exchange-rate expectations, and cost-push, demand and exchange-rate shocks. With the help of Dynare, we simulate the equilibrium solutions using previously given parameters values. We present the detailed operations to obtain the final transition functions in terms of past values of inflation, output gap and exchange rate, and current values of private expectations and shocks. We establish an equivalence between these transition functions and the ALMs using (5)-(7) yielding (see Appendix A.3. for detailed calculation):

$$
\begin{bmatrix} \pi_t \\ x_t \\ e_t \\ r_t \end{bmatrix} = \begin{bmatrix} \tilde{d}_\pi^{cg} & \tilde{d}_x^{cg} & \tilde{d}_e^{cg} \\ \tilde{k}_\pi^{cg} & \tilde{k}_x^{cg} & \tilde{k}_e^{cg} \\ \tilde{j}_\pi^{cg} & \tilde{j}_x^{cg} & \tilde{j}_e^{cg} \\ \tilde{m}_\pi^{cg} & \tilde{m}_x^{cg} & \tilde{m}_e^{cg} \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ x_{t-1} \\ e_{t-1} \end{bmatrix} + \begin{bmatrix} \tilde{d}_a^{cg} & \tilde{d}_b^{cg} & \tilde{d}_z^{cg} \\ \tilde{k}_a^{cg} & \tilde{k}_b^{cg} & \tilde{k}_z^{cg} \\ \tilde{j}_a^{cg} & \tilde{j}_b^{cg} & \tilde{j}_z^{cg} \\ \tilde{m}_a^{cg} & \tilde{m}_b^{cg} & \tilde{m}_z^{cg} \end{bmatrix} \begin{bmatrix} a_t \\ b_t \\ z_t \end{bmatrix} + \begin{bmatrix} d_{\varepsilon^\pi}^{cg} & d_{\varepsilon^x}^{cg} & d_{\varepsilon^e}^{cg} \\ k_{\varepsilon^\pi}^{cg} & k_{\varepsilon^x}^{cg} & k_{\varepsilon^e}^{cg} \\ j_{\varepsilon^\pi}^{cg} & j_{\varepsilon^x}^{cg} & j_{\varepsilon^e}^{cg} \\ m_{\varepsilon^\pi}^{cg} & m_{\varepsilon^x}^{cg} & m_{\varepsilon^e}^{cg} \end{bmatrix} \begin{bmatrix} \varepsilon_t^\pi \\ \varepsilon_t^x \\ \varepsilon_t^e \end{bmatrix} \quad (24)
$$

where $\tilde{\ell}_\pi^{cg} \equiv \frac{(1-\gamma)\ell_\pi^{cg} - \gamma\ell_a^{cg}}{1-\gamma}$, $\tilde{\ell}_x^{cg} \equiv \frac{(1-\gamma)\ell_x^{cg} - \gamma\ell_b^{cg}}{1-\gamma}$, and $\tilde{\ell}_e^{cg} \equiv \frac{(1-\gamma)\ell_e^{cg} - \gamma\ell_z^{cg}}{1-\gamma}$, $\tilde{\ell}_n^{cg} \equiv \frac{\ell_n^{cg}}{1-\gamma}$ with $\ell = d, k, j, m$, and $n = a, b, z$. We numerically check that the absolute values of the composite coefficients on π_{t-1}, x_{t-1}, e_{t-1} and r_{t-1} in (24) are extremely close to zero, and more precisely they are smaller than 1×10^{-5} for $\gamma \in (0, 1)$, so that the terms associated with π_{t-1}, x_{t-1}, e_{t-1} and r_{t-1} are negligible and equations in (24) can be considered as the ALMs of endogenous variables. The numerical simulations show that the ALMs obtained under learning displays substantial history dependence, meaning that there is inertia in the dynamics of inflation, the output gap and the exchange rate.

We would like to compare the rules obtained under RE and with those obtained under learning when $\gamma = 0$. This means that the transition function detailed in A.3. is identical to the ALM for learning gain $\gamma = 0$, i.e., using the definition of a_t, b_t and z_t, the endogenous variables do not depend on private agents' expectations.

Result 2. *In the open economy, the ALMs for inflation, the output gap, the exchange rate and the interest rate are a function of expected inflation, expected output gap, expected exchange rate, and cost-push, demand and exchange-rate shocks.*

This result sharply contrasts with the one obtained in the closed economy since the ALMs for inflation and the output gap in the closed economy only depend on expected inflation and cost-push shocks. Regarding the ALM for the interest rate, the difference is also remarkable since in a closed economy model, it depends on expected inflation, expected output gap, and

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cost-push shocks (André and Dai 2018). Indeed, in the closed economy, the demand shocks can be entirely offset by an optimal adjustment of the interest rate. However, this is not verified in the open economy. Notably, an adjustment of the interest rate affects not only the aggregate demand but also the exchange rate while the latter affects the Phillips curve and hence the CB's intratemporal and intertemporal trade-offs. Therefore, it is not optimal for the CB to entirely offset the effects of demand and exchange-rate shocks on the equilibrium.

The effects of learning on the feedback coefficients in the ALM for inflation are illustrated in Figure 1.[22] We take a large value for the penalty parameters controlling the CB's focus in favor for robustness, i.e., $\theta^\pi = \theta^x = \theta^e = 1867240$, to ensure the determinacy of the equilibrium, for $\gamma \in (0, 1)$. We notice that $\tilde{d}_a^{cg} > 0$, $\tilde{d}_b^{cg} > 0$, $\tilde{d}_z^{cg} < 0$, $d_{\varepsilon^\pi}^{cg} > 0$, $d_{\varepsilon^x}^{cg} > 0$, and $d_{\varepsilon^e}^{cg} < 0$. Compared to the equilibrium with RE, which can be proxied by $\gamma = 0$, an increase in the learning gain always attenuates the response of inflation to b_t, z_t, ε_t^π, ε_t^x and ε_t^e, for $\gamma \in (0, 1)$ except for a_t. The feedback coefficient on a_t sharply decreases for small values of γ and continues to decrease until $\gamma = 0.25$, and slightly increases with γ for $\gamma > 0.25$. The value of $\tilde{d}_a^{cg}$ is in the interval $(0, 1)$, meaning that current inflation increases with inflation expectations (a_t) less than proportionally.
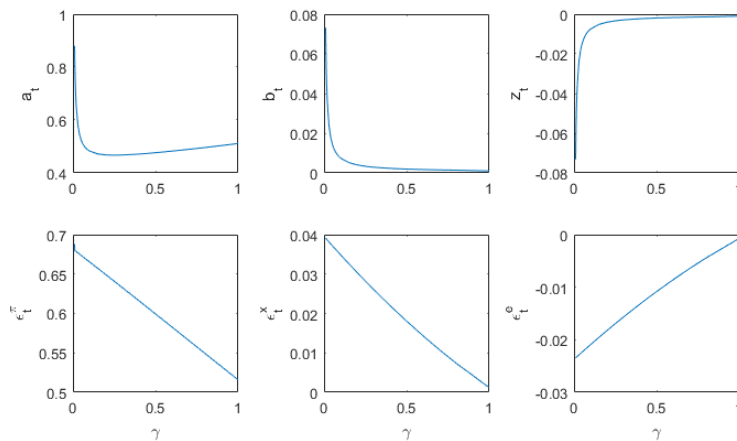

Figure 1: The effect of learning on the feedback coefficients in the ALM for π_t.

Current inflation depends indirectly on the CB's interest rate policy response to past shocks and, hence, on γ. In the closed economy, an increase in γ has two opposite effects on $\tilde{d}_a^{cg}$, i.e., the feedback from a_t to π_t in (24). The first effect consists in that a higher learning algorithm, i.e. γ, in (5) increases the positive correlation between current inflation π_t and future inflation expectations a_{t+1}, and hence the incentive for the CB to lower $\tilde{d}_a^{cg}$. The second

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[22]Figures 1 to 8 are obtained with Dynare, upon authors' own calculation, using the above calibration.

effect on $\tilde{d}_a^{cg}$ occurs also through an increase in γ that weakens the feedback of a_t on a_{t+1}, thus allowing an improvement in social welfare through an increase in $\tilde{d}_a^{cg}$.

In the closed economy, the first effect dominates so that $\tilde{d}_a^{cg}$ is decreasing in γ as demonstrated in André and Dai (2017, 2018). This means that the interest rate policy is increasingly responsive to inflation to limit future movements in inflation expectations that deteriorate the short-term intratemporal trade-offs. In the open economy, the interactions between inflation and the exchange rate in the Phillips curve (2) imply that the CB has also to take into account how out-of-equilibrium inflation expectations (a_t) affect the output gap and the real exchange rate and hence their expectations in the future (b_{t+1} and z_{t+1}). Therefore, such interactions could break the monotonicity of $\tilde{d}_a^{cg}$ by changing the incentives governing the CB's choice of $\tilde{d}_a^{cg}$ as γ increases. This also explains the non-monotonicity in $\tilde{k}_a^{cg}$, $\tilde{j}_a^{cg}$ and $\tilde{m}_a^{cg}$ in Figures 2-4.
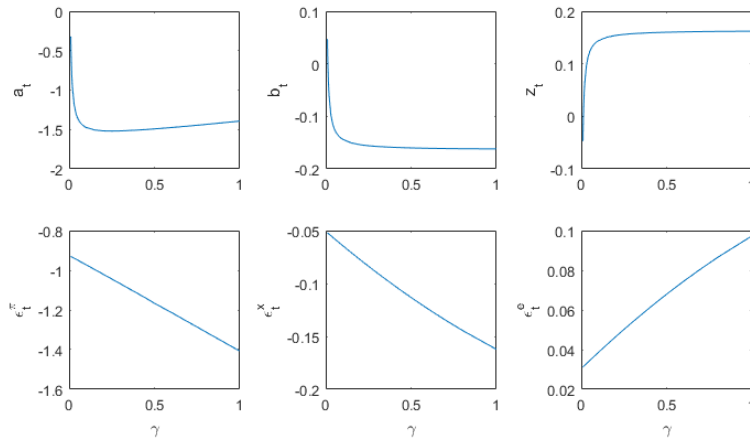

Figure 2: The effect of learning on the feedback coefficients in the ALM for x_t.

Figure 2 displays the effects of learning on the feedback coefficients in the ALM for the output gap and shows that $\tilde{k}_a^{cg} < 0$, $k_{\varepsilon^\pi}^{cg} < 0$, $k_{\varepsilon^x}^{cg} < 0$, and $k_{\varepsilon^e}^{cg} > 0$ except for $\tilde{k}_b^{cg}$ and $\tilde{k}_z^{cg}$ whose sign changes with γ. More precisely, $\tilde{k}_b^{cg} > 0$ for $\tilde{k}_z^{cg} < 0$ for $\gamma < 0.013$ and $\tilde{k}_b^{cg} < 0$ for $\tilde{k}_z^{cg} > 0$ for $\gamma \geq 0.013$. Compared to the RE equilibrium, a higher learning gain implies a reinforcement in the response of the output gap to ε_t^π, ε_t^x and ε_t^e, for $\gamma \in (0, 1)$ except for a_t, b_t, z_t. In the open economy, a positive learning gain always implies a greater optimal output contraction in response to out-of-equilibrium inflation expectations than under RE and discretion.

However, the effect of out-of-equilibrium inflation expectations is not monotonic contrary to the closed economy. The feedback coefficient on a_t sharply decreases for small values of

γ and then slightly decreases until $\gamma = 0.25$, and moderately increases with γ making more complex for $\gamma > 0.25$. The feedback coefficient on b_t is positive for $\gamma < 0.013$, and becomes negative for $\gamma \geq 0.013$ and increasingly so as the learning gain increases. The direct positive effect of an increase in b_t on x_t is more than compensated by the indirect negative effect on x_t due to the decrease in the real exchange rate that is induced by an increase in b_t (see Figure 3) as the learning gain becomes significantly high. The same mechanism explains the evolution of the feedback coefficient on z_t with γ.
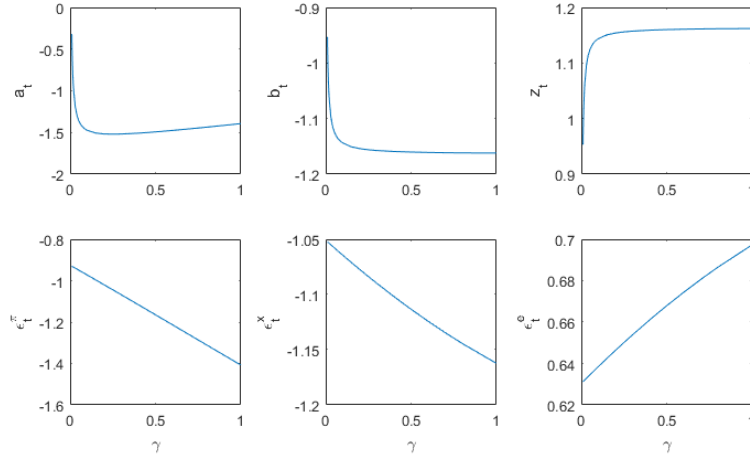

Figure 3: The effect of learning on the feedback coefficients in the ALM for e_t.

Figure 3 depics the effects of learning on the feedback coefficients in the ALM for the exchange rate, $\tilde{j}_a^{cg} < 0$, $\tilde{j}_b^{cg} < 0$, $\tilde{j}_z^{cg} > 0$, $j_{\varepsilon^\pi}^{cg} < 0$, $j_{\varepsilon^x}^{cg} < 0$, and $j_{\varepsilon^e}^{cg} > 0$. A positive learning gain calls the CB to be more aggressive against inflation in response to an increase in inflation expectations and hence a stronger appreciation of the real exchange rate. An increase in the learning gain strengthens (attenuates) the response of the exchange rate to a_t for $\gamma \leq 0.25$ ($\gamma > 0.25$), and strengthens the response of the exchange rate to b_t, z_t, ε_t^π, ε_t^x and ε_t^e for any γ. Notice that the response of the exchange rate to b_t and z_t is amplified by an increase in γ but the amplification effect decelerates for $\gamma > 0.25$.
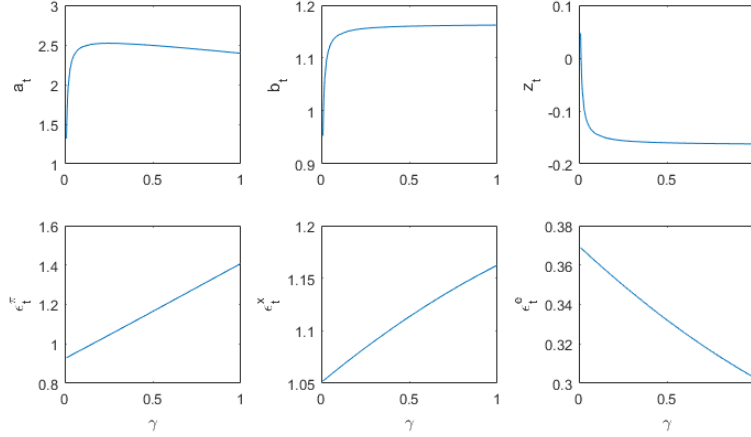
Figure 4: The effect of learning on the feedback coefficients in the ALM for r_t.

Figure 4 shows the effects of learning on the feedback coefficients in the ALM for the interest rate with $\tilde{m}_a^{cg} > 0$, $\tilde{m}_b^{cg} > 0$, $\tilde{m}_z^{cg} < 0$, $m_{\varepsilon^\pi}^{cg} > 0$, $m_{\varepsilon^x}^{cg} > 0$ and $m_{\varepsilon^e}^{cg} > 0$. An increase in learning gain leads the CB to amplify (attenuate) the response of the interest rate to a_t for $\gamma \leq 0.25$, b_t, z_t, ε_t^π, ε_t^x for any γ (to a_t for $\gamma > 0.25$, and ε_t^e for any γ). The non-monotonicity of the feedback coefficient on inflation expectations in the ALM for the interest rate is due to the exchange rate channel in the transmission mechanism of monetary policy.

Result 3. *Adaptive learning makes robust monetary policy less (more) accommodative compared to RE in its response to cost-push and demand (exchange-rate) shocks. In the worst-case model, the fact that the CB exploits the intertemporal trade-off resulting from the learning behavior of private agents globally leads to an attenuation (amplification) in the response of inflation (the output gap and the exchange rate) to inflation, output-gap and exchange-rate expectations, and cost-push, demand and exchange-rate shocks for the range of parameters used in the simulation.*

Private agents' learning behavior offers the CB the possibility to influence private expectations through its policy. In general, the higher the value of learning, the more (less) aggressive the monetary policy should be in response to cost-push and demand (exchange rate) shocks. An increase in both ε_t^π and ε_t^x (ε_t^e) is inflationary (disinflationary) and feeds thus into higher (lower) future expected inflation, hence calling for a more (less) aggressive policy for positive ε_t^π and ε_t^x (ε_t^e). Such a policy results in short-term output-gap losses that are more than compensated by the gain in fighting inflation when learning gain that is present in inflation expectations becomes small enough.

4.4 Effects of robustness on the equilibrium

In the closed economy model under RE, an increase in the CB's focus on robustness against inflation misspecification (decrease in θ^π) results in a more aggressive response to inflation compared to the model without misspecification, while an increase in the CB's focus on robustness against output-gap misspecification (in the IS equation) has no implication for monetary policy (Leitemo and Söderström 2008a, Dai and Spyromitros 2012).

In the open economy model with RE, Leitemo and Söderström (2008b) investigate the effect of an increase in the CB's focus against misspecification in model's equations and find that a stronger focus on robustness against inflation and output-gap misspecification makes monetary policy respond more aggressively to inflation and output shocks, but less aggressively to exchange rate shocks, whereas a stronger focus on robustness against exchange rate misspecification has opposite effects. These effects are reinforced by the private agents' learning behavior. In other words, adaptive learning makes the CB more cautious in the sense of Söderström (2002) and Giannoni (2007) but less cautious in the sense of Brainard (1967), i.e., being more aggressive to inflation.

However, opting for a more robust monetary policy is prevented by the need of influencing private beliefs based on learning because adaptive learning imposes a much more restrictive constraint on monetary policy robustness to ensure the determinacy of the equilibrium than under RE. This constraint is also true in a closed economy model but to a lesser extent (André and Dai 2018). Opening the economy with learning agents sharply reinforces the constraint on the focus on robustness against a particular misspecification, and more so as the learning gain increases, due to non linearity of learning gain for $\gamma < 0.1$ as shown in Figures 1 to 4 in the previous section.. The misspecification the malevolent agent can introduce becomes insignificant and is hence quite insensitive to the change in the focus on robustness.

We perform additional simulations to study how the degree of openness of an economy can change the minimum possible level of robust monetary policy, conditional on fixed learning gains and pass-through of the exchange rate in the Phillips curve, that are exposed in Section 5.

There is a monotonic relationship between the openness of the economy measured by δ (or ω) and the degree of maximum robustness that the CB can implement for $0.02 < \gamma < 1$, that is the more open the economy is, the larger is the space where the robust monetary policy can be set without having an indeterminate equilibrium under learning. This is true for negative or positive pass-through ϕ of the exchange rate to inflation in the Phillips curve.

However, the openness of the economy measured by ϕ, i.e., the pass-through of the ex-

change rate to inflation in the Phillips curve, plays an even greater role in determining the thresholds for the CB's focus in favor for robustness than δ, because once $\phi = 0$, any change in δ (or ω) will not have any impact on the these thresholds. This is because how close ϕ is to zero determines how much the small open economy is isomorphic the close economy. Note that the higher the learning gain, the lower is the possibility to conduct a robust monetary policy. This is valid for any value of the pass-through of the exchange rate to inflation in the Phillips curve, increasingly so as it departs from zero. Consequently, the more negative the pass-through of the exchange rate is in the Phillips curve, the less possible it is to conduct a robust monetary policy (higher $\underline{\theta}^\pi, \underline{\theta}^x, \underline{\theta}^e$ when ϕ goes from positive to negative values). Indeed, the negative pass through of the exchange rate ensures, according to Leitemo and Söderström (2008b), that the total effect of a depreciation of domestic currency on inflation is negative, for an advanced economy parameter set.

In Figures 5-8, simulations show the evolution of feedback coefficients of inflation, the output gap, the exchange rate and the interest rate according to θ_j, $j = \pi, x, e$, for three values of learning gain, i.e., $\gamma = 0.01$, $\gamma = 0.2$, $\gamma = 0.99$, represented by the red solid line, green dotted line, and blue dashed line, respectively. For different values of γ and the corresponding threshold of θ given in Table 1, we simulate how a change in θ affects the feedback coefficients in the ALMs for inflation, the output gap, the exchange rate and the interest rate.

We first consider that all θ_j are equal and examine how their evolution can impact the feedback coefficients in the ALMs derived for the worst-case model. The robustness of the simulation results is then checked by investigating the effect of an increase in the focus on robustness against one source of misspecification, i.e., a decrease in θ_i, while keeping $\theta_{j \neq i}$ fixed at a large value on these feedback coefficients. We find that the alternative scenarios have no incidence on the results.

In Figures 5-8, the red solid and green dotted lines begin with relatively small values of θ whereas the blue dashed line begins with a very high value of θ since this is required for the determinacy of the equilibrium. Figures 5-8 are consistent with the observations we made about the non-monotonicity of $\tilde{d}_a^{cg}$, $\tilde{k}_a^{cg}$, $\tilde{j}_a^{cg}$ and $\tilde{m}_a^{cg}$ in the open economy (see Figures 1-4), meaning that the solid, dotted and dashed lines corresponding to different values of γ might not appear in the same order in the sub-figures of Figures 5-8.
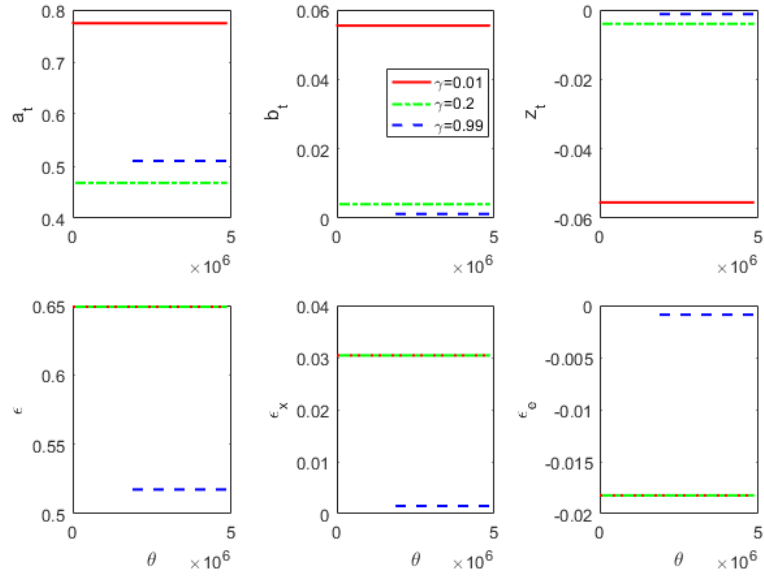
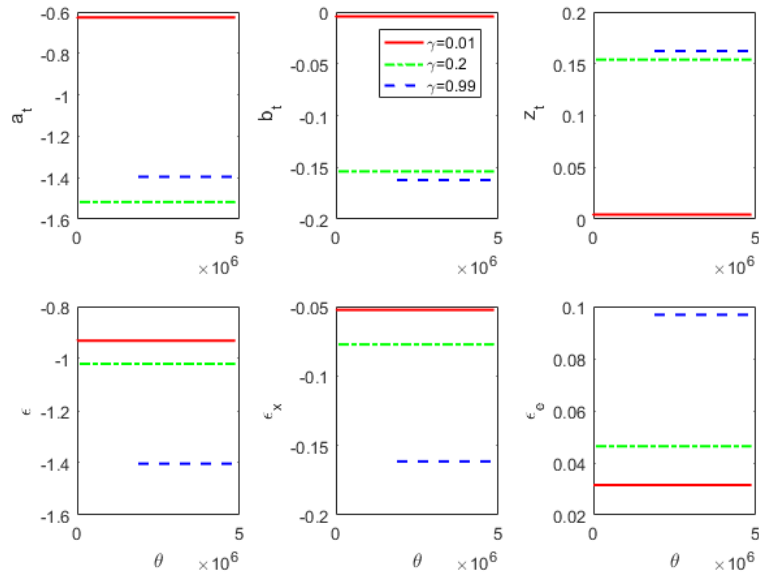Figure 5: Feedback coefficients in the ALM for inflation.

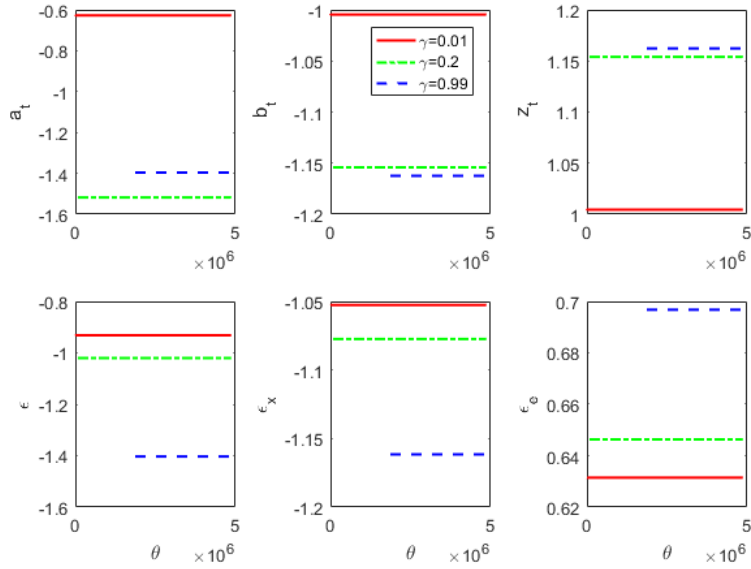Figure 6: Feedback coefficients in the ALM for the output gap.

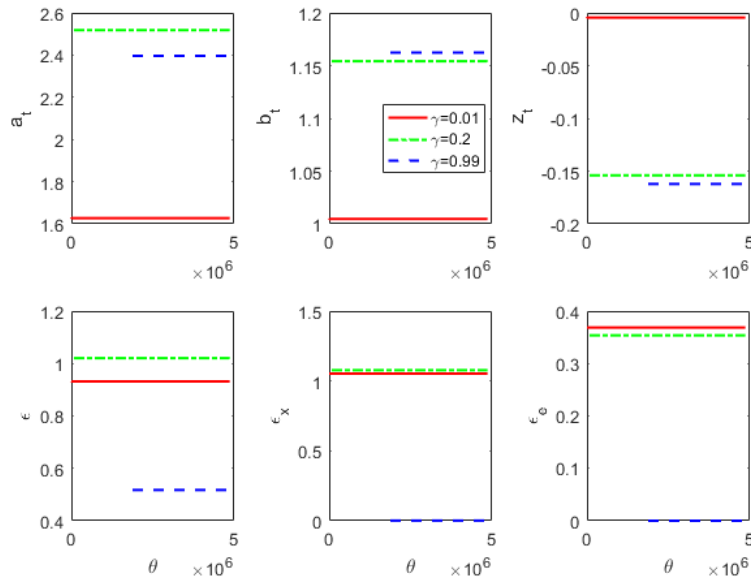Figure 7: Feedback coefficients in the ALM for the exchange rate.

Figure 8: Feedback coefficients in the ALM for the interest rate.

Result 4. *The CB's focus on robustness against model misspecification has no significant impact on the value of feedback coefficients once its focus is bounded to ensure the determinacy of the equilibrium.*

This result suggests that in the open economy, the CB exploiting the intertemporal trade-off due to learning cannot introduce much misspecification that accounts for worst-case scenarios. Furthermore, due to the lower bound imposed on the penalty parameters controlling the CB's focus in favor for robustness, increasing such focus has almost no effect on the dynamic path of the economy. This result is obtained only when the exchange rate affects the Phillips curve and is true even for a very small parameter value of ϕ (the values retained in this paper are $\phi = 0.057$ and $\phi = -0.006$). In contrast, in the closed economy, an increase in the CB's focus on robustness (smaller θ_π) can have a significant impact on the feedback coefficients particularly when θ_π approaches its threshold value ensuring the determinacy of the economy while the CB should optimally set $\theta_x \rightarrow +\infty$ (André and Dai 2018).

5   Robustness of the Results

In the above sections, to facilitate the comparison with the results of Leitemo and Söderström, we first posit (sections 2 to 4) that the exchange-rate pass through in the Phillips curve is negative. These results have been obtained for a small open economy calibration for advanced economies, corresponding to a negative exchange-rate pass through to inflation. Without reporting the numerical results, we just discuss their implications here

The negative relationship between inflation and the real exchange rate obtained by Leitemo and Söderström (2008b) is somewhat counter-intuitive. Walsh (1999), and Razin and Yuen (2002) among others obtain a positive relationship between these two variables. However, both types of relationships could find empirical justification. Estimating a variant of the Phillips curve in Galí and Monacelli (2005), Mihailov, Rumler and Scharler (2011) find that inflation can be either positively or negatively correlated with the expected change in the real exchange rate with the coefficients ranging from −0.26 to 0.47 for different European countries.

To ensure the robustness of the results, we then analyze how the presence of misspecification may change the monetary policy design in an emerging small open economy like México, where empirical evidence concludes that the exchange-rate pass-through is positive.[23] The fact that the exchange-rate pass through is positive implies another transmission mechanism for monetary policy. We therefore use a different calibration in equation (2) for the coefficient of exchange-rate pass through observed for the Mexican economy between 2001 and 2015 (Banco de México 2016).

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[23]See Banco de México (2016), and López-Martín (2019) among others.

γ^{26}	0	0.01	0.05	0.10	0.20	0.50	0.99
$\underline{\theta}^{\pi},\underline{\theta}^{x},\underline{\theta}^{e}$	6.218	13.344	13.344	13.345	13.345	13.346	13.347

Table 2: The thresholds for the CB's focus in favor for robustness for $\phi = 0.006$ and $\omega = 0.53$.

In the Mexican economy, movements in the exchange rate positively affect inflation, for a given output gap.[24] An increase in the exchange rate (depreciation of domestic currency) raises domestic consumer prices and reduces the real wage for a given nominal wage. Given households' marginal rate of substitution between leisure and consumption, this incites households to supply less labor and enjoy more leisure. Firms must increase the real wage to offset the reduction in the households' real wage, leading to higher marginal cost and inflation. Meanwhile, the depreciation increases the relative price of foreign goods in terms of domestic goods, which makes domestic goods more attractive since we assume that emerging economies such as Mexico are confronted to a price competition, that yields an increase of export goods. Domestic activity is stimulated, which strengthens inflationary pressures. While the depreciation increases the relative price of domestic goods, domestic consumption is reduced initially. The condition $(2-\omega)\zeta\sigma > 1$ no longer holds, which was verified in sections 2 to 4 according to the calibrations of Leitemo and Söderström (2008b) such that now, in the Mexican economy, the trade effects dominate the domestic ones. The total effect of an increase in the real exchange rate on inflation is therefore positive.

In this case, a positive pass-through of the exchange rate to core inflation then implies that an increase in the risk premium generates depreciation of the domestic currency and a higher inflation, which is common for an emerging economy.

For the robustness check, we use for the calibration: a positive pass-through of the exchange rate to core inflation, $\phi = 0.006$, in the Mexican economy for the period 2001-2015 (Banco de México 2016), it requires that $\sigma = 1.667$ that gives for $\hat{\sigma} = 0.78349$, $\eta = 3.2$ according to Leyva and Urrutia (2018), $\omega = 0.53$, $\delta = -0.123$ and $\zeta = 0.25$ following López-Martín (2019), $\vartheta = 0.85$ from Klenow and Malin (2010), and $\kappa = 0.017$ from its definition using the previous calibrations.[25] Table 2 shows us the lower-bound thresholds of $\underline{\theta}^{\pi},\underline{\theta}^{x},\underline{\theta}^{e}$ reflecting the maximum level of CB's focus in favor for robustness in the Mexican economy, varying with the learning gain γ.

From Table 2, we confirm Result 3, that learning decreases the CB's focus on robustness

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[24]This is more generally a characteristic of emerging economies exporting price-competitive goods.

[25]López-Martín (2019) uses the value of $\omega = 0.53$, $\delta = -0.123$ and $\zeta = 0.25$, following Ramanarayan's calibration (2017) for Chile.

[26]This table has been obtained with Dynare, upon authors' own calculation, using the above calibration for Mexico.

to ensure a stable path of the economy. For $\gamma = 0$, which is a proxy for the RE equilibrium, we have approximately $\underline{\theta}^{\pi}, \underline{\theta}^{x}, \underline{\theta}^{e} = 1.8139$ for a negative pass-through of the exchange rate to the inflation, $\phi = -0.057$, (Table 1) and $\underline{\theta}^{\pi}, \underline{\theta}^{x}, \underline{\theta}^{e} = 6.218$ for a positive pass-through $\phi = -0.006$. Those limits for the robustness tend to increase with the learning gain, meaning that the CB's focus in favor for robustness that can be introduced into the model under RE are higher than when private agents have an increasing learning gain, i.e. form expectations that are increasingly more backward-looking.

However, for a positive pass-through, the monetary policy can take into account a higher degree of misspecification when conducted (lower $\underline{\theta}^{\pi}, \underline{\theta}^{x}, \underline{\theta}^{e}$) for any $\gamma > 0$. This result is explained by the fact that here, with our calibration, the open economy presents a high degree of isomorphism compared to the closed economy, whose degree depends entirely on how the parameter ϕ is getting closer to zero. For $\gamma = 0$, $\underline{\theta}^{\pi}, \underline{\theta}^{x}, \underline{\theta}^{e} = 6.218$ for a positive pass-through $\phi = -0.006$. Furthermore, the focus on robustness stays high (lower $\underline{\theta}^{\pi}, \underline{\theta}^{x}, \underline{\theta}^{e}$) for any $\gamma > 0$ in this particular case, where the exchange-rate pass through is positive and close to zero.

6 Conclusion

Using a stylized New Keynesian model of a small open economy, this paper finds that robust monetary policy should be more (less) aggressive to cost-push and demand (exchange-rate) shocks when private agents are learning compared to the case under RE, when the central bank is confronted to challenges arising from uncertain economic environment, openness to trade and capital flows.

The mechanism underlying our main finding is that the real exchange rate affects the Phillips curve through the wage-setting process, hence making the intertemporal trade-off for the central bank due to learning more difficult, i.e., the social cost is higher when choosing between stabilizing inflation today or tomorrow. This radically changes the conduct of monetary policy and the interactions between endogenous variables and shocks compared to the closed economy. Notably, opening the economy reduces significantly the plausible set of worst-case scenarios against which monetary policy should be robust, and drastically more so as the learning gain rises because of the need for the central bank to properly anchor private expectations. This finding is expected to hold as long as the central bank influences inflation only through aggregate demand, while interest rate fluctuations do not directly affect social loss. In a closed or open economy with a Phillips curve not being affected by the exchange rate, the central bank can offset all shocks other than cost-push shocks. Compared to such

economies, the equilibrium values of endogenous variables in the type of economy examined in this paper are affected by all sorts of disturbances. Indeed, model misspecification can affect the IS equation and the uncovered interest rate parity, as the central bank cannot offset demand shocks without affecting inflation through the real exchange rate. However, due to the effect of learning and the openness of the economy, the central bank's focus on robustness against model misspecification has no significant impact on the equilibrium once its focus is bounded to ensure the determinacy of the equilibrium.

The main results obtained in this paper are based on the assumption of constant-gain learning. Nevertheless, agents could start learning with a decreasing gain before adopting a constant gain. The first can be seen as the preliminary expectations process adopted by most economic agents whereas the second is more suitable for time-varying environments. One immediate extension to this paper is to consider that private agents are learning under a decreasing gain over time, as studied in Molnár and Santoro (2014). In general, the equilibria under decreasing-gain learning replicate the equilibria under learning with different constant gains, this extension would confirm the main results obtained in this paper.

This paper focuses on the worst-case model, meaning that the malevolent agent chooses model misspecification to be as damaging as possible while the central bank's policy rule and private agents' expectations reflect this misspecification. An interesting extension is to examine the case where the central bank uses the robust control approach to design the policy interest rate rule but the economy functions according to an approximating model as in Leitemo and Söderström (2008b). Since only the interest rate rule is disturbed to account of model misspecification while the true model of the economy remains undisturbed, it seems that the economy has a smaller risk of being destabilized, meaning that the central bank could have higher focus in favor for model misspecification than in the worst-case model.

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A Appendix

A.1 The equilibrium of the worst-case model under RE

We present here the main results derived by Leitemo and Söderström (2008b) under RE and discretion for the worst-case model. The solutions given in the subsection 3.2 are obtained as follows.

The state variables are the shocks ε_t^π, ε_t^x and ε_t^e. The solution of the worst-case model with the method of undetermined coefficients (McCallum1983) is assumed to be function of the state variables. This solution is given by (16).

Eliminating x_t and h_t^π in the Phillips curve (2) using the targeting rule (11) and equation (12), and substituting h_t^e and r_t given respectively by equation (14) and the optimal interest rate rule (15) into the UIP condition (4) yield

$$C\pi_t = \beta E_t \pi_{t+1} + \phi e_t + \varepsilon_t^\pi, \tag{A.1}$$

$$(1+\sigma\delta)e_t = (1+\sigma\delta)E_t e_{t+1} - D\pi_t + \sigma\Gamma E_t \pi_{t+1} - \sigma\varepsilon_t^x + \varepsilon_t^e, \tag{A.2}$$

where $C \equiv \theta^\pi(1+\Gamma\kappa) - 1 > 0$ and $D \equiv \left[\Gamma\sigma + \frac{\phi\sigma^2}{\theta^x(1+\sigma\delta)} + \frac{\phi}{\theta^e(1+\sigma\delta)}\right] > 0$ if θ^j with $j = \pi, x, e$ are sufficiently large (i.e., when the focus in favor for robustness is sufficiently low).

Using the assumed solution of π_t and e_t given in (16) and the assumption that all shocks are serially uncorrelated, i.e., $E_t \varepsilon_{t+1}^\pi = E_t \varepsilon_{t+1}^x = E_t \varepsilon_{t+1}^e = 0$, it follows that $E_t \pi_{t+1} = d_\pi^{RE} E_t \varepsilon_{t+1}^\pi + d_x^{RE} E_t \varepsilon_{t+1}^x + d_e^{RE} E_t \varepsilon_{t+1}^e = 0$ and that $E_t e_{t+1} = j_\pi^{RE} E_t \varepsilon_{t+1}^\pi + j_x^{RE} E_t \varepsilon_{t+1}^x + j_e^{RE} E_t \varepsilon_{t+1}^e = 0$. Substituting $E_t \pi_{t+1} = 0$ and $E_t e_{t+1} = 0$ into (A.1)-(A.2), solving the resulting system of equations to obtain the solutions of π_t and e_t, and then comparing the latter with the assumed solution of π_t and e_t given in (16), we obtain

$$d_\pi^{RE} = \frac{(1+\sigma\delta)}{(1+\sigma\delta)C + \phi D}, \ d_x^{RE} = \frac{\phi\sigma}{(1+\sigma\delta)C + \phi D}, \ d_e^{RE} = -\frac{\phi}{(1+\sigma\delta)C + \phi D}, \tag{A.3}$$

$$j_\pi^{RE} = -\frac{D}{(1+\sigma\delta)C + \phi D}, \ j_x^{RE} = -\frac{C\sigma}{(1+\sigma\delta)C + \phi D}, \ j_e^{RE} = \frac{C}{(1+\sigma\delta)C + \phi D}, \tag{A.4}$$

where $(1+\sigma\delta)C - \phi D > 0$ for sufficiently large θ^j, with $j = \pi, x, e$, implying that there is a lower bound for the degree of model robustness that the CB can introduce into the model. Otherwise, if θ^j are such that $(1+\sigma\delta)C - \phi D < 0$, inflation would decrease following a positive cost-push shock , which is counterfactual.

Using (11) and (A.3), we get the coefficients of the assumed solution of x_t:

$$k_\pi^{RE} = -\Gamma d_\pi^{RE} = -\frac{\Gamma(1+\sigma\delta)}{(1+\sigma\delta)C+\phi D}, \tag{A.5}$$

$$k_x^{RE} = -\Gamma d_x^{RE} = -\frac{\Gamma\phi\sigma}{(1+\sigma\delta)C+\phi D}, \tag{A.6}$$

$$k_e^{RE} = -\Gamma d_e^{RE} = \frac{\Gamma\phi}{(1+\sigma\delta)C+\phi D}. \tag{A.7}$$

Substituting the final solution of π_t, e_t, $E_t\pi_{t+1}$ and $E_t e_{t+1}$ into (15) yields

$$m_\pi^{RE} = \sigma\left[Bd_\pi^{RE} + \delta j_\pi^{RE}\right], \; m_x^{RE} = \sigma\left[Bd_x^{RE} + \delta j_x^{RE} + 1\right], \; m_e^{RE} = \sigma\left[Bd_e^{RE} + \delta j_e^{RE}\right], \quad \text{(A.8)}$$

where $B \equiv \Gamma + \frac{\sigma\phi}{\theta^x(1+\sigma\delta)}$. For sufficiently large θ^j, with $j = \pi, x, e$, i.e., when the CB's focus in favor for robustness is sufficiently low, it is straightforward to show that $m_\pi^{RE} > 0$, $m_x^{RE} > 0$ and $m_e^{RE} > 0$.

Equations (12)-(14) imply that the coefficients in the solution of misspecification h^j with $j = \pi, x, e$, in the CB's worst-case scenario are related to the coefficients in the final solution of inflation, the output gap and the exchange rate given by (A.3)-(A.7):

$$\hat{d}_j^{RE} = \frac{1}{\theta^\pi} d_j^{RE}, \tag{A.9}$$

$$\widehat{k}_j^{RE} = \frac{\phi\sigma}{\theta^x(1+\sigma\delta)} d_j^{RE}, \tag{A.10}$$

$$\hat{j}_j^{RE} = -\frac{\phi}{\theta^e(1+\sigma\delta)} d_j^{RE}. \tag{A.11}$$

This ultimately leads to the equilibrium of the worst-case model corresponding to the system of equations given by (A.12).

The equilibrium of the worst-case model could be obtained by solving the system of equations (2), (4), and (11)-(15). The state variables are the shocks ε_t^π, ε_t^x and ε_t^e. The solution of the worst-case model with the method of undetermined coefficients (McCallum1983) is

assumed to be function of state variables:

$$
\begin{bmatrix}
\pi_t \\
x_t \\
e_t \\
r_t \\
h_t^\pi \\
h_t^x \\
h_t^e
\end{bmatrix}
=
\begin{bmatrix}
d_\pi^{RE} & d_x^{RE} & d_e^{RE} \\
k_\pi^{RE} & k_x^{RE} & k_e^{RE} \\
j_\pi^{RE} & j_x^{RE} & j_e^{RE} \\
m_\pi^{RE} & m_x^{RE} & m_e^{RE} \\
\hat{d}_\pi^{RE} & \hat{d}_x^{RE} & \hat{d}_e^{RE} \\
\widehat{k}_\pi^{RE} & \hat{k}_x^{RE} & \widehat{k}_e^{RE} \\
\hat{j}_\pi^{RE} & \hat{j}_x^{RE} & \hat{j}_e^{RE}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_t^\pi \\
\varepsilon_t^x \\
\varepsilon_t^e
\end{bmatrix}.
\tag{A.12}
$$

Using (16) together with (2), (4), and (11)-(15) yields:

$$
d_\pi^{RE} = (1 + \sigma\delta)\Omega, \qquad d_x^{RE} = \phi\sigma\Omega, \qquad d_e^{RE} = -\phi\Omega;
\tag{A.13}
$$

$$
k_\pi^{RE} = -\Gamma(1 + \sigma\delta)\Omega, \quad k_e^{RE} = -\Gamma\phi\sigma\Omega, \qquad k_x^{RE} = \Gamma\phi\Omega;
\tag{A.14}
$$

$$
j_\pi^{RE} = -D\Omega, \qquad j_x^{RE} = -C\sigma\Omega, \qquad j_e^{RE} = C\Omega;
\tag{A.15}
$$

$$
m_\pi^{RE} = \sigma\left[Bd_\pi^{RE} + \delta j_\pi^{RE}\right], \; m_x^{RE} = \sigma\left[Bd_x^{RE} + \delta j_x^{RE} + 1\right], \; m_e^{RE} = \sigma\left[Bd_e^{RE} + \delta j_e^{RE}\right];
\tag{A.16}
$$

$$
\hat{d}_\pi^{RE} = \frac{1 + \sigma\delta}{\theta^\pi}\Omega, \qquad \hat{d}_x^{RE} = \frac{\phi\sigma}{\theta^\pi}\Omega, \qquad \hat{d}_e^{RE} = -\frac{\phi}{\theta^\pi}\Omega;
\tag{A.17}
$$

$$
\widehat{k}_\pi^{RE} = \frac{\phi\sigma}{\theta^x}\Omega, \qquad \widehat{k}_x^{RE} = \frac{\phi^2\sigma^2}{\theta^x(1 + \sigma\delta)}\Omega, \qquad \widehat{k}_e^{RE} = -\frac{\phi^2\sigma}{\theta^x(1 + \sigma\delta)}\Omega;
\tag{A.18}
$$

$$
\hat{j}_\pi^{RE} = -\frac{\phi}{\theta^e}\Omega, \qquad \hat{j}_x^{RE} = -\frac{\phi^2\sigma}{\theta^e(1 + \sigma\delta)}\Omega, \qquad \hat{j}_e^{RE} = \frac{\phi^2}{\theta^e(1 + \sigma\delta)}\Omega.
\tag{A.19}
$$

where $B \equiv \Gamma + \frac{\sigma\phi}{\theta^x(1 + \sigma\delta)} > 0$, $C \equiv \theta^\pi(1 + \Gamma\kappa) - 1 > 0$, $D \equiv \left[\Gamma\sigma + \frac{\phi\sigma^2}{\theta^x(1 + \sigma\delta)} + \frac{\phi}{\theta^e(1 + \sigma\delta)}\right]$, and $\Omega = \frac{1}{(1 + \sigma\delta)C - \phi D}$. Note that $D > 0$ if θ^j with $j = \pi, x, e$ are sufficiently large (i.e., when the focus in favor for robustness is sufficiently low).

A.2   Min-max approach under learning

We look for the optimal monetary policy when private agents are learning. Unfortunately, it is not possible to find closed-form solutions

Deriving the CB's optimal robust policy under discretion amounts to solving the La-

grangian of the following min-max problem:

$$\min_{\Psi} \max_{h_t^j} \mathscr{L}_t^{CB} = E_t \sum_{i=0}^{+\infty} \beta^i \left\{ \tfrac{1}{2} \left[\pi_{t+i}^2 + \alpha x_{t+i}^2 - \theta^\pi h_{t+i}^{\pi}{}^2 - \theta^x h_{t+i}^{x}{}^2 - \theta^e h_{t+i}^{e}{}^2 \right] \right.$$

$$- \lambda_{1,t+i} \left[\pi_{t+i} - \beta a_{t+i} - \kappa x_{t+i} - \phi e_{t+i} - h_{t+i}^{\pi} - \varepsilon_{t+i}^{\pi} \right]$$

$$- \lambda_{2,t+i} \left[x_{t+i} - b_{t+i} + \sigma^{-1}(r_{t+i} - a_{t+i}) + \delta(z_{t+i} - e_t) - h_{t+i}^{x} - \varepsilon_{t+i}^{x} \right]$$

$$- \lambda_{3,t+i} \left[e_{t+i} - z_{t+i} + (r_{t+i} - a_{t+i}) - h_{t+i}^{e} - \varepsilon_{t+i}^{e} \right]$$

$$- \lambda_{4,t+i} \left[a_{t+1+i} - a_{t+i} - \gamma_{t+i}(\pi_{t+i} - a_{t+i}) \right]$$

$$- \lambda_{5,t+i} \left[b_{t+1+i} - b_{t+i} - \gamma_{t+i}(x_{t+i} - b_{t+i}) \right]$$

$$\left. - \lambda_{6,t+i} \left[z_{t+1+i} - z_{t+i} - \gamma_{t+i}(e_{t+i} - z_{t+i}) \right] \right\}. \tag{A.20}$$

where $\Psi \equiv \{r_t, \pi_t, x_t, e_t, a_{t+1}, b_{t+1}, z_{t+1}\}$, $j = \pi, x, e$, and $\lambda_{n,t}$ with $n = 1, 2, \dots 6$ are Lagrange multipliers that are respectively associated with (2)-(4) in which we substitute $E_t^* \pi_{t+1} = a_t$, $E_t^* x_{t+1} = b_t$ and $E_t^* e_{t+1} = z_t$, and (5)-(7). Compared to the decision problem in the benchmark, there are additional first-order conditions with respect to private expectations because as expectations deviate from full rationality, they become state variables but also new control variables for the CB as long as the latter desires to influence private beliefs.

Differentiating the Lagrangian (A.20) with respect to r_t, π_t, x_t, e_t, a_{t+1}, b_{t+1}, z_{t+1}, h_t^{π}, h_t^{x} and h_t^{e} leads to the following first-order conditions:

$$\frac{\partial \mathscr{L}_t^{CB}}{\partial r_t} = 0 \quad \Rightarrow \quad -\sigma^{-1} \lambda_{2,t} - \lambda_{3,t} = 0, \tag{A.21}$$

$$\frac{\partial \mathscr{L}_t^{CB}}{\partial \pi_t} = 0 \quad \Rightarrow \quad \pi_t - \lambda_{1,t} + \gamma \lambda_{4,t} = 0, \tag{A.22}$$

$$\frac{\partial \mathscr{L}_t^{CB}}{\partial x_t} = 0 \quad \Rightarrow \quad \alpha x_t + \kappa \lambda_{1,t} - \lambda_{2,t} + \gamma \lambda_{5,t} = 0, \tag{A.23}$$

$$\frac{\partial \mathscr{L}_t^{CB}}{\partial e_t} = 0 \quad \Rightarrow \quad \phi \lambda_{1,t} + \delta \lambda_{2,t} - \lambda_{3,t} + \gamma \lambda_{6,t} = 0, \tag{A.24}$$

$$\frac{\partial \mathscr{L}_t^{CB}}{\partial a_{t+1}} = 0 \quad \Rightarrow \quad -\lambda_{4,t} + \beta E_t \left[\beta \lambda_{1,t+1} + \sigma^{-1} \lambda_{2,t+1} + \lambda_{3,t+1} + \lambda_{4,t+1}(1-\gamma) \right] = 0, \tag{A.25}$$

$$\frac{\partial \mathscr{L}_t^{CB}}{\partial b_{t+1}} = 0 \quad \Rightarrow \quad -\lambda_{5,t} + \beta E_t \left[\lambda_{2,t+1} + \lambda_{5,t+1}(1-\gamma) \right] = 0, \tag{A.26}$$

$$\frac{\partial \mathscr{L}_t^{CB}}{\partial z_{t+1}} = 0 \quad \Rightarrow \quad -\lambda_{6,t} + \beta E_t \left[-\delta \lambda_{2,t+1} + \lambda_{3,t+1} + \lambda_{6,t+1}(1-\gamma) \right] = 0, \tag{A.27}$$

$$\frac{\partial \mathscr{L}_t^{CB}}{\partial h_t^\pi} = 0 \quad \Rightarrow \quad \lambda_{1,t} = \theta^\pi h_t^\pi, \tag{A.28}$$

$$\frac{\partial \mathscr{L}_t^{CB}}{\partial h_t^x} = 0 \quad \Rightarrow \quad -\theta^x h_t^x + \lambda_{2,t} = 0, \tag{A.29}$$

$$\frac{\partial \mathscr{L}_t^{CB}}{\partial h_t^e} = 0 \quad \Rightarrow \quad -\theta^e h_t^e + \lambda_{3,t} = 0. \tag{A.30}$$

Equation (A.21) implies that $\lambda_{3,t+1} = -\sigma^{-1}\lambda_{2,t+1}$. Substituting $\lambda_{3,t+1}$ into (A.27) gives $-\lambda_{6,t} + \beta E_t \left[-(\delta + \sigma^{-1})\lambda_{2,t+1} + \lambda_{6,t+1}(1-\gamma) \right] = 0$. From the previous equation and (A.26), we find that a possible set of solutions for $\lambda_{5,t}$ and $\lambda_{6,t}$ must verify the following condition:

$$\lambda_{6,t} = -(\delta + \sigma^{-1})\lambda_{5,t}. \tag{A.31}$$

After having substituted $\lambda_{6,t}$ given by (A.31) into (A.24), and $\lambda_{3,t}$ by $\lambda_{3,t} = -\sigma^{-1}\lambda_{2,t}$, we deduce from the resulting equations and (A.23) that: $\lambda_{1,t} = -\frac{\alpha(1+\sigma\delta)}{\theta^\pi[\kappa(1+\sigma\delta)+\sigma\phi]}x_t$ and $\lambda_{5,t} = \frac{1}{\gamma}\lambda_{2,t} + \frac{\alpha\sigma\phi}{\gamma[\kappa(1+\sigma\delta)+\sigma\phi]}x_t$. Using the expression of $\lambda_{1,t}$ into (A.22) yields that $\lambda_{4,t} = -\frac{1}{\gamma}\left\{ \pi_t + \frac{\alpha(1+\sigma\delta)}{[\kappa(1+\sigma\delta)+\sigma\phi]}x_t \right\}$.

We now look for the intertemporal trade-off condition implied by (A.21) and (A.25). Substituting $\lambda_{1,t}$ and $\lambda_{4,t}$ obtained in the above into (A.25) and using (A.21) to eliminate $\lambda_{2,t+1}$ and $\lambda_{3,t+1}$ lead to the intertemporal optimal trade-off condition for the CB between stabilizing inflation and the output gap in periods t and $t+1$:

$$\pi_t + \frac{\alpha(\delta + \sigma^{-1})}{\left[\kappa(\delta + \sigma^{-1}) + \phi\right]}x_t = \beta(1-\gamma)E_t\pi_{t+1} + \frac{\alpha\beta(\delta + \sigma^{-1})[1 - \gamma(1-\beta)]}{\left[\kappa(\delta + \sigma^{-1}) + \phi\right]}E_t x_{t+1}. \tag{A.32}$$

For $\gamma = 0$, the optimality condition $\lambda_{4,t} = -\frac{1}{\gamma}\left\{ \pi_t + \frac{\alpha(1+\sigma\delta)}{[\kappa(1+\sigma\delta)+\sigma\phi]}x_t \right\}$ is verified only when $\pi_t + \frac{\alpha(1+\sigma\delta)}{[\kappa(1+\sigma\delta)+\sigma\phi]}x_t = 0$.

For $\gamma > 0$, the intertemporal trade-off is reflected by the terms associated with $E_t\pi_{t+1}$ and $E_t x_{t+1}$ at the right-hand side of (A.32). Given that inflation and output-gap targets are both zero, the right-hand side of (A.32) could be either positive or negative with its value depending on γ. When the right-hand side of (A.32) is positive, it is optimal to contract more sharply the output gap in the present compared to the policy under RE and discretion and *vice versa*.

The system of equations (5)-(7) and (A.35)-(20) can be solved to find the equilibrium

solutions of a_t, b_t, z_t, π_t, x_t, r_t, e_t and h_t^x and then using (21) and (23) to obtain the equilibrium solutions of h_t^π and h_t^e.

Replacing $\lambda_{1,t} = -\frac{\alpha(1+\sigma\delta)}{\theta^\pi[\kappa(1+\sigma\delta)+\sigma\phi]}x_t$ into (A.28) yields:

$$h_t^\pi = -\frac{\alpha(1+\sigma\delta)}{\theta^\pi[\kappa(1+\sigma\delta)+\sigma\phi]}x_t. \tag{A.33}$$

Notice that we cannot obtain a simple relationship between h_t^π and π_t under learning in the open economy.

Conditions (A.21) and (A.29)-(A.30) can be arranged to obtain $\frac{\lambda_{2,t}}{\lambda_{3,t}} = \frac{\theta^x h_t^x}{\theta^e h_t^e} = -\sigma$, implying

$$h_t^e = -\frac{\theta^x}{\sigma\theta^e}h_t^x, \tag{A.34}$$

meaning that coefficients in the solution of h_t^e are proportional to those in the final solution of h_t^x.

We now replace h_t^π given by (A.33) and $E_t^*\pi_{t+1} = a_t$ into the Phillips curve (2) to obtain

$$\pi_t = \beta a_t + \left[\kappa - \frac{\alpha(1+\sigma\delta)}{\theta^\pi[\kappa(1+\sigma\delta)+\sigma\phi]}\right]x_t + \phi e_t + \varepsilon_t^\pi. \tag{A.35}$$

Then substituting into (A.32) the expressions of x_t and $E_t x_{t+1}$ that are drawn from (A.35) and using $a_{t+1} = a_t + \gamma(\pi_t - a_t)$ implied by the learning algorithm (5) give:

$$E_t\pi_{t+1} = A_{11}\pi_t + A_{12}a_t + A_{13}E_t e_{t+1} + A_{14}e_t + P_1\varepsilon_t^\pi, \tag{A.36}$$

where

$$A_{11} = \frac{\kappa\theta^\pi[\kappa(1+\sigma\delta)+\sigma\phi]-\alpha(1-\theta^\pi)(1+\sigma\delta)+\theta^\pi\alpha\gamma\beta^2(1+\sigma\delta)[1-\gamma(1-\beta)]}{\beta(1-\gamma)\{\kappa\theta^\pi[\kappa(1+\sigma\delta)+\sigma\phi]-\alpha(1+\sigma\delta)\}+\theta^\pi\alpha(1+\sigma\delta)[1-\gamma(1-\beta)]},$$

$$A_{12} = \frac{\beta\theta^\pi(1-\gamma)(1+\sigma\delta)\left[\alpha\beta^2\gamma+\alpha\beta(1-\gamma)\right]-\alpha\beta\theta^\pi(1+\sigma\delta)}{\beta(1-\gamma)\{\kappa\theta^\pi[\kappa(1+\sigma\delta)+\sigma\phi]-\alpha(1+\sigma\delta)\}+\theta^\pi\alpha(1+\sigma\delta)[1-\gamma(1-\beta)]},$$

$$A_{13} = \frac{-\phi\theta^\pi(1+\sigma\delta)\left[\alpha\beta^2\gamma+\alpha\beta(1-\gamma)\right]}{\beta(1-\gamma)\{\kappa\theta^\pi[\kappa(1+\sigma\delta)+\sigma\phi]-\alpha(1+\sigma\delta)\}+\theta^\pi\alpha(1+\sigma\delta)[1-\gamma(1-\beta)]},$$

$$A_{14} = \frac{\alpha\phi\theta^\pi(1+\sigma\delta)}{\beta(1-\gamma)\{\kappa\theta^\pi[\kappa(1+\sigma\delta)+\sigma\phi]-\alpha(1+\sigma\delta)\}+\theta^\pi\alpha(1+\sigma\delta)[1-\gamma(1-\beta)]},$$

$$P_1 = -\frac{\alpha(1+\sigma\delta)\theta^\pi}{\beta(1-\gamma)\{\kappa\theta^\pi[\kappa(1+\sigma\delta)+\sigma\phi]-\alpha(1+\sigma\delta)\}+\theta^\pi\alpha(1+\sigma\delta)[1-\gamma(1-\beta)]}.$$

Inserting $r_t - E_t^* \pi_{t+1}$ given by (4), h_t^e given by (A.34), $E_t^* x_{t+1} = b_t$ and $E_t^* e_{t+1} = z_t$ into the IS equation (3) and rearranging the terms lead to

$$x_t = b_t - (\delta + \sigma^{-1})(z_t - e_t) + \left(1 + \frac{\theta^x}{\sigma^2 \theta^e}\right) h_t^x - \sigma^{-1} \varepsilon_t^e + \varepsilon_t^x. \tag{A.37}$$

Using (A.23) and (A.26), $\lambda_{5,t} = \frac{1}{\gamma} \lambda_{2,t} + \frac{\alpha \sigma \phi}{\gamma [\kappa (1+\delta \sigma) + \sigma \phi]} x_t$, and (A.29) to eliminate the Lagrange multipliers, we get

$$h_t^x = \frac{\alpha \sigma \phi}{\theta^x [\kappa (1 + \sigma \delta) + \sigma \phi]} [\beta (1 - \gamma) E_t x_{t+1} - x_t] + \beta E_t h_{t+1}^x. \tag{A.38}$$

Substituting $E_t^* \pi_{t+1} = a_t$, $E_t^* e_{t+1} = e_t$ and h_t^e given by (A.34) into (4) yields

$$r_t = a_t + z_t - e_t - \frac{\theta^x}{\sigma \theta^e} h_t^x + \varepsilon_t^e. \tag{A.39}$$

The equilibrium solutions can be solved using a reduced but still complex system of ten equations, i.e., (5)-(7) and (A.33)-(A.39), given that learning algorithm adds three equations.

The system of equations (5)-(7) and (A.35)-(A.39) can be solved to find the equilibrium solutions of a_t, b_t, z_t, π_t, x_t, r_t, e_t and h_t^x and then using (A.33) and (A.34) to obtain the equilibrium solutions of h_t^π and h_t^e.

The dynamic nature of the above-mentioned system and the complex interactions between endogenous variables induced by learning and the openness of the economy imply that it is impossible to reduce this system to smaller and tractable subsystems that allow obtaining reasonably simple analytical solutions.

Consequently, we numerically simulate the model using calibrations proposed by Galí and Monacelli (2005) and Leitemo and Söderström (2008b) for the baseline small open economy model.

A.3 Determining transition functions under learning

The ALMs for inflation, the output gap, the exchange rate and the interest rate are function of inflation, output-gap, exchange-rate expectations, and cost-push, demand and exchange-rate shocks. With the help of Dynare, we simulate the equilibrium solutions using previously given parameters values. Note that we simulate the model with values of $\theta^\pi = \theta^x = \theta^e$ ensur-

ing the determinacy of the equilibrium for the entire set of $\gamma \in (0, 1)$. Dynare gives transition functions of inflation, the output gap, the exchange rate and the interest rate with one period lag as follows:

$$
\begin{bmatrix} \pi_t \\ x_t \\ e_t \\ r_t \end{bmatrix} = \begin{bmatrix} d_\pi^{cg} & d_x^{cg} & d_e^{cg} \\ k_\pi^{cg} & k_x^{cg} & k_e^{cg} \\ j_\pi^{cg} & j_x^{cg} & j_e^{cg} \\ m_\pi^{cg} & m_x^{cg} & m_e^{cg} \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ x_{t-1} \\ e_{t-1} \end{bmatrix} + \begin{bmatrix} d_a^{cg} & d_b^{cg} & d_z^{cg} \\ k_a^{cg} & k_b^{cg} & k_z^{cg} \\ j_a^{cg} & j_b^{cg} & j_z^{cg} \\ m_a^{cg} & m_b^{cg} & m_z^{cg} \end{bmatrix} \begin{bmatrix} a_{t-1} \\ b_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} d_{\varepsilon^\pi}^{cg} & d_{\varepsilon^x}^{cg} & d_{\varepsilon^e}^{cg} \\ k_{\varepsilon^\pi}^{cg} & k_{\varepsilon^x}^{cg} & k_{\varepsilon^e}^{cg} \\ j_{\varepsilon^\pi}^{cg} & j_{\varepsilon^x}^{cg} & j_{\varepsilon^e}^{cg} \\ m_{\varepsilon^\pi}^{cg} & m_{\varepsilon^x}^{cg} & m_{\varepsilon^e}^{cg} \end{bmatrix} \begin{bmatrix} \varepsilon_t^\pi \\ \varepsilon_t^x \\ \varepsilon_t^e \end{bmatrix}.
$$
(A.40)

The numerical simulations show that the ALMs obtained under learning displays substantial history dependence, meaning that there is inertia in the dynamics of inflation and the output gap.

The transition functions differ from the ALMs as defined by Evans and Honkapohja (2012). The latter are defined in terms of current values of private expectations and shocks.[27] We establish an equivalence between these transition functions and the ALMs using (5)-(7). The latter yield $a_{t-1} = \frac{1}{(1-\gamma)} a_t - \frac{\gamma}{(1-\gamma)} \pi_{t-1}$, $b_{t-1} = \frac{1}{(1-\gamma)} b_t - \frac{\gamma}{(1-\gamma)} x_{t-1}$, $z_{t-1} = \frac{1}{(1-\gamma)} z_t - \frac{\gamma}{(1-\gamma)} e_{t-1}$. Substituting a_{t-1}, b_{t-1} and z_{t-1} by their expressions into the transition functions (A.40), we obtain

$$
\begin{bmatrix} \pi_t \\ x_t \\ e_t \\ r_t \end{bmatrix} = \begin{bmatrix} \tilde{d}_\pi^{cg} & \tilde{d}_x^{cg} & \tilde{d}_e^{cg} \\ \tilde{k}_\pi^{cg} & \tilde{k}_x^{cg} & \tilde{k}_e^{cg} \\ \tilde{j}_\pi^{cg} & \tilde{j}_x^{cg} & \tilde{j}_e^{cg} \\ \tilde{m}_\pi^{cg} & \tilde{m}_x^{cg} & \tilde{m}_e^{cg} \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ x_{t-1} \\ e_{t-1} \end{bmatrix} + \begin{bmatrix} \tilde{d}_a^{cg} & \tilde{d}_b^{cg} & \tilde{d}_z^{cg} \\ \tilde{k}_a^{cg} & \tilde{k}_b^{cg} & \tilde{k}_z^{cg} \\ \tilde{j}_a^{cg} & \tilde{j}_b^{cg} & \tilde{j}_z^{cg} \\ \tilde{m}_a^{cg} & \tilde{m}_b^{cg} & \tilde{m}_z^{cg} \end{bmatrix} \begin{bmatrix} a_t \\ b_t \\ z_t \end{bmatrix} + \begin{bmatrix} d_{\varepsilon^\pi}^{cg} & d_{\varepsilon^x}^{cg} & d_{\varepsilon^e}^{cg} \\ k_{\varepsilon^\pi}^{cg} & k_{\varepsilon^x}^{cg} & k_{\varepsilon^e}^{cg} \\ j_{\varepsilon^\pi}^{cg} & j_{\varepsilon^x}^{cg} & j_{\varepsilon^e}^{cg} \\ m_{\varepsilon^\pi}^{cg} & m_{\varepsilon^x}^{cg} & m_{\varepsilon^e}^{cg} \end{bmatrix} \begin{bmatrix} \varepsilon_t^\pi \\ \varepsilon_t^x \\ \varepsilon_t^e \end{bmatrix}.
$$
(A.41)

where $\tilde{\ell}_\pi^{cg} \equiv \frac{(1-\gamma)\ell_\pi^{cg} - \gamma \ell_a^{cg}}{1-\gamma}$, $\tilde{\ell}_x^{cg} \equiv \frac{(1-\gamma)\ell_x^{cg} - \gamma \ell_b^{cg}}{1-\gamma}$, and $\tilde{\ell}_e^{cg} \equiv \frac{(1-\gamma)\ell_e^{cg} - \gamma \ell_z^{cg}}{1-\gamma}$, $\tilde{\ell}_n^{cg} \equiv \frac{\ell_n^{cg}}{1-\gamma}$ with $\ell = d, k, j, m$, and $n = a, b, z$. We numerically check that the absolute values of the composite coefficients on π_{t-1}, x_{t-1}, e_{t-1} and r_{t-1} in (A.41) are extremely close to zero and more pre-

---

[27]For example, the ALM for inflation would take the following form: $\pi_t = \tilde{d}_a^{cg} a_t + \tilde{d}_b^{cg} b_t + \tilde{d}_z^{cg} z_t + \tilde{d}_{\varepsilon^\pi}^{cg} \varepsilon_t^\pi + \tilde{d}_{\varepsilon^x}^{cg} \varepsilon_t^x + \tilde{d}_{\varepsilon^e}^{cg} \varepsilon_t^e$. Despite this difference, equations in (A.40) allow us to see clearly the effects of learning on the equilibrium values of endogenous variables.

cisely they are generally smaller than 1×10^{-5} for $\gamma \in (0, 1)$ so that the terms associated with π_{t-1}, x_{t-1}, e_{t-1} and r_{t-1} are negligible and equations in (A.41) can be considered as the ALMs of endogenous variables.