Shared-Appreciation Mortgages and Uninsurable Idiosyncratic Shocks

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Abstract: Shared-appreciation mortgage (SAM) contracts, which display payments indexed to a local house price, have been proposed as an alternative to alleviate the costs of recessions. Using a heterogeneous agent model with two types of agents (Borrowers and Savers), uninsurable idiosyncratic income risk, and calibrated to the US, this paper studies the effects, on both macroeconomic variables and welfare, of introducing such contracts. I find that equilibrium default rates, house price volatility, and welfare losses of both Borrowers and Savers following an unexpected negative shock on aggregate income, are smaller. Also, while this policy benefits Savers, only Borrowers with moderate/low mortgage and housing wealth levels are better-off (61% of Borrowers under the main calibration). Finally, if Borrowers are less patient, the fraction that benefits may never be above 50%.

Keywords: Mortgage design, Heterogeneous agents, Housing policy.
JEL Classification: G00, C61, E44

Resumen: Las hipotecas de apreciación compartida (SAM por sus siglas en inglés), las cuales poseen pagos indexados a un índice local de precio de vivienda, fueron concebidas para aminorar los impactos de recesiones. Utilizando un modelo de agentes heterogéneos con dos tipos hogares (Prestatarios y Ahorradores), riesgo idiosincrásico no asegurable, y calibrado para EUA, este documento estudia los efectos de la introducción de contratos SAM sobre variables macroeconómicas y el nivel de bienestar de los hogares. Se encuentra que la tasa de impago, la volatilidad del precio de viviendas y las pérdidas de bienestar de prestatarios y ahorradores tras un choque negativo no esperado en el ingreso agregado, son menores. Asimismo, si bien esta política beneficia a los Ahorradores, únicamente los Prestatarios con hipotecas y riqueza inmobiliaria moderadas/bajas se benefician (61% de Prestatarios con la calibración principal). Finalmente, si los Prestatarios son menos pacientes, la fracción beneficiada podría nunca superar el 50%.

Palabras Clave: Diseño de contratos hipotecarios, Agentes heterogéneos, Política de vivienda.

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1 Introduction

Both housing and mortgage markets were at the center of the Great Recession. Excessive mortgage debt, along with collapsing house prices led to painful household deleveraging and costly foreclosure. As a result, with the aim of mitigating the effects of that recession and future ones, policy makers around the world introduced a series of macro prudential policies.¹ Demand-side measures include limits on loan-to-value ratios and debt-service-to-income ratios. Limits on bank credit growth, loan contract restrictions, and loan loss provisions are examples of supply-side measures.

Increasing the flexibility of mortgage contracts may also help lessen the negative consequences of recessions. For instance, risk sharing between mortgage lenders and borrowers could be widened, which is the goal of shared-appreciation mortgage (SAM) contracts advocated by Shiller (2014) and Mian and Sufi (2014). In this state-contingent equity-like design of mortgage contracts, the principal or payments are written down if a local house price index falls, whereas they increase if such price index rises.²

This paper studies both the effects of introducing SAM contracts on house prices and other macroeconomic variables, and the welfare consequences of such policy. Specifically, I consider an economy that switches from traditional fixed-rate mortgages to contracts in which payments display certain degree of indexation to a local house price, in the context of a general equilibrium model with housing, lack of commitment, which also displays default in equilibrium. The welfare implications for households with different levels of assets and housing wealth are examined in detail.

The model follows the tradition of Kiyotaki and Moore (1997) and Aiyagari (1994), in that it has two types of agents with different levels of patience, who are also subject to idiosyncratic income shocks.³ In equilibrium, the average impatient household borrows (Borrower) whereas the average patient household saves (Saver). In addition, and departing from the two former papers, households cannot commit to honor their mortgage obligations: If they choose to default, they lose their housing stock. I also introduce an idiosyncratic house depreciation shock which, along with the lack of commitment, results in a fraction of households defaulting every period. I assume that payments in SAM contracts are indexed to the realization of

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²This contract is different from one available in the UK in the early 2000s, in which homeowners share all house price appreciations with lenders, but do not get any payment relief during house price falls. See Sanders and Slawson 2005).
³As in Aiyagari (1994), there is no aggregate uncertainty.
this idiosyncratic depreciation shock, but only up to 25 percent. Finally, all mortgages are one-period contracts.

The SAM contract considered in this paper renders insurance against negative shocks that decrease the value of a house, as it provides payment relief when a household faces bad realizations of the idiosyncratic depreciation shock. In addition, higher degrees of indexation generally reduce the probability of default for a given level of housing stock and mortgage debt; this can translate into lower mortgage interest rates and facilitates the taking of more debt to finance larger houses. However, this contract also entails higher mortgage payments for good shock realizations, which could represent a considerable fraction of income when a household is highly indebted.

Using a calibration that matches U.S. data, I find that house prices are monotonically increasing with indexation, which is driven by a higher housing demand from Borrowers given that they take more debt. However, despite being increasing in the indexation level, following an unanticipated one-period-lived shock on aggregate income, higher levels of indexation reduce the volatility in the response of house prices, at least for indexation levels below 25 percent. Additionally, I find that default rates are monotonically decreasing in the degree of indexation.

With SAM contracts, welfare losses of both the average Borrower and the average Saver following a negative shock on aggregate income are smaller and decreasing with the level of indexation. Also, switching to SAM contracts generates net welfare gains for Savers, relative to the case with non state-contingent mortgage contracts. However, welfare gains for the

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4Greenwald et al. (2018) refers to this case as local indexation, as opposed to an aggregate indexation scheme in which payments are indexed to changes in the aggregate house price. I only consider local indexation as my model does not exhibit aggregate uncertainty. A further discussion of these two types of indexation is given in the Related Literature section.

5This is because this shock plays the role of both a pure idiosyncratic shock and a local depreciation shock. Greenwald et al. (2018), who consider two separate shocks, estimate that local shocks account for 25 percent of the total standard deviation of individual house prices. Taking that contribution of local shocks as given, I explore whether using one depreciation shock instead of two changes the results considerably, in a simplified version of the model that generates similar aggregate dynamics. I find that having two shocks and full indexation to the shock with 25-percent volatility contribution, has very similar results to the case of 25-percent indexation of a unique shock.

6Results are qualitatively similar if long-term contracts are considered.

7This is not always the case, as it is shown in Greenwald et at. (2018): only in the so-called local indexation setups, lower default probabilities translate into lower mortgage interest rates.

8I define volatility of certain variable, in the context of a model without aggregate uncertainty, as the largest deviation of that variable with respect to its steady state level, along the transition path following a one-period-lived shock on the mean of the income process.
average Borrower are positive only for indexation levels below 22 percent. In fact, gains peak at 10-percent indexation (SAM-10) and fall monotonically afterwards. It is also important to mention that, with higher degrees of indexation, the moral hazard risk of underinvestment in maintenance, which this paper abstracts from, becomes larger.\textsuperscript{9} As a consequence, as this model considers one depreciation shock, only results with low levels of indexation should be considered.\textsuperscript{10}

Despite the fact that the average Borrower benefits from SAM contracts for most reasonable indexation degrees, not every household of this type is strictly better off: only those with moderate/low debt and housing wealth levels, who add up to 61 percent of Borrowers with SAM-10, benefit from the introduction of SAM contracts. There is one main force driving this result. In this model, impatient households finance housing expenditures through mortgages, which is reflected in the fact that the stationary equilibrium of the model exhibits a strong correlation between mortgage debt and housing holdings, a correlation that is also found in the data.\textsuperscript{11} Thus, Borrowers with high housing wealth also have high mortgage debt. And, as was previously mentioned, when the level of mortgage debt is higher, the payment burden is large in the face of good realizations of the depreciation shock.

Motivated by the heterogeneity on welfare gains when switching to SAM contracts, I also examine the fraction of Borrowers with positive gains for different levels of indexation. I find that more than half of Borrowers exhibit positive gains for indexation degrees below 21 percent. This fraction peaks at 5-percent indexation and falls monotonically afterwards. This result suggests that, under the main calibration used in this paper, welfare gains of the average Borrower are correlated with the fraction of Borrowers who benefit from SAM contracts. Moreover, the sign of the average Borrower's welfare change may predict whether this fraction is above or below 50 percent.

However, a sensitivity analysis shows that the latter is not always the case. For instance, if Borrowers are impatient enough, the fraction of Borrowers with positive gains may never be above 50 percent. Conversely, if Borrowers are more patient, the mass of Borrowers who benefit is higher for every level of indexation, even though the average Borrower mostly

\textsuperscript{9}See Sanders and Slawson (2005) for further details on this point.
\textsuperscript{10}This moral hazard risk is the main reason behind the fact that advocates of SAM contracts suggest indexation to a local price as opposed to an individual house price. As local prices account for 25 percent of the standard deviation of individual house prices, only indexations up to 25 percent could be considered reasonable in this model.
\textsuperscript{11}More technically, almost all the mass of probability in the stationary distribution is concentrated on an "diagonal" set that displays a positive relation between mortgage debt and housing wealth.
displays welfare losses. Thus, this finding highlights the importance of explicitly modelling the heterogeneity of households of the same type to have a better understanding of the welfare effects of such policy.

**Related Literature**

This paper primarily relates to the strand of literature studying mortgage design; in particular, to the one that analyzes state-contingent mortgage contracts, pioneered by Shiller and Weiss (1999). These authors propose home-equity insurance that resembles both ordinary homeowner insurance and financial hedging vehicles. Shiller et al. (2013a) work the pricing of SAMs that are tied to a local home price and show how this local indexing solves the moral hazard problem associated to tying payments to individual home prices. Shiller et al. (2013b) implement these contracts in a partial equilibrium framework and find substantial welfare gains.

Piskorski and Tchistyi (2010; 2011), using an optimal contract framework, find that the optimal mortgage resembles an adjustable-rate mortgage, in an environment with stochastic interest rates and house prices. Guren, Krishnamurthy, and McQuade (2018) study the ways in which, adding simple state-contingency can improve the performance of fixed-rate mortgages during recessions. They find that, if the central bank reduces interest rates during a crisis, fixed-rate mortgages with the option to convert costlessly into adjustable-rate mortgages do this job best.

Greenwald et al. (2018), which is the closest to this paper, study the effects of introducing SAM contracts, with a focus on system-wide risk management, in a rich framework with big families of Borrowers, Savers, and financial intermediaries, and perfect insurance inside each family. They find that indexing mortgage debt to aggregate house prices increases financial fragility, despite the fact that default rates are lower. On the other hand, indexation to a local house price reduces both mortgage defaults and financial fragility. Moreover, local indexation increases the welfare of both Borrowers and Savers, but reduces it for Intermediaries.

Relative to their work, my contribution consists in lifting the big-family assumption in a framework otherwise similar to theirs and study the distributional effects of introducing

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12 Early works also include Caplin, Chan, Freeman, and Tracy (1997); Caplin, Carr, Pollock, and Tong (2007); and Caplin, Cunningham, Engler, and Pollock (2008).

13 This means that all households belonging to the family of Borrowers get the same level of consumption. The same applies for Savers and Intermediaries.

14 This is implemented by the introduction uninsurable idiosyncratic income risk for Borrowers.
a locally indexed contract. In particular, I find that skipping the big-family assumption unveils a considerable amount of welfare gains heterogeneity and that the gains of the average member of one type of households do not necessarily reflect the welfare effects of most of them. In that sense, this paper complements their work.

Piskorski and Tchistyi (2018) study state-contingent mortgage contracts in a two-period model with informational asymmetries and endogenous mortgage design. They find that when lenders are uncertain about the private value borrowers attach to their homes, the equilibrium contract only depends on house prices and takes the form of a home equity insurance mortgage. They also find that when an economy switches to those contracts, the effects on household welfare depend on how severe economic downturns are: the more severe they are, the more welfare-improving the equilibrium state-contingent contract is over a standard fixed-rate contract. The quantitative approach of this paper, which a focus on household risk management and volatility of house prices, complements their work.

Other theoretical and quantitative works study potential reasons behind the fact that risk-sharing mortgage contracts are not widely used in the real world, even though they seem to Pareto-dominate non-state-contingent contracts. Hartman-Glaser and Hébert (2019) build a framework in which the price index used in SAM contracts could be a poor measure of the true state Borrowers face. They provide conditions for which this effect is strong enough so that Borrowers choose contracts without insurance over indexed contracts. Fazilet (2017) develops a model with heterogeneous agents subject to idiosyncratic risk and both local and aggregate house price shocks, and finds that, in a world in which the government does not implicitly subsidize fixed-rate mortgages, mortgage contracts that are contingent on house prices are chosen over FRMs.

More generally, this paper relates to several strands of literature on housing and financial macroeconomics. This paper is also related to the literature that studies financial frictions

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15 Savers in this paper encompasses both Savers and Intermediaries in theirs. Also, unlike their work, the model in this paper does not display aggregate uncertainty. Finally, while they consider 30-year FRM contracts with exogenously determined rates (that include default risk and are pinned downed in equilibrium), mortgage contracts in this paper are one-year FRMs and give households access to a whole menu of amount-of-debt/amount-of-collateral/interest-rate combinations.


17 Mian and Sufi (2014) argue that SAM contracts are not observed because of government policies that favor fixed-rate contracts such as: mortgage interest tax deductions, Government Sponsored Enterprises actions in the secondary mortgage markets (an implicit interest-rate subsidy), and bank regulations on risk weighted capital requirements.

18 See Davis and Van Nieuwerburgh (2015) for a review of this literature.
and the business cycle, pioneered by Bernanke, Gertler, and Gilchrist (1996) and Kiyotaki and Moore (1997).\textsuperscript{19} Finally, modeling-wise, this paper is connected to works that rely on general equilibrium models with heterogeneous agents subject to uninsurable idiosyncratic risk, housing, and defaultable mortgage contracts with endogenous interest rates. These include Corbae and Quintin (2014), Chatterjee and Eyigungor (2011), Guler (2015) and Jeske, Krueger, and Mitman (2013).

The rest of the document is organized as follows. The model is presented in Section 2, while details on the calibration are presented in Section 3. Section 4 reports the main results, whereas section 5 concludes.

2 The Model

2.1 Environment

\textbf{Endowments.} The economy has two types of goods. First, there is an idiosyncratic endowment of a non-durable good $\mu_y$, where $y$ follows a Markov process with unconditional mean equal to one, and $\mu_y$ is a constant scale factor. As $\mu_y$ is assumed to be non-stochastic, there is no aggregate uncertainty, which implies by the Law of Large Numbers that the aggregate endowment of the non-durable good is equal to $\mu_y$. This endowment can be interpreted as labor income when labor supply is fixed.

Second, there is perfectly divisible durable good (housing) in fixed supplied normalized to $H_s$.

\textbf{Preferences.} There are two types of households, a measure $\psi$ of Impatient households ("Borrowers") with discount factor $\beta$; and a measure $(1-\psi)$ of Patient households ("Savers") with discount factor $\tilde{\beta}$, where $\beta < \tilde{\beta}$. Throughout the paper, for decision variables common to both types of households, $x$ denotes Borrowers choices while $\tilde{x}$ represents Savers choices.

Borrowers get a fraction $\kappa$ of the nondurable aggregate endowment $\mu_y$, while Savers get the remaining fraction $1 - \kappa$. Households derive period utility $u()$ and $\widetilde{u}()$ from nondurable consumption ($c$ for Borrowers, $\widetilde{c}$ for Savers) and housing consumption which is proportional

\textsuperscript{19}See Jermann and Quadrini (2012) for a synthesis of the main types of financial frictions proposed in the literature applied in stylized model.
to the housing stock owned in that period ($h$ for Borrowers, $\widetilde{h}$ for Savers). The housing good can be purchased every period at price $p$ (relative to the nondurable good).

There is also a competitive bank, which is owned by Patient households ("Savers").

**Assets.** Households can buy, from the competitive bank, one-period deposits $d'$ that pay a risk-free rate $r^d$. Households can also purchase houses at price $p$, set in terms of the nondurable good. Houses are risky assets, subject to both aggregate risk (given by the endowment $y$) and idiosyncratic depreciation shock $\omega$. At the beginning of each period, each household faces a realization of $\omega$ so that the effective housing stock is $\omega h_{-1}$. The depreciation shock $\omega$ is i.i.d. across households, has lognormal cumulative distribution $F(\omega)$, $E(\omega) = 1$, and $\sigma = \text{var}(\ln \omega)$.

**Mortgages.** Households have access to a one-period fixed-rate mortgage contract offered by the competitive bank. Let $Q$ denote the price schedule of such contract. If a household takes a new mortgage, she gets $Qm'$ in the current period and agrees to make a payment $m'$ on the next period.

In addition to the FRM defined above, a SAM contract is introduced. In this contract, payments are indexed to the idiosyncratic depreciation shock $\omega$. Specifically, next period payment is given by $(\omega')^{\iota}m'$, where $\iota$ is an indexation parameter. Notice that, compared to a purely linear indexation scheme, this non-linearly indexed contract is more beneficial to households, because payment relief is greater for bad shocks, and additional payments are smaller for good shocks (see Figure 1 for a comparison of a 10-percent SAM indexation and 10-percent linear indexation). Also, when $\iota = 0$, the contract becomes a one-period fixed-rate contract. As the SAM contract encompasses the FRM, all definitions will be stated in terms of the former.

Finally, every household has the option to default on her mortgage obligations after observing the realization of its depreciation shock $\omega$. When default is chosen, a household loses its entire housing stock which is seized by the bank. There are no other costs for the household after default. The bank then sells the house incurring in a proportional cost $\mu$.

**Big Family of Savers.** As Borrowers are the main focus, it is also assumed that savers belong to large representative family of savers, so that they can diversify away any idiosyncratic risk.

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20 $u$ and $\widetilde{u}$ must meet the minimum requirements for utility functions: nondecreasing and quasi-concave.
21 The mortgage interest rate $r^m$ is then given by $(1 + r^m) = 1/Q$
22 No market exclusion and no recourse.
As a result, each household inside a family of savers consumes exactly the same amount of the durable good $\bar{c}$ and housing services $\bar{h}$. Also, at the end of each period, the family pools all its assets among its members.

**Aggregate state.** The aggregate state is given by the beginning-of-period distribution of housing stock and deposit/mortgages among the two types of households. I choose the asset distributions of Borrowers $(\Theta(h_{-1}, m))$, where $h_{-1}$ denotes the initial housing stock of a Borrowers and $m$ accounts for the initial promised mortgage payments for the period. Let $X = \{\Theta(h_{-1}, m)\}$ be the aggregate state of the economy.

![Figure 1: SAM and Linear Indexation](image)

**Note:** Percent of original mortgage payments as a function of omega ($\omega$), for 10-percent linear and SAM indexation.

### 2.2 Borrower’s Problem

Each Borrower starts the period with a portfolio $x = \{h_{-1}, m\}$ of housing stock and promised mortgage payments for that period. Each household also gets a realization of the non-durable good endowment $y$ and learns what its idiosyncratic depreciation shock $\omega$ is.

Borrowers make default/payment decisions regarding current period’s mortgage payments $\omega' m$, then chooses consumption $(c, h)$ along with next period’s total mortgage obligations $m'$ taking as given the mortgage contract with indexation $t$ and price schedule $Q$. I guess and
later verify that the default decision at the family level is characterized by a threshold $\bar{\omega}$. That is, the household honors the promised payment when $\omega > \bar{\omega}$ and defaults otherwise. Let $D(\omega)$ be the default function associated with a threshold $\bar{\omega}$.

Given the mortgage contract with indexation $i$ and price schedule $Q(h, m', X; i)$, house price $p(X, i)$, and future decision rules, the recursive problem of a Borrower consists of choosing nondurable consumption $c$, housing stock $h$, total promised mortgage payments $m'$ and a default decision $D(\omega)$ to solve

$$V(h_{-1}, m, y, X; i) = \max_{c, h, m', D} u(c, h) + \beta E_{/} V(h, m', y', X', i)$$

$$c + p(X, i)h + (1 - D(\omega))\omega'm = \frac{K}{\mu_y} \mu_y y + (1 - D(\omega))\omega ph_{-1} + Q(h, m', X; i) m'$$

Notice how functions explicitly depend on both the indexation parameter $i$. The left hand side of the budget constraint consists of nondurable consumption and housing consumption, as well as the promised mortgage payments $\omega'm$ conditional on the default decision $(1 - D(\omega))$. The right hand side includes the endowment of the nondurable good $y$, the value of houses kept conditional on the default decision $(1 - D(\omega))\omega ph_{-1}$, and the resources from additional mortgages taken in the current period, which are determined by tomorrow’s additional coupon payments $Q(h, m', X; i) m'$.

### 2.3 Saver’s Problem

Inside the representative family of Savers, each household starts the period with the same portfolio $(\h_{-1}, d)$ of housing stock and one-period deposits. The family collects the endowments of the non-durable goods from all members, $\mu_y$, which is equal to unconditional mean of $y$. It also collects the initial housing stock from all members, which is given by $\int \omega h_{-1} d F(\omega) = E(\omega)h_{-1} = \h_{-1}$.\footnote{Because $E(\omega) = 1$, the initial stock of housing, after all $\omega$ are realized, remains constant. Notice that, at this stage, there is heterogeneity at the member’s level. However, the family pools its total housing stock among its members, and the heterogeneity disappears.} Given the house price $p(X; i)$ and the risk-free interest rate $r^d(X, i)$, the recursive problem of a representative family of Savers consists of choosing nondurable consumption $\bar{c}$, housing stock $\h$, and new deposits $d'$ to solve
Notice that even though households in the representative family of Savers are also subject to idiosyncratic depreciation shocks, they are completely unaffected from this because, in equilibrium, they do not take any debt.

2.4 Banks and the mortgage price schedule

The competitive bank is owned by Savers, so when choosing a mortgage price schedule, they take into account Savers’ stochastic discount factor (SDF). However, since there is no aggregate uncertainty, the SDF is always equal to one. Banks also take as given Borrowers’ future decision rules, including the default decision. In equilibrium, given administrative costs $\theta$, the mortgage price schedule $Q(h, m', X; i)$ satisfies:

$$Q(h, m', X; i) = \frac{\Gamma(h, m', X'; i)}{(1 + r^d(X))(1 + \theta)}$$

where $\Gamma$ satisfies

$$\Gamma(h, m', X'; i)m' = \int_{\omega_i}^{\infty} \omega d F(\omega) m' + (1 - \mu) \int_{0}^{\infty} \omega d F(\omega) p(X'; i)h$$

The function $\Gamma$ accounts for the resources the bank gets for every unit of next period’s promised coupon payment, given the household’s total collateral $h$ and the total promised coupon $(\omega')'m'$. It consists of two parts. The first one accounts for the non-defaulted fraction $\int_{\omega_i}^{\infty} \omega d F(\omega)$ of next period’s coupon payment $m'$. The second part is the value of the houses associated with defaulted mortgages $\int_{0}^{\infty} \omega d F(\omega) p(X'; i)h$, net of the foreclosure cost $\mu$.

Finally, because there is no aggregate uncertainty, dividends are equal to zero.

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$^{24}$One way alternative way to interpret this payoff function is by assuming that banks live for two periods. In the first period, they get deposits from Savers to buy a diversified portfolio of mortgages. In the second period, banks meet their deposit obligations with funds collected from non-defaulted coupon payments and from selling the sized houses.
2.5 Stationary Equilibrium

Let $s = \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+$ denote the individual state space of Borrowers, $\tilde{s} = \mathbb{R}_+ \times \mathbb{R}_+$ the individual state space for Savers, and $S$ be the aggregate state space.

A stationary recursive competitive equilibrium is a collection of decision rules of Borrowers $c, m', h, \bar{c} : s \times S \rightarrow \mathbb{R}$; decision rules of Savers $\bar{c}, \bar{h}, \bar{d} : \tilde{s} \times S \rightarrow \mathbb{R}$; associated value functions $V : s \times S \rightarrow \mathbb{R}$ and $\bar{V} : \tilde{s} \times S \rightarrow \mathbb{R}$; future decision rules $g^c, g^m, g^h, g^\bar{c} : s \times S \rightarrow \mathbb{R}$; prices $p, r^d : S \rightarrow \mathbb{R}$, mortgage price schedule $Q : s \times S \rightarrow \mathbb{R}$ and distribution $\Theta$ such that:

1. Decision rules and value functions solve both households’ problems, taking future decision rules, $p, r^d$, and $Q$ as given.

2. All markets clear

$$
\psi \left[ \int \left( c + \mu \int D(\omega)\omega ph_{-1} d F(\omega) + \theta Qm' \right) d \Theta \right] + (1 - \psi)\bar{c} = \mu_y
$$

$$
\psi \int h \ d \Theta + (1 - \psi)\bar{h} = H_s
$$

$$
(1 + \theta)\psi \int Qm' \ d \Theta = (1 - \psi)\frac{d'}{(1 + r^d)}
$$

3. $\Theta$ is a stationary probability measure.


2.6 Characterization of Equilibrium

This section develops the equilibrium conditions of some of the decision variables. In the case of Borrowers, the optimal default decision satisfies:

$$
\bar{c} ph_{-1} = \bar{c}' m
$$
This condition is just equating the current cost of defaulting, which is given by the loss of housing stock of value $\bar{\sigma} ph_{-1}$, with the of honoring the mortgage obligation, $\bar{\sigma}m$. On the other hand, the FOCs for the family of Savers reads:

$$\begin{align*}
\tilde{u}_{\gamma} &= \beta(1 + r^d)\tilde{u}_{\gamma} \\
\tilde{p}\tilde{u}_{\gamma} &= \eta u_{\tilde{\gamma}} + \tilde{p}\beta\tilde{u}_{\gamma}
\end{align*}$$

where, in the case of the stationary equilibrium, $p = p'$ and $u_{\gamma} = u_{\gamma'}$. From the first equation, the risk-free interest rate can be pinned down in the stationary equilibrium as $1 + r^d = 1/\beta$.

## 3 Calibration

A summary of the calibration for an annual frequency is shown in Table 1. Details are discussed below.

**Income Process.** The idiosyncratic non-durable good endowment $y$ is assumed to be an AR(1) process of the form:

$$\log y = \rho \log y_{-1} + (1 - \rho^2)^{1/2} \varepsilon$$

where $E(\varepsilon) = 0$, $E(\varepsilon^2) = \sigma^2$, and $\rho$ is the one-period autocorrelation, whereas $\sigma$ is the unconditional standard deviation. Notice that with this functional form, the unconditional mean of $y$ is equal to 1. Recent estimates\textsuperscript{25} of the income process for heterogeneous-agent models report $\rho = 0.98$ and $\sigma = 0.3$ on average. I choose those values, and approximate this AR(1) process with a 5-state Markov chain using Tauchen and Hussey’s (1991) algorithm.

Finally, $\mu_y$ is equal to 1.0 to calculate to stationary distribution, and set to 0.99 in the first period of a transition path and to 1.0 from period two onwards to emulate a one-period-lived shock to the mean of the income process.

**Foreclosure Cost.** A value a 0.22 is chosen for the foreclosure parameter $\mu$, following the work of Pennington-Cross (2006) studying the liquidation sales revenue from foreclosed houses using national data.

**Depreciation Shock.** The depreciation shock $\omega$ follows a lognormal distribution with mean one and $\sigma = \text{var}(\ln \omega)$. Notice that, in the model, both default and foreclosure take place.

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\textsuperscript{25}See Storesletten et al. (2004).
in the same period. In the real world, only a fraction of delinquent mortgages ends up being foreclosed two years after the initial date of default on average. Using data collected by ATTOM Data Solutions, the average delinquency rate on single-family residential mortgages during the period 2001-2017 was 5.25 percent, whereas the average foreclosure rate was 1.05 percent. A target of 3.0 percent for the default rate is chosen, which results in a value of 0.103 for $\sigma$.

**Demographics and Income Shares.** The mass of Borrowers ($\psi$) and Savers ($1 - \psi$) is pinned down by calculating a net financial-asset position for households in the 2016’s Survey of Consumer Finance (SCF-16). This net position equals total value of financial assets minus total value of debt. Borrowers are defined as those with a negative position, and represent 48 percent of households in the survey. Also, with this definition, they account for 32.5 percent of total household income, which is the value assigned to $\kappa$.

**Preferences.** The period utility functions have the form

$$u(c, h) = \ln c + \eta \ln h$$

$$\tilde{u}(\tilde{c}, \tilde{h}) = \ln \tilde{c} + \tilde{\eta} \ln \tilde{h}$$

Parameters $\eta$ and $\tilde{\eta}$ are chosen to match the average housing-wealth-to-income ratios of Borrowers and Savers on SCF-16, under the definition previously given. The average ratio for Borrowers is 4.14, while that of Savers is 4.51, which imply values of $\eta$ and $\tilde{\eta}$ of 0.063 and 0.060 respectively. The discount factor of Savers, $\tilde{\beta}$, is set at 0.99 to match an equilibrium risk-free rate of 1%. The discount factor of Borrowers, $\beta$, is set at 0.92 to match a mortgage-to-income ratio of 2.42 for Borrowers.

**Mortgage.** The administrative cost per unit of mortgage issued, $\theta$, is set at 40 basis points, following Jeske et al. (2013).

**Housing stock.** The median house-price-to-income ratio during the period 2015-2019 was 3.2. On the other hand, the median rent-to-income ratio was of only 0.2. An average of the two is chosen, 1.70, which is generated by a fixed housing stock $H_s$ of 2.5.

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26 Alternatively, $\eta$ and $\tilde{\eta}$ could be equated and set to match the share of housing in total consumption expenditures from NIPA. The average share is 13.9% for the period 2012-2016, which would imply a value of $\eta$ and $\tilde{\eta}$ of 0.161. However, such value generates housing-wealth-to-income ratio 4 times as high as those in SFC-16.  
27 In their paper, banks have to pay 10 basis points for administrative fees and 30 basis points for insurance.
Table 1: Calibration

<table>
<thead>
<tr>
<th>Exogenously Calibrated Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Param</strong></td>
</tr>
<tr>
<td>$\rho$</td>
</tr>
<tr>
<td>$\sigma_e$</td>
</tr>
<tr>
<td>$\mu$</td>
</tr>
<tr>
<td>$\theta$</td>
</tr>
<tr>
<td>$\psi$</td>
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<tr>
<td>$\kappa$</td>
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<tr>
<td>$H_s$</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Endogenously Calibrated Parameters</th>
</tr>
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<tbody>
<tr>
<td><strong>Parameter</strong></td>
</tr>
<tr>
<td>$\sigma$</td>
</tr>
<tr>
<td>$\tilde{\eta}$</td>
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<tr>
<td>$\eta$</td>
</tr>
<tr>
<td>$\tilde{\beta}$</td>
</tr>
<tr>
<td>$\beta$</td>
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</tbody>
</table>

4 Results

4.1 Aggregate Results

The main results of implementing 10-percent indexation SAM contracts (SAM-10), in aggregate terms, are shown in Table 2. A column containing the corresponding moments in data is also included. All variables, with the exception of welfare changes, are computed using values from the stationary distribution.

The baseline calibration with FRMs generates a mortgage interest rate 0.71 percentage points below the one observed in data (1.77% vs 2.48%), an expected result given that FRMs in the model are one-period, whereas in the data, the dominant product is the 30-year FRM. When switching to a SAM-10, the mortgage interest rate is 0.09 percentage points lower on average, which is explained in part by a lower default rate (0.36 percentage points below), despite the fact that SAM-10 contracts generates higher aggregate mortgage debt (4.5 percent above). House prices are 2.0 percent higher, which is mainly driven by a larger Borrowers’ housing...
demand. In fact, Borrowers’ housing wealth is 4.6 percent higher, sustained by a larger level of mortgage debt.

Even though the model does not exhibit aggregate uncertainty, the economy can be subject to unanticipated one-period-lived shocks to the mean of the income process \( \mu_y \), so that the response on different aggregate variables can be examined throughout the transition path. I define volatility in the context of this model as the peak response of a variable, following the aforementioned one-period-lived shock, over the level of that variable in the stationary equilibrium. With this definition in hand, house prices are slightly less volatile with SAM contracts.\(^{28}\) More importantly, net welfare losses upon an unexpected shock to \( \mu_y \) for both Borrowers and Savers are smaller.\(^{29}\)

I also compute aggregate welfare changes of switching to SAM contracts. To this end, I solve for the transition path of the economy, which at time 0 is at the stationary equilibrium with fixed-rate mortgages, and at time 1 switches to SAM-10 contracts. Then, the value functions at time 1 are compared to those in time 0.\(^{30}\) I find that, on average, both Borrowers and Savers benefit from switching to SAM contracts: Borrower’s welfare increase 0.25 percent, whereas Savers’ is 0.01 percent higher, both in consumption-equivalent terms.

\(^{28}\)Notice that the reduction in volatility is small in part because the house-price volatility generated by the model with FRMs (0.9%) is less than half of that observed in data during the period 2015-2019 (2.0%), where volatility of house prices in data is defined as the standard deviation over the mean of the ratio of house price over income.

\(^{29}\)Borrowers’ value function (utility level) is negative with FRM and becomes positive as indexation increases, which distorts welfare loss computations (peak response over initial level). However, as equilibrium results are invariant to monotonic transformations of the utility function, a positive constant can be added to the per-period utility of Borrowers to ensure that it is positive even for zero indexation. Results vary depending on the constant chosen, but all indicate a reduction in welfare losses of Borrowers in the face of a negative shock on \( \mu_y \). Taking this into account, a constant was chosen such that relative welfare losses are the same for Borrowers and Savers with FRMs.

\(^{30}\)In other words, I compare the value function when the economy stays with fixed-rate contracts (which is equal to that in time 0), with the value function of an economy switching to SAM contracts.
Table 2: Main Results

<table>
<thead>
<tr>
<th>Main Moments</th>
<th>Data(^a)</th>
<th>FRM</th>
<th>10-pct</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. House to Income Savers</td>
<td>4.51</td>
<td>4.51</td>
<td>4.47</td>
</tr>
<tr>
<td>2. House to Income Borrowers</td>
<td>4.14</td>
<td>4.14</td>
<td>4.34</td>
</tr>
<tr>
<td>3. Borrowers’ housing share</td>
<td>0.39</td>
<td>0.32</td>
<td>0.32</td>
</tr>
<tr>
<td>4. Mortgage to Income Borrowers</td>
<td>2.42</td>
<td>2.42</td>
<td>2.53</td>
</tr>
<tr>
<td>5. Mortgage rate</td>
<td>2.48%</td>
<td>1.77%</td>
<td>1.68%</td>
</tr>
<tr>
<td>6. Default rate</td>
<td>3.00%</td>
<td>2.99%</td>
<td>2.63%</td>
</tr>
<tr>
<td>7. House price</td>
<td>1.70</td>
<td>1.70</td>
<td>1.74</td>
</tr>
<tr>
<td>8. House price volatility</td>
<td>2.0%</td>
<td>0.981%</td>
<td>0.980%</td>
</tr>
<tr>
<td>9. Saver’s welfare loss upon (\mu_y) shock</td>
<td>-</td>
<td>0.9488%</td>
<td>0.9478%</td>
</tr>
<tr>
<td>10. Average Borrower’s welfare loss (\mu_y) shock</td>
<td>-</td>
<td>0.9488%</td>
<td>0.9483%</td>
</tr>
<tr>
<td>11. Saver’s CEV(^b) of switching</td>
<td>-</td>
<td>-</td>
<td>0.01%</td>
</tr>
<tr>
<td>12. Average Borrower’s CEV(^b) of switching</td>
<td>-</td>
<td>-</td>
<td>0.25%</td>
</tr>
</tbody>
</table>

\(^a\) Source: Survey of Consumer Finance (2016) and FRED-St. Louis FED.

\(^b\) CEV: Consumption Equivalent Variation

4.2 Distributional Results

In this subsection, the welfare change of Borrowers is studied in detail. To facilitate the analysis, Borrowers are split into 9 groups, according to their level of mortgage debt and housing wealth.\(^{31}\) The results are shown in Table 3. Despite the fact that the average Borrower benefits with an indexed contract, not every household of this type is strictly better off: only 61.4 percent of Borrowers have positive gains. In particular, Borrowers with high mortgage debt levels and high housing wealth are worse-off, as shown in Table 3C. The reason behind this result is as follows.

By construction, impatient households in the model finance housing expenditures through mortgages, which is reflected in the fact that the stationary equilibrium of the model exhibits a strong correlation between mortgage debt and housing holdings. More specifically, most of

\(^{31}\)These categories correspond to percentile intervals of either mortgage debt or housing wealth: low (0-33), medium (34-67), and high (68-100). The “low M” category also includes negative values of mortgage debt; that is, positive deposit holdings that pay the risk-free rate.
the mass of probability in the stationary distribution is concentrated on an "diagonal" set that displays a positive relation between mortgage debt and housing wealth. This correlation can be seen in Table 3B, which shows the initial stationary distribution considering the 9 groups previously defined, and highlights the aforementioned diagonal that concentrates most of the mass.\(^{32}\) This strong correlation is also found in the data, as Table 3A reports. As a result, Borrowers with high housing wealth also have high mortgage debt. And, as was previously discussed, high mortgage debt implies a larger payment burden in the face of good realizations of the depreciation shock.\(^{33}\)

### Table 3: Borrowers’ Welfare Change by level of Debt and Housing

<table>
<thead>
<tr>
<th></th>
<th>low H</th>
<th>medium H</th>
<th>high H</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>3A: Distribution in SFC-16(^a)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>low M</td>
<td>0.24</td>
<td>0.07</td>
<td>0.03</td>
</tr>
<tr>
<td>medium M</td>
<td>0.09</td>
<td>0.19</td>
<td>0.06</td>
</tr>
<tr>
<td>high M</td>
<td>0.00</td>
<td>0.08</td>
<td>0.25</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
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<th>low H</th>
<th>medium H</th>
<th>high H</th>
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</thead>
<tbody>
<tr>
<td><strong>3B: Distribution in Baseline Model</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>low M</td>
<td>0.32</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>medium M</td>
<td>0.01</td>
<td>0.33</td>
<td>0.00</td>
</tr>
<tr>
<td>high M</td>
<td>0.00</td>
<td>0.00</td>
<td>0.33</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>low H</th>
<th>medium H</th>
<th>high H</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>3C : Welfare Change in CEV(^b) (percent)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>low M</td>
<td>0.11</td>
<td>-0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>medium M</td>
<td>0.00</td>
<td>0.15</td>
<td>0.07</td>
</tr>
<tr>
<td>high</td>
<td>-0.34</td>
<td>0.13</td>
<td>-0.10</td>
</tr>
</tbody>
</table>

\(^a\) Source: Survey of Consumer Finance (2016), for households with negative net financial-asset positions. This position equals total value of financial assets minus total debt.

\(^b\) CEV: Consumption Equivalent Variation.

\(^{32}\) When further splitting the "low M" category, I find households with positive mortgage debt accounts for most of the probability mass in this group. In other words, virtually no impatient household has deposit holdings. This result, which is not always the case, holds because the discount factors of Borrowers and Savers are different enough.

\(^{33}\) Notice that this effect would be larger if linear indexation schemes were chosen instead.
The higher housing demand of Borrowers that results from switching to SAM-10 contracts is mainly driven by households with initially low and medium housing and mortgage debt, given that interest rates are now lower. This pushes the house price up so that, when facing a good realization of the depreciation shock, Borrowers with initially high mortgage debt find it optimal to sell some of their housing to afford the larger mortgage payment burden. As a result, the distribution of housing wealth becomes less unequal as it is shown in the left panel of Figure 2. Mortgage debt follows a similar pattern: households with initial low and medium debt take more of it, while those initially highly indebted take less of it. Notice that the patterns in housing wealth and mortgage debt have opposite effects on the Borrower’s distribution of net wealth (total assets minus total debt). For the main calibration, the total effect is negligible, as it is displayed on the right panel of Figure 2.

**Figure 2: Borrowers’ Lorenz’s Curves: Data vs Model**

With the goal of exploring a potentially optimal degree of indexation, I perform the same calculations for different levels of indexation. The results are shown in Figure 3, and include degrees of indexation up to 50 percent for reference purposes only.
Figure 3: Results for different levels of Indexation

Savers' Housing Wealth (Ratio over Income)

Borrowers' Housing Wealth (Ratio over Income, average)

Borrowers' Mortgage (Ratio over Income, average)

Mortgage Interest Rate (Percent)

Default Rate (Percent)

House Price (Level)

House Price Volatility (Percent)

Savers' Welfare (CEV, Percent)

Borrowers' Welfare (Percent)

Note: Housing wealth, mortgage debt, mortgage interest rate, default rate, and house price correspond to their stationary equilibrium levels. House price volatility is computed as the largest relative deviation of house price, along the transition path, following an unexpected one-period shock to aggregate income of -1.0%. Welfare changes, expressed as Consumption Equivalent Variation (CEV), are computed at time 1 of the transition path from the initial stationary equilibrium with fixed-rate mortgages to one with SAM contracts of different indexation levels.
The equilibrium default rate is decreasing with the level of indexation. This allows for the possibility of taking more mortgage debt to finance larger houses when indexation is higher, which ultimately happens in equilibrium: both mortgage debt and Borrowers’ housing consumptions are increasing in the level of indexation. This increasing housing demand of Borrowers translate into increasing house prices. However, whereas house prices increase with the indexation level, their volatility is decreasing.\textsuperscript{34} Also, switching to SAM contracts generates mostly increasing net welfare gains for Savers, relative to the case with non state-contingent mortgage contracts. However, welfare gains for the average Borrower are positive only for indexation levels below 22 percent (see lower right panel of Figure 3). In fact, gains peak at 10-percent indexation and fall monotonically afterwards.

Additionally, I examine how the fraction of Borrowers with positive gains varies with the level of indexation. I find that more than half of Borrowers exhibit positive gains for indexation degrees below 21 percent (see lower right panel of Figure 3). This fraction peaks at 5-percent indexation and falls monotonically afterwards. As a consequence, any indexation level between 5 and 10 percent would maximize the gains of the average Borrower and the fraction of Borrowers with positive gains. Moreover, this result implies that, for the calibration used in this paper, the welfare gains of the average Borrower are strongly correlated with the fact that the mass of Borrowers with positive gains is above 50 percent.

However, a sensitivity analysis shows that this last result is not always the case. This pattern similarity changes, for instance, when the level of patience of Borrowers is modified. Figure 4 shows two opposite cases: one in which Borrowers are more impatient ($\beta = 0.90$) and another which they are more patient ($\beta = 0.94$), both with respect to the main calibration ($\beta = 0.92$). If Borrowers are more impatient, the fraction of Borrowers with positive gains is never be above 50 percent. Conversely, if Borrowers are more patient, the mass of Borrowers who benefit is higher for every level of indexation, even though the average Borrower mostly displays welfare losses. Thus, this finding underscores the importance of explicitly modelling the heterogeneity of households of the same type to thoroughly evaluate the effects of introducing SAM contracts.

\textsuperscript{34}Volatility starts to increase with indexation levels higher than 60 percent. However, this result is not quantitatively relevant because the moral hazard risk of underinvestment in maintenance.
5 Conclusion

Shared-appreciation mortgage (SAM) contracts have been proposed as an alternative to alleviate the negative consequences of recessions. These contracts typically have payments that are indexed to a local house price. Using a heterogeneous agent model with two types of agents (Borrowers and Savers) and uninsurable idiosyncratic risk, this paper studies the effects, on the macroeconomy and welfare, of introducing SAM contracts. I find that, with SAM contracts, equilibrium default rates, the volatility of house prices, and welfare losses of both Borrowers and Savers (upon a negative aggregate income shock) are smaller. Also, switching to SAM contracts benefits the average Borrower and Saver. However, not every Borrower is strictly better-off: Borrowers with high mortgage levels and high housing wealth are worse-off.

This paper contributes to the strand of literature studying mortgage design, by analyzing the distributional effects of switching to SAM contracts, in the context of a general equilibrium model with uninsurable idiosyncratic income risk. Several extensions could be studied in this framework. These include: introducing long-term contracts to match data more precisely, adding aggregate uncertainty to better study the effects on volatility and welfare losses during recessions, and explicitly addressing the moral hazard risk of underinvestment in maintenance. These extensions are left for future research.
References


