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A Functional Approach to Test Trending Volatility*

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Abstract: In this paper we extend the traditional GARCH(1,1) model by including a functional trend term in the conditional volatility of a time series. We derive the main properties of the model and apply it to all agricultural commodities in the Mexican CPI basket, as well as to the international prices of maize, wheat, pork, poultry and beef products for three different time periods that implied changes in price regulations and behavior. The proposed model seems to adequately fit the volatility process and, according to homoscedasticity tests, outperforms the ARCH(1) and GARCH(1,1) models, some of the most popular approaches used in the literature to analyze price volatility.
Keywords: Agricultural prices, volatility, GARCH models
JEL Classification: C22, C51, E31, Q18

Resumen: En este documento extendemos el modelo tradicional GARCH(1,1) para incluir un término funcional de tendencia en la volatilidad condicional de una serie de tiempo. Derivamos las principales propiedades del modelo y lo aplicamos a todos los productos agrícolas de la canasta del INPC, así como a los precios internacionales del maíz, trigo, cerdo, aves de corral y productos de res para tres periodos diferentes que implicaron un cambio en las regulaciones y comportamiento de los precios. El modelo propuesto parece modelar de manera adecuada la volatilidad y de acuerdo a pruebas de homocedasticidad supera a los modelos ARCH(1) y GARCH(1,1), los cuales son algunos de los métodos más populares en la literatura para analizar la volatilidad de precios.
Palabras Clave: Precios de productos agrícolas, volatilidad, modelos GARCH

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1 Introduction

In 2001 Mexico adopted an inflation targeting regime which has been successful at reducing Consumer Price Index (CPI) inflation and bringing it to its objective of 3%. However, agricultural price inflation still remains as one of the main upside risks for inflation in the short run.[1] In particular, agricultural products (fruits and vegetables) show large fluctuations compared to the rest of agricultural products (Figure 1).

A confirmation of an increasing trend in the price volatility of agricultural products is a clear sign of an increase of short-run inflation risks. Moreover, having an assessment of the evolution of price volatility is important for policy makers. Policy strategies can be targeted according to the risks that each commodity possesses in terms of its own price volatility process. A commodity that exhibits an explosive volatility process may be of particular concern due to its potential effects on poverty and welfare.

To our knowledge, no studies analyze price volatility of Mexican agricultural products. Moreover, although a few studies test for a trend in the volatility of international commodity prices (Balcombe, 2009; Jacks, O'Rourke and Williamson, 2009; Gilbert and Morgan, 2010 and Huchet-Bourdon, 2011), none of them use a methodology that can directly address the question: Is commodity price volatility increasing? Most of the papers that intend to provide answers to this question either perform simple mean comparisons of standard deviation of prices across different time periods (Balcombe, 2009; Jacks, O'Rourke and Williamson, 2009; Huchet-Bourdon, 2011) or, in more sophisticated cases, perform a GARCH model with dummy variables for specific time periods (Gilbert and Morgan, 2010). Whereas the first method can

[1] Even though the weight of agricultural products on the CPI is relatively small (8.47%, February 1995-May 2015), their combined inflation incidence is 1.4 times larger than their weight, whereas for the rest of the products in the CPI basket, inflation incidence is only 0.95 times their weight.

be a simple approximation to the question, it may not be conclusive since it will always depend on the reference period. It is also limited since it cannot provide the explicit behavior of volatility (explosive, decreasing, stable, etc.). With regard to the second method, it is much closer to our method, however, it also has some limitations. First, the way the authors test for differences in volatility is by introducing a dummy variable that takes a value of one after 2007 in a GARCH(1,1) model. In practice, this is equivalent to the first method described here, of comparing volatilities across periods and, as such, it is susceptible to the reference period. Second, and perhaps more important, the authors do not provide the mathematical properties of the proposed model, so they cannot characterize the moments of the stochastic process.

This research makes several contributions to the literature on commodity price volatility and on volatility in general. First, based upon the classic GARCH model introduced by Engle (1982) and Bollerslev (1986), we develop a model in which volatility has a trend and we derive the properties of its stochastic process. To the best of our knowledge, except for the work by Bauer (2007), there are no other theoretical derivations of the GARCH model that test for trends in the volatility of a process. Compared to Bauer's work, our method allows us to directly test for the presence of trends in the volatility of a time series. Second, our method allows us to rank commodities according to the volatility characteristics of their processes. Compared to the traditional GARCH model, our model allows us to characterize price volatility not only in terms of its clustering and persistence, but also in terms of its trend.[2]

Using our model, we analyze the price volatility of all agricultural commodities of the

[2]Clustering refers to the fact that periods of high (low) volatility are followed by periods of high (low) volatility; persistence has to do with the fact that lagged volatility explains a considerable fraction of current volatility.

non-core CPI basket, the composite agricultural and livestock CPIs and the international prices of maize, wheat, sugar, beef, swine and poultry for the periods: 1987-1993, 1994-2005 and 2006-2014. We chose those periods based on price policies' considerations. Our results are, in general, similar for domestic commodity prices and for international prices: the trends and clustering of price volatility for most agricultural products suffered large increases from the period 1987-1993 to 1994-2005 and decreased afterwards in the period 2006-2014. However, what characterizes the most recent period analyzed, commonly referred to as the "commodity supercycle" is the large increase in the persistence of the volatility of most of the price series analyzed. In other words, volatility since 2006 has a larger memory, which implies that episodes with large volatility last for longer periods. Additionally, regarding domestic products, such as avocado, chicken and beans, present positive and statistically significant trends, that are many times larger than they were before 1994. These products can be of special concern for policy makers not only because their volatilities are increasing but also because they represent almost a tenth of the food CPI basket. Finally, we also compare the statistical properties of our model vs. Bauer's model and show that our model is more parsimonious and relatively easier to solve.

The paper is organized as follows: Section 2 describes the price series used in the analysis and the selection of the time periods. Section 3 derives the model and its stochastic properties. Section 4 shows the results and presents some graphical analyses to document the evolution of price volatility trends as well as some fitness tests that compare our model to the classic ARCH and GARCH models. Finally Section 5 concludes.

2 The Data

We apply our model to 51 monthly time series: 42 monthly CPI series of agricultural commodities that conform the non-core agricultural CPI basket,[3] 3 composite CPI series, one for agricultural products, one for livestock and one agricultural and livestock products, and 6 series of international prices (wheat, maize, poultry, beef and sugar). Time series of domestic price indexes were obtained from the National Institute of Statistics and Geography of Mexico (INEGI, by its Spanish acronym). All indexes are base December 2010=100 and represent prices paid by consumers at the retail level. International prices were obtained from the IMF and are deflated by the US CPI.[4]

We have data for the period 1987M1-2014M9 and conduct the analysis for each of the aforementioned commodities in three sub-periods: 1987-1993, 1994-2005 and 2006-2014. These sub-periods were chosen based on historical considerations of price policies, which were further confirmed in the analysis.

In order to fit the model, we transform the series by means of the following steps: first, we take the first difference of the logarithms of the level series; second, on the transformed series we fit an Autoregressive Model of up to 12 lags (AR(12)) to control for possible periodic components and other deterministic factors; finally, we check that the residual is white-noise

[3]The commodities included in the basket are apple, avocado, bananas, beans, carrot, cucumber, dry chili, grapes, green beans, green tomato, guava, lettuce and cabbage, lime, melon, nopales, onion, orange, other fresh chilies, other fruits, other legumes, other dry legumes, papaya, peas, peach, pear, pineapple, poblano chili, potato and other tubers, serrano chili, squash, tomato, watermelon, zucchini, pasteurized and fresh milk, beef, beef offal, chicken, eggs, fish and seafood, other seafood, pork and shrimp.

[4]The price of sugar refers to the "Sugar, Free Market, Coffee Sugar and Cocoa Exchange (CSCE) contract no.11 nearest future position, US cents per pound"; the price of poultry is the "Poultry (chicken), Whole bird spot price, Ready-to-cook, whole, iced, Georgia docks, US cents per pound"; the price of swine is defined as "Swine (pork), 51-52% lean Hogs, U.S. price, US cents per pound"; the price of maize is the "Maize (corn), U.S. No.2 Yellow, FOB Gulf of Mexico, U.S. price, US$ per metric ton"; the price of beef refers to the "Beef, Australian and New Zealand 85% lean fores, CIF U.S. import price, US cents per pound" and the price of wheat is defined as "Wheat, No.1 Hard Red Winter, ordinary protein, FOB Gulf of Mexico, US$ per metric ton". See http://www.imf.org/external/np/res/commod/index.aspx.

through its autocorrelation function. We perform these steps for each one of the analyzed series in the periods for which we estimate the GARCH with Trend model.

2.1 A Summary of Agricultural Price Policies in Mexico

Since the early 1970's and up to mid 1980's agricultural policies in Mexico were protectionist and intervened not only in the production, but also in the distribution, marketing, storage, credit, investment and research of agricultural products[5]. Production was regulated through diverse mechanisms: the setting and control of prices, certificates of origin and production permits to grow specific products such as coffee, cacao, tobacco and vegetables. Trade was limited by import permits and import tariffs. The National Company for Popular Subsistence (CONASUPO, by its Spanish acronym), a state-owned firm, was in charge of post-harvest handling, commercilizing and storage of 12 main crops (corn, beans, wheat, barley, sorghum, rice, soybeans and pulses, cotton, carthamus, safflower, sesame and copra) (OCDE, 1997). Prices were fixed by the government through *precios de garantía*. Prices of products such as vegetables and fruits were not subject to controls, and the government's role in those markets was to provide technical services of market information and commercialization advise to producers.

From 1987 to 1993, as part of the stabilization and adjustment programs, which aimed at minimizing the government's role in markets, the government started a series of structural economic reforms and a trade openness process. Governmental intervention in the agricultural sector was reduced: CONASUPO's role as commercializer of main crops was eliminated; price controls were gradually removed; imports tariffs and import permits were gradually reduced

[5]Organisation for Economic Co-operation and Development (OECD). 1997. Examen de las políticas agrícolas en México. Políticas nacionales y comercio agrícola. France, 236 pp.

and eliminated; licenses for production and certificates of origin for fruits and vegetables were not longer required; and subsidies were eliminated or reallocated. In 1994, Mexico opened up to trade with the US and Canada via the NAFTA. As a result, domestic agricultural prices were more exposed to international prices due to gradual reduction of tariffs and suppression of trade tariffs. Regarding agricultural subsidies, most of them were decoupled and substituted by conditional cash transfers (PROCAMPO) and counter-cyclical payments to manage price risks (Targeted-Income Program). Hence, the period 1995-2005 was characterized by a gradual integration of Mexican domestic food markets to international markets, which reduced domestic prices but were more exposed to external fluctuations. In the same period, many developing countries were reducing government support schemes to agriculture, which changed the global supply and demand of commodities. In the supply side, the reduction of government support schemes in developing countries implied lower investment and the decline of research and development in agricultural activities, lowering output growth (Mittal, 2009).

In the domestic market, to cope with low prices, since 2000 the Mexican government started two of the most important sponsored programs to mitigate price risks for crop producers: 1) the minimum price program, formally called "ingreso objetivo", which worked as a deficiency payment in which the government pays for the difference between the market price and a "minimum price"; and 2) "contract agriculture" where the government operates as an intermediary in contracts between producers and retailers.

The period 2006-2014, usually known as the "commodity supercycle", is characterized by upward trends in commodity prices and increased volatility. Several factors have been referred to as potential sources for the commodity price behavior observed in that period: 1) higher usage of food commodities to produce energy; 2) financialization of commodity markets; 3)

an increase in food demand by emerging markets, mostly from India and China; and 4) more frequent extreme weather events associated to climate change such as droughts, floods, and frosts.

In this period, food prices in Mexico were also affected by international fluctuations. Between 2006 and 2011 the Ministry of Agriculture encouraged the use of market-based mechanisms for price hedging, by subsidizing the purchase of price options in the CBOT market for some commodities such as corn, wheat, sorghum, among others, in order to reduce price volatility in those markets. In addition, domestic events also affected prices of some commodities. In particular zoo-sanitarian and meteorological events had severe effects in the supply of some products: in June of 2012 an outbreak of avian flu in Western Mexico produced dramatic increases in egg and chicken prices. In 2011 and 2013, severe frosts affected the supply of some grains, fruits and vegetables.

3 The Model

In this section we extend the GARCH model (Bollerslev, 1986) to directly tests for a trend in the volatility (other extensions of the GARCH include IGARCH (Engle and Bollerslev, 1986), FIGARCH (Bollerslev et. al., 1996), EGARCH (Nelson, 1991), AGARCH (Engle and Ng, 1993), TGARCH (Glosten et. al., 1993), GARCH-M (Domowitz and Hakkio, 1985)). In this respect, an alternative model for testing for trends in the volatility of a process is the Trend GARCH (Bauer, 2007). In particular, throughout the following lines we will discuss their differences.

To be specific, our model consists of introducing a linear trend into a GARCH(1,1) process, since some rational expectations models of commodity markets imply that commodity prices

follow an GARCH process (Beck, 1993, 2001; Muth, 1961).

Let the *returns* of the price of a given commodity, denoted by ε_t, have the following dynamics:

$$\begin{aligned} \varepsilon_t &= \sigma_t w_t, \qquad w_t \overset{\text{i.i.d.}}{\sim} \mathbb{N}(0,1) \\ \sigma_t^2 &= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta t + \gamma \sigma_{t-1}^2, \end{aligned}$$

where the coefficient β captures the effect of a trending volatility. Furthermore, setting

$$\lambda_t := \mathbb{E}[\varepsilon_t^2]$$

one can show—see Appendix A—that the variance of the ε process, with $\mathfrak{c} := \lambda_0$, equals

$$\lambda_t = (\alpha_1 + \gamma)^t \mathfrak{c} + \sum_{k=1}^{t} (\alpha_0 + \beta k)(\alpha_1 + \gamma)^{t-k}, \quad \text{for all } t \geq 1.$$

Moreover, by setting $\alpha_2 := \alpha_1 + \gamma$, the previous identity is equivalent to

$$\lambda_t = \frac{\alpha_0(\alpha_2 - 1)(\alpha_2^t - 1) + \alpha_2^t(\alpha_2 \beta + (\alpha_2 - 1)^2 \mathfrak{c}) - \beta(\alpha_2 + (\alpha_2 - 1)t)}{(\alpha_2 - 1)^2}. \tag{1}$$

In general, we have that the odd moments of ε are zero and the even moments can be calculated making use of the following expression (see Appendix A)

$$\mathbb{E}[\varepsilon_t^{2k}] = \prod_{j=1}^{k} (2j-1) \sum_{m=0}^{k} \begin{pmatrix} k \\ m \end{pmatrix} (\alpha_0 + \beta t)^{k-m} \mathbb{E}\left[\left((\alpha_1 w_{t-1}^2 + \gamma)\sigma_{t-1}^2\right)^m\right].$$

From expression (1), we can derive the following asymptotic properties of λ: If $\beta = 0$ the model reduces to the GARCH(1,1). Alternatively, if $\beta \neq 0$ and $|\alpha_1 + \gamma| < 1$ then for large t it follows that

$$\lambda_t \approx -\frac{\alpha_0}{\alpha_1 + \gamma - 1} - \frac{\beta\alpha_1 + \gamma}{(\alpha_1 + \gamma - 1)^2} - \frac{\beta}{\alpha_1 + \gamma - 1} t,$$

where

$$-\frac{1}{\alpha_1 + \gamma - 1} > \frac{1}{2}.$$

That is, the unconditional variance of ε grows linearly.

When both $\beta \neq 0$ and $\alpha_1 + \gamma > 1$ the unconditional variance grows exponentially. In general, the term not explained by the standard GARCH(1,1) model equals

$$\beta \frac{(\alpha_1 + \gamma)^{t+1} - (\alpha_1 + \gamma + (\alpha_1 + \gamma - 1)t)}{(\alpha_1 + \gamma - 1)^2}.$$

Hence, when analyzing commodity prices we will mostly be concerned with parameters that lead to explosive variances, i.e. $\beta \neq 0$ and $\alpha_1 + \gamma > 1$.

To estimate the coefficients of the model we will use MLE, since we assume that conditionally

$$\varepsilon_t | \mathcal{F}_{t-1} \sim \mathbb{N}(0, \sigma_t^2).$$

In particular in this work we use standard Newton's method.

In order to fit the model we filter the price indexes and price series of international prices

via an AR(12) (Section 2), which is appropriate to filter for periodic components (see Appendix B for a formal proof).

3.1 Volatility comparison with Bauer (2007)

Given that Bauer (2007) also tests for trends in the volatility process, in this subsection we compare our model to Bauer's. He proposes the following model,

$$\begin{aligned} \varepsilon_t &= \sigma_t w_t \\ \sigma_t^2 &= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 + \gamma \left(\frac{1}{s} \sum_{i=1}^{s} \varepsilon_{t-i} \right)^2 . \end{aligned}$$

In order to analyze its variance, let us first define the trend term as Bauer does:

$$trend_{t-1} := \frac{1}{s} \sum_{i=1}^{s} \varepsilon_{t-i}$$

which in turn yields

$$\mathbb{E}[trend_{t-1}^2] = \frac{1}{s^2} \sum_{i=i}^{s} \mathbb{E}[\sigma_{t-i}^2].$$

Notice that the trend is given by the effect that past realizations up to s lags of the stochastic process has on the variance of the process. After some calculations —see Appendix C— we are able to derive the volatility of Bauer's model in terms of the solution of the following difference equation

$$\mathbb{E}[\sigma_t^2] = \alpha_0 + (\alpha_1 + \beta + \gamma/s^2)\mathbb{E}[\sigma_{t-1}^2] + \frac{\gamma}{s^2} \sum_{i=2}^{s} \mathbb{E}[\sigma_{t-i}^2]$$

which is a non-homogeneous s-order difference equation with constant coefficients, and hence it has a solution. First, in contrast to our model (1), the solution to this equation is much more complicated and difficult to characterize with a couple of linear combinations of parameters, as our model allows. Second, the way the trend is included Bauer's model is highly dependent on the number of lags s of the stochastic process one is willing to include and therefore it adds an extra degree of subjectivity that is avoided in the GARCH with Trend model we propose.[6]

4 Results

To summarize the results, Figures 2, 4 and 6 plot the clustering (α_1), trend (β) and persistence (γ) estimated parameters for each domestic commodity and for the three studied periods.[7] Figures 3, 5 and 7 depict estimated parameters for international prices. Because the purpose of the paper is to extend the GARCH model by including a trend term, we focus our results on the significance of the trend and the persistence parameters. Persistence (γ) is plotted in the y-axis, trend (β) in the x-axis and clustering (α_1) in the z-axis. Asterisks pin heads denote positive and statistically significant persistence, triangles represent negative and statistically significant persistence, whereas circles show non- statistically significant persistence parameters. Solid dark lines represent positive and statistically significant trends, dotted black lines denote negative and statistically significant trends, whereas dashed lines show non-statistically significant trends. Tables 1 to 3 show the constant (α_0), clustering (α_1), trend (β) and persistence (γ) estimated parameters for selected commodities and for each time period analyzed.

[6] Bauer uses up to 5 lags of the process, but it is arguably debatable whether the past five observations are enough to characterize the trend of the process.

[7] For the period 1987-1993 there are only 36 products with complete time series because some goods were added in further periods to the CPI basket.

Before 1994 most of these products were located in the west-central part of the graph (see Figure 2), that is, their volatility presented low clustering levels, their trends were mostly negative and many products exhibited positive and statistically significant persistence parameters. The price behavior of domestic commodities mostly reflected the normalization of commodity prices after a large period of high volatility, registered during the early 80's.[8] The aggregate Agricultural and Livestock CPI and Livestock CPI show positive and significant clustering and no statistically significant trends or persistence terms. In contrast with domestic prices, international livestock prices of products such as beef and swine show positive and statistically significant persistence parameters and negative and statistically significant clustering parameters (Table 1). For domestic livestock products, this process may be explained by the nature of the domestic market during that period where livestock products were supplied only by national producers handled with low quality standards, frequently ungraded, and heavily supported by government programs.

The period after 1994 covers almost 20 years including the international commodity supercycle period, that started in the mid 2000s. For the sake of the analysis, we partitioned this period: before the commodities supercycle period (1994-2005) and during the commodities supercycle (2006-2014).

During the period 1994-2005, most of the domestic price series increased their price volatility clustering and trends and, some of them increased their persistence parameters (compare Figures 2 and 4). The price volatility trend parameter of the Agricultural and Livestock CPI increased 25% from the period 1987-1993 to 1994-2005, whereas the price volatility trend of

[8] We conducted a parallel analysis for the the monthly variations of the aggregate domestic CPI indexes and the international prices, where the volatility was calculated as a moving average of standard deviations during the same period following previous studies (Balcombe, 2009; Huchet-Bourdon, 2011). The results confirm that most of the standard deviations show negative trends in the period 1987-1993, as our model predicts. Results of this analysis are available upon request.

the same series increased 190% and the persistence declined 153% (see Tables 1 and 2).

Hence, trade liberalization seems to have increased price volatility trends. The behavior of the aggregate agricultural products CPI may reflect the patterns of fruit, vegetables and horticultural markets, which were highly benefited by trade openness, since tariffs for fruits like lime, strawberry, bananas and mangoes were completely eliminated in 1994. For other fruits such as peaches, watermelon, grapes, apples and avocado, import tariffs were gradually reduced. With regards to vegetables, tariffs were reduced in 1994 and other products, such as tomatoes and green pepper, experienced gradual tariff reductions until 1998 when they were completely eliminated; for other products like zucchini, peppers, onions and potato, tariffs remained seasonal until 2003, when they were eliminated. Products like grapes, apples, oranges and strawberries were in the top of the exports list; vegetables like tomatoes and potatoes leaded exports to the United States.

International prices experienced a similar process: trends increased, although not enough to become positive and statistically significant for any of the analyzed commodities, whereas persistence declined except for beef that showed positive and statistically significant persistence. Beef and poultry prices presented statistically significant and positive clustering, as in the previous period.

Although the liberalization period increased the price volatility clustering of domestic agricultural commodities and reduced its persistence, more recently, in the commodity supercycle period (2006-2014), price volatility trends have declined, whereas persistence has increased. Hence, during this episode, agricultural and livestock price volatility is mostly characterized by an increase in its persistence levels, which essentially implies that large levels of volatility are likely to persist and last longer. In the domestic market, products such as avocado, beans

and chicken may be problematic given their positive volatility trends and, in the case of the potato, due to its large persistence parameter (Table 3). These products are also important in the food basket as their combined weight is around 10%.

For international markets, we can observe a similar story: price trends declined and price persistence significantly increased (Figures 5 and 7). During the period 2006-2014 many commodities moved to the left and central parts of the graph, indicating a decrease in both price volatility clustering and an increase in volatility trends and persistence. On average, for international prices, the persistence of the volatility increased 33% from the period 1994-2005 to the period 2006-2104. In international markets, particular attention should be paid to products such as maize, swine and sugar since their price fluctuations may be less likely to be reduced in the short run; in the case of beef, an increasing volatility trend also signals larger price fluctuations in the near future.

4.1 Goodness of Fit

To finalize this Section we perform a Breusch–Pagan test on the aggregate Agricultural and Livestock CPI index and some selected commodities for which trends or persistence terms were statistically significant in different time periods. Recall from Section 3 that the model we propose is as follows:

$$\begin{aligned} \varepsilon_t &= \sigma_t w_t, \qquad w_t \overset{\text{i.i.d.}}{\sim} \mathbb{N}(0,1) \\ \sigma_t^2 &= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta t + \gamma \sigma_t^2 \end{aligned}$$

Hence, the term $\varepsilon_t / \hat{\sigma}_t$, should be homoscedastic, given the GARCH model is correctly

specified. Table 4 shows the percentage of analyzed time series for which the null hypothesis of homoscedastic errors of the Breusch–Pagan test could not be rejected. The results show that, our model outperforms the ARCH (1) and GARCH (1,1) models, since for at least 86% of the products in a given period, our model successfully addresses the heteroscedasticity of the errors, compared to 43% and 54% obtained after fitting the ARCH(1) and GARCH(1,1) models. For the most recent period analyzed, our model produces 100% of time series with homoscedastic errors, whereas the GARCH(1,1) yields 72%.

5 Concluding remarks

In this work we propose a novel extension of the classic GARCH(1,1) model, where the conditional variance has a linear trend. Our model can be a useful tool for testing for price volatility trends in different applications. Moreover, via a Breusch-Pagan test of homocedasticity we show our model to outperforms the ARCH(1) and GARCH(1,1) models. In this paper, we apply the model to 51 time series of domestic and international agricultural and livestock products for three different time periods: 1987-1993, 1994-2005, 2006-2014.

Our results show that, before 1994, many products exhibited no price volatility trends or negative trends. From 1994-2005, price volatility trends increased for most products and their persistence declined. During the period 2006-2014, price volatility trends decreased but persistence increased. Our model helps to identify products that could be problematic in terms of their price volatilities. In particular, domestic products such as avocado, chicken and beans showed positive price volatility trends in the most recent period of study. From a policy perspective, those products may be problematic since they have a non-negligible weight in the CPI. Particular attention should be paid to those markets in the near future to control

their volatilities. This results may also be useful to anticipate changes in price volatility of markets that are in the process of deregulation.

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Appendix A Statistical properties of the GARCH with Trend model

Let $\mathcal{F}_t$ be the σ-algebra generated by ε_s for all $s = 0, \ldots, t$, the conditional moments of ε are

$$\begin{aligned} \mathbb{E}[\varepsilon_t|\mathcal{F}_{t-1}] &= 0 \\ \mathbb{E}[\varepsilon_t^2|\mathcal{F}_{t-1}] &= \sigma_t^2. \end{aligned} \tag{2}$$

In turn, from (2) and the law of total expectation

$$\mathbb{E}[\varepsilon_t^2] = \mathbb{E}\left[\mathbb{E}[\varepsilon_t^2|\mathcal{F}_{t-1}]\right] = \mathbb{E}[\sigma_t^2].$$

Therefore the unconditional moments of ε, given the previous identity, are

$$\begin{aligned} \mathbb{E}[\varepsilon_t] &= \mathbb{E}[\mathbb{E}[\varepsilon_t|\mathcal{F}_{t-1}]] \\ &= 0 \\ \mathbb{E}[\varepsilon_t^2] &= \mathbb{E}[\mathbb{E}[\varepsilon_t^2|\mathcal{F}_{t-1}]] \\ &= \alpha_0 + \beta t + \alpha_1 \mathbb{E}[\varepsilon_{t-1}^2] + \gamma \mathbb{E}[\sigma_{t-1}^2] \\ &= \alpha_0 + \beta t + \alpha_1 \mathbb{E}[\varepsilon_{t-1}^2] + \gamma \mathbb{E}[\varepsilon_{t-1}^2] \\ &= \alpha_0 + \beta t + (\alpha_1 + \gamma)\, \mathbb{E}[\varepsilon_{t-1}^2]. \end{aligned} \tag{3}$$

If we set

$$\lambda_t := \mathbb{E}[\varepsilon_t^2]$$

it follows, from (3), that the second moment of ε_t can be obtained by solving the following first order difference equation

$$\lambda_t = \alpha_0 + \beta t + (\alpha_1 + \gamma)\lambda_{t-1}. \tag{4}$$

Hence, assuming that $\lambda_0 = \mathfrak{c}$ it follows by recursion that

$$\lambda_t = (\alpha_1 + \gamma)^t \mathfrak{c} + \sum_{k=1}^{t} (\alpha_0 + \beta k)(\alpha_1 + \gamma)^{t-k}, \quad \text{for all } t \geq 1.$$

Alternatively, the difference equation in (4) can be transformed into a linear difference equation with constant coefficients by sutbracting

$$\lambda_{t-1} = \alpha_0 + \beta(t-1) + (\alpha_1 + \gamma)\lambda_{t-2}$$

on each side of (4). That is

$$\lambda_t - (\alpha_1 + \gamma + 1)\lambda_{t-1} + (\alpha_1 + \gamma)\lambda_{t-2} = \beta.$$

This last expression, and letting $\alpha_2 := \alpha_1 + \gamma$, has the following solution

$$\lambda_t = \frac{\alpha_0(\alpha_2 - 1)(\alpha_2^t - 1) + \alpha_2^t(\alpha_2\beta + (\alpha_2 - 1)^2\mathfrak{c}) - \beta(\alpha_2 + (\alpha_2 - 1)t)}{(\alpha_2 - 1)^2}.$$

In general, for $k = 1, 2, \ldots$ it follows that

$$
\begin{aligned}
\mathbb{E}[\varepsilon_t^{2k}] &= \mathbb{E}[\mathbb{E}[\varepsilon_t^{2k}|\mathcal{F}_{t-1}]] \\
&= \mathbb{E}[\mathbb{E}[\sigma_t^{2k} w_t^{2k}|\mathcal{F}_{t-1}]] \\
&= \mathbb{E}[\sigma_t^{2k}\mathbb{E}[w_t^{2k}|\mathcal{F}_{t-1}]] \text{ since } \sigma_t^{2k} \text{ is } \mathcal{F}_{t-1}\text{-measurable} \\
&= \mathbb{E}[\sigma_t^{2k}\mathbb{E}[w_t^{2k}]] \text{ since } w_t^{2k} \text{ is independent of } \mathcal{F}_{t-1} \\
&= \prod_{j=1}^{k}(2j-1)\mathbb{E}[(\alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta t + \gamma\sigma_{t-1}^2)^k] \text{ since } w_t \sim \mathbb{N}(0,1) \\
&= \prod_{j=1}^{k}(2j-1)\sum_{m=0}^{k}\begin{pmatrix} k \\ m \end{pmatrix}(\alpha_0 + \beta t)^{k-m}\mathbb{E}\left[\left((\alpha_1 w_{t-1}^2 + \gamma)\sigma_{t-1}^2\right)^m\right],
\end{aligned}
$$

where the last identity follows from the binomial Theorem. Furthermore, applying a conditioning argument on the expectation above, yields

$$
\begin{aligned}
\mathbb{E}\left[(\alpha_1 w_{t-1}^2 + \gamma)^m \sigma_{t-1}^{2m}\right] &= \mathbb{E}[\mathbb{E}\left[(\alpha_1 w_{t-1}^2 + \gamma)^m \sigma_{t-1}^{2m}\right]|\mathcal{F}_{t-1}] \\
&= \alpha_1^m \mathbb{E}[(w_{t-1}^2 + \gamma/\alpha_1)^m]\mathbb{E}[\sigma_{t-1}^{2m}]
\end{aligned}
$$

Hence, the even moments of ε_t can be computed recursively. This is true since $w_{t-1}^2 + \gamma/\alpha_1$ is a non-central chi-squared with non-centrality parameter equal to γ/α.

It is also straightforward to show that the autocovariance function φ of ε at any lag $h \geq 0$

is identically zero:

$$\begin{aligned}
\varphi(h) &:= \mathbb{E}[\varepsilon_t \varepsilon_{t-h}] \\
&= \mathbb{E}[\mathbb{E}[\varepsilon_t \varepsilon_{t-h} | \mathcal{F}_{t-h}]] \\
&= \mathbb{E}[\varepsilon_{t-h} \mathbb{E}[\varepsilon_t | \mathcal{F}_{t-h}]] \\
&= \mathbb{E}[\varepsilon_{t-h} \mathbb{E}[\mathbb{E}[\varepsilon_t | \mathcal{F}_{t-1}] | \mathcal{F}_{t-h}]] \\
&= 0
\end{aligned}$$

Appendix B AR(p) models and periodic series

Given that the residuals of some of the products seem to possess periodic componentes, in this appendix we show the relationship between an AR(2) filter and a periodic function. In fact this argument can be extended for an arbitrary number of frequencies and thus justifies the use of an AR filter on the residuals.

Let π, ϕ, β be respectively the frequency, period and amplitude respectively of a periodic function. Furthermore let W be a stationary process, then the following process is not stationary

$$\begin{aligned}
X_t &= \beta \sin(\phi + 2\pi\nu t) + W_t \\
&= \beta \sin(\phi) \cos(2\pi\nu t) + \beta \cos(\phi) \sin(2\pi\nu t) + W_t \\
&= A^\top S_t + W_t.
\end{aligned}$$

The second identity follows from the properties of sinusoidal function and the third, by setting

$A^\top = (\beta\sin(\phi), \beta\cos(\phi))$ and

$$S_t = \Sigma S_{t-1}, \;\; S_0^\top = (1,0), \;\; \Sigma = \begin{pmatrix} \cos(2\pi\nu) & -\sin(2\pi\nu) \\ \sin(2\pi\nu) & \cos(2\pi\nu) \end{pmatrix}.$$

Furthermore

$$\Sigma = UVU^{-1}, \qquad V = \begin{pmatrix} e^{-2\pi i\nu} & 0 \\ 0 & e^{2\pi i\nu} \end{pmatrix}, \;\; U = \begin{pmatrix} -i & i \\ 1 & 1 \end{pmatrix}$$

Hence,

$$\begin{aligned} X_t + \varphi_1 X_{t-1} + \varphi_2 X_{t-2} &= A^\top S_t + \varphi_1 A^\top S_{t-1} + \varphi_2 A^\top S_{t-2} \\ &\quad + W_t + \varphi_1 W_{t-1} + \varphi_2 W_{t-2} \\ &= A^\top \left(\Sigma^2 + \varphi_1\Sigma + \varphi_2 I\right) S_{t-2} \\ &\quad + W_t + \varphi_1 W_{t-1} + \varphi_2 W_{t-2} \\ &= A^\top U \left(V^2 + \varphi_1 V + \varphi_2 I\right) U^\top S_{t-2} \\ &\quad + W_t + \varphi_1 W_{t-1} + \varphi_2 W_{t-2}. \end{aligned}$$

Thus, in order to obtain a stationary transformation it is sufficient that

$$V^2 + \varphi_1 V + \varphi_2 I = 0 \quad \Rightarrow \begin{pmatrix} \cos(2\pi\nu) & 1 \\ \sin(2\pi\nu) & 0 \end{pmatrix} \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} = \begin{pmatrix} -\cos(4\pi\nu) \\ \sin(4\pi\nu) \end{pmatrix}$$

which is equivalent to $\varphi_1 = -2\cos(2\pi\nu)$ and $\varphi_2 = 1$.

In summary, what we have shown is that

$$X_t - 2\cos(2\pi\nu)X_{t-1} + X_{t-2}$$

is stationary. This same line of reasoning can be applied for an arbitrary number of sinusoidal functions.

Appendix C Statistical properties of Bauer's model

In Bauer (2007) the author presents the following model,

$$\begin{aligned} \varepsilon_t &= \sigma_t w_t \\ \sigma_t^2 &= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 + \gamma \left(\frac{1}{s} \sum_{i=1}^{s} \varepsilon_{t-i} \right)^2 . \end{aligned}$$

If we set:

$$trend_{t-1} := \frac{1}{s} \sum_{i=1}^{s} \varepsilon_{t-i}$$

it follows that

$$
\begin{aligned}
\mathbb{E}[trend_{t-1}^2] &= \mathbb{E}\left[\mathbb{E}[trend_{t-1}^2 \big| \mathcal{F}_{t-2}]\right] \\
&= \mathbb{E}\left[\frac{1}{s^2}\mathbb{E}\left[\sum_{i=1}^{s}\varepsilon_{t-i}^2 + 2\sum_{1\le i<j\le s}\varepsilon_{t-i}\varepsilon_{t-j}\Bigg|\mathcal{F}_{t-2}\right]\right] \\
&= \mathbb{E}\left[\frac{1}{s^2}\left(\sum_{i=2}^{s}\varepsilon_{t-i}^2 + 2\sum_{2\le i<j\le s}\varepsilon_{t-i}\varepsilon_{t-j}\right)\right] \\
&\quad + \frac{1}{s^2}\mathbb{E}\left[\mathbb{E}[\varepsilon_{t-1}^2|\mathcal{F}_{t-2}] + 2\mathbb{E}[\varepsilon_{t-1}\varepsilon_{t-2}|\mathcal{F}_{t-2}] + \cdots + \mathbb{E}[\varepsilon_{t-1}\varepsilon_{t-s}|\mathcal{F}_{t-2}]\right] \\
&= \mathbb{E}\left[\frac{1}{s^2}\left(\sum_{i=2}^{s}\varepsilon_{t-i}^2 + 2\sum_{2\le i<j\le s}\varepsilon_{t-i}\varepsilon_{t-j}\right)\right] \\
&\quad + \frac{1}{s^2}\mathbb{E}\left[\sigma_{t-2}^2\right] \\
&\vdots \\
&= \frac{1}{s^2}\sum_{i=i}^{s}\mathbb{E}[\sigma_{t-i}^2].
\end{aligned}
$$

Which in turn yields

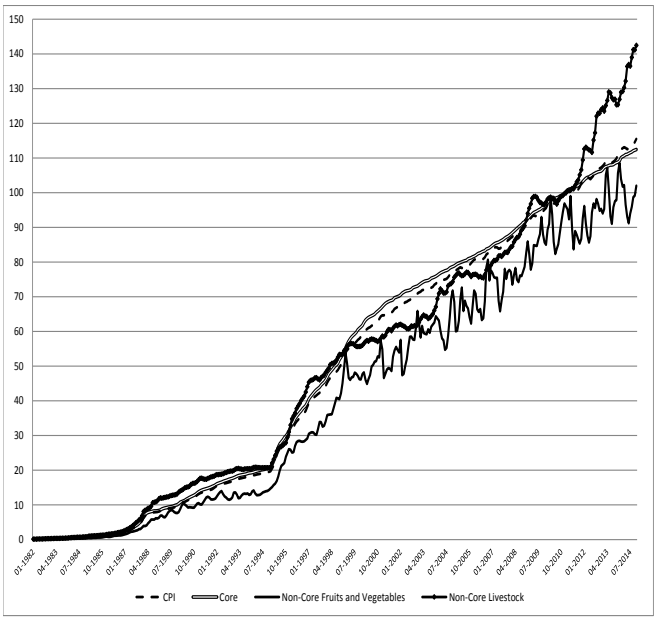
$$
\begin{aligned}
\mathbb{E}[\sigma_t^2] &= \mathbb{E}\left[\alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 + \gamma\left(\frac{1}{s}\sum_{i=1}^{s}\varepsilon_{t-i}\right)^2\right] \\
&= \alpha_0 + (\alpha_1+\beta)\mathbb{E}[\sigma_{t-1}^2] + \frac{\gamma}{s^2}\sum_{i=1}^{s}\mathbb{E}[\sigma_{t-i}^2] \\
&= \alpha_0 + (\alpha_1+\beta+\gamma/s^2)\mathbb{E}[\sigma_{t-1}^2] + \frac{\gamma}{s^2}\sum_{i=2}^{s}\mathbb{E}[\sigma_{t-i}^2].
\end{aligned}
$$

This expression can be solved by noting that the volatility is a non-homogeneous s-order difference equation with constant coefficients with a possible general solution of the form

$$
\mathbb{E}[\sigma_t^2] = \frac{\alpha_0}{1+c_1+(s-1)\cdot d} + (\hat{z}_1)^t + \cdots + (\hat{z}_s)^t
$$

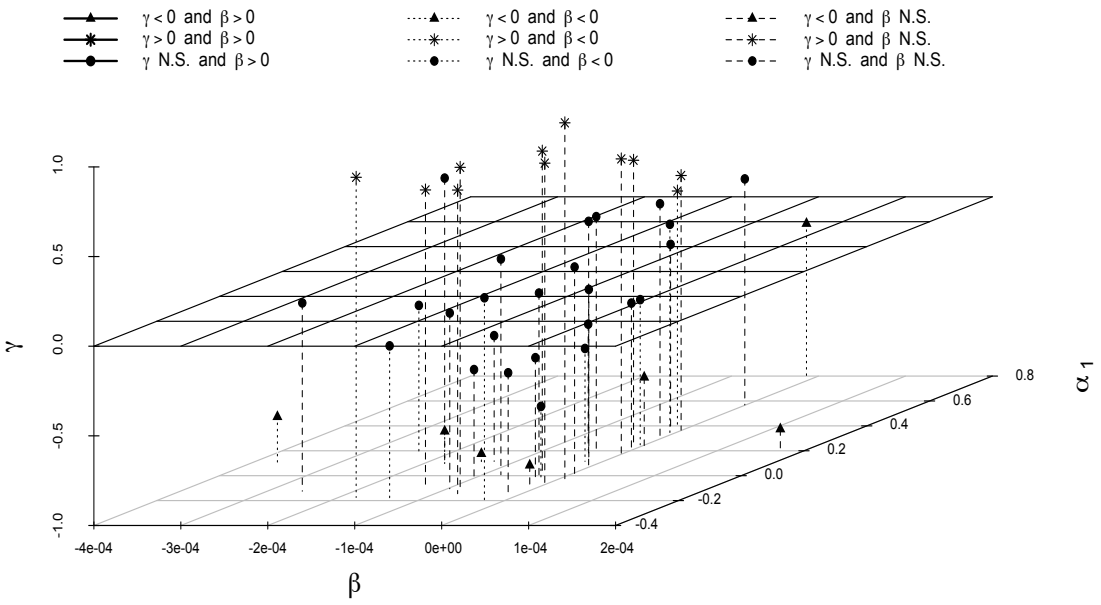
for some given constants $\hat{z}_1, \ldots, \hat{z}_s$ which depend on the coefficients of the model.

Figure 1: Consumer Price Indexes



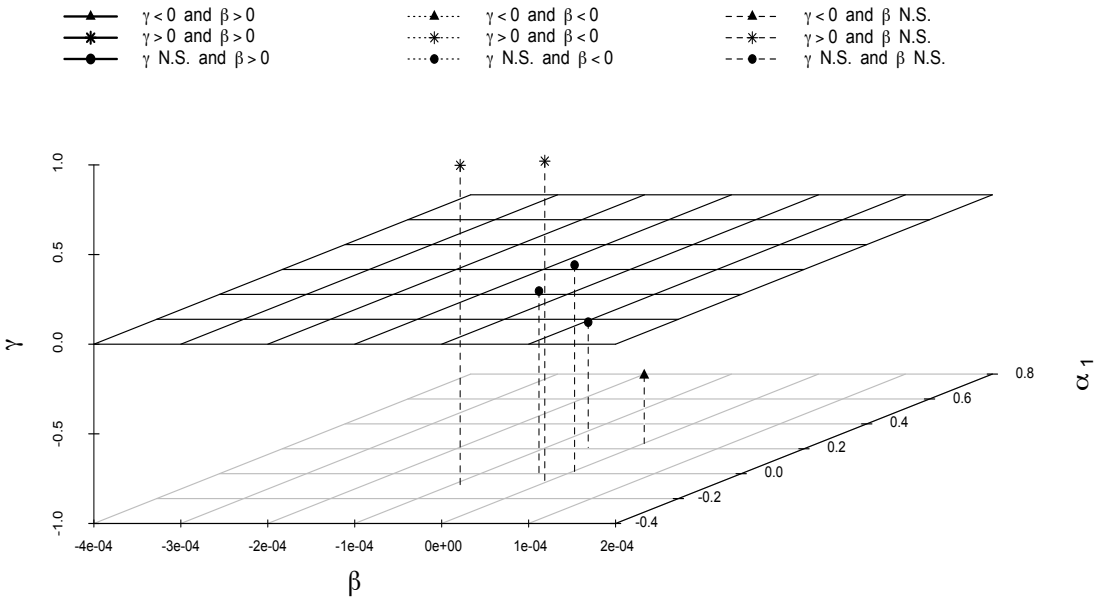
Source: INEGI. All indexes are base December 2010=100.

Figure 2: 1987-1993 Price Volatility Clustering, Trends and Persistence Parameters for Domestic Prices



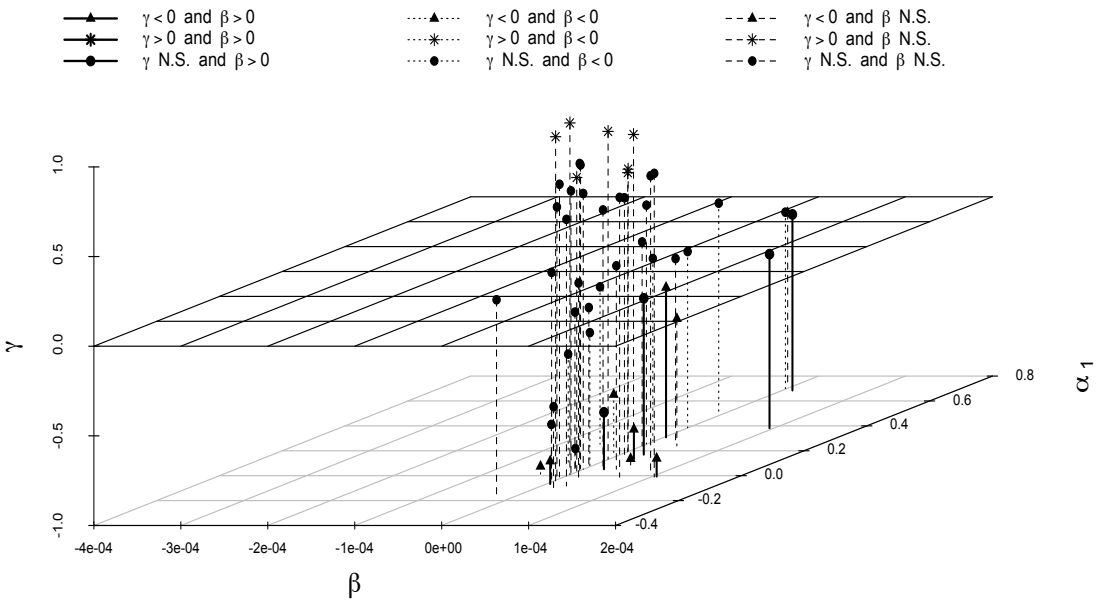
Note: In the x-axis we plot the trend (β), in the y-axis the persistence parameter (γ) and in the z-axis the clustering parameter (α_1) estimates of the GARCH with Trend Model. Asterisks pin heads denote positive and statistically significant persistence, triangles represent negative and statistically significant persistence, whereas circles show non- statistically significant persistence parameters. Solid dark lines represent negative and statistically significant trends, dotted lines denote negative and statistically significant trends, whereas dashed lines show non-statistically significant trends.

Figure 3: 1987-1993 Price Volatility Clustering, Trends and Persistence Parameters for International Prices



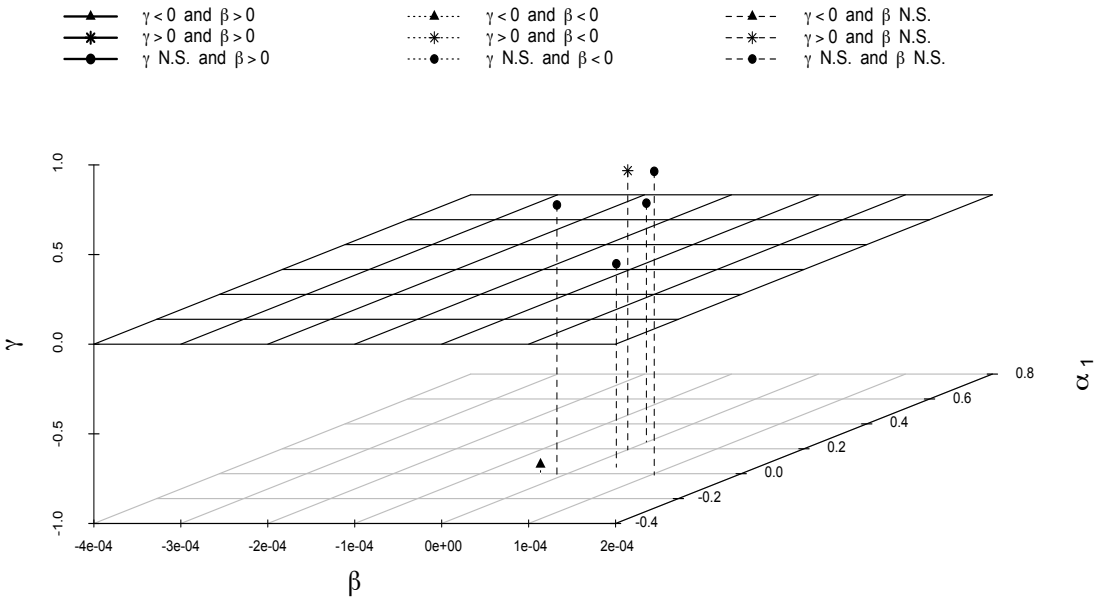
Note: In the x-axis we plot the trend (β), in the y-axis the persistence parameter (γ) and in the z-axis the clustering parameter (α_1) estimates of the GARCH with Trend Model. Asterisks pin heads denote positive and statistically significant persistence, triangles represent negative and statistically significant persistence, whereas circles show non- statistically significant persistence parameters. Solid dark lines represent negative and statistically significant trends, dotted lines denote negative and statistically significant trends, whereas dashed lines show non-statistically significant trends.

Figure 4: 1994-2005 Price Volatility Clustering, Trends and Persistence Parameters for Domestic Prices



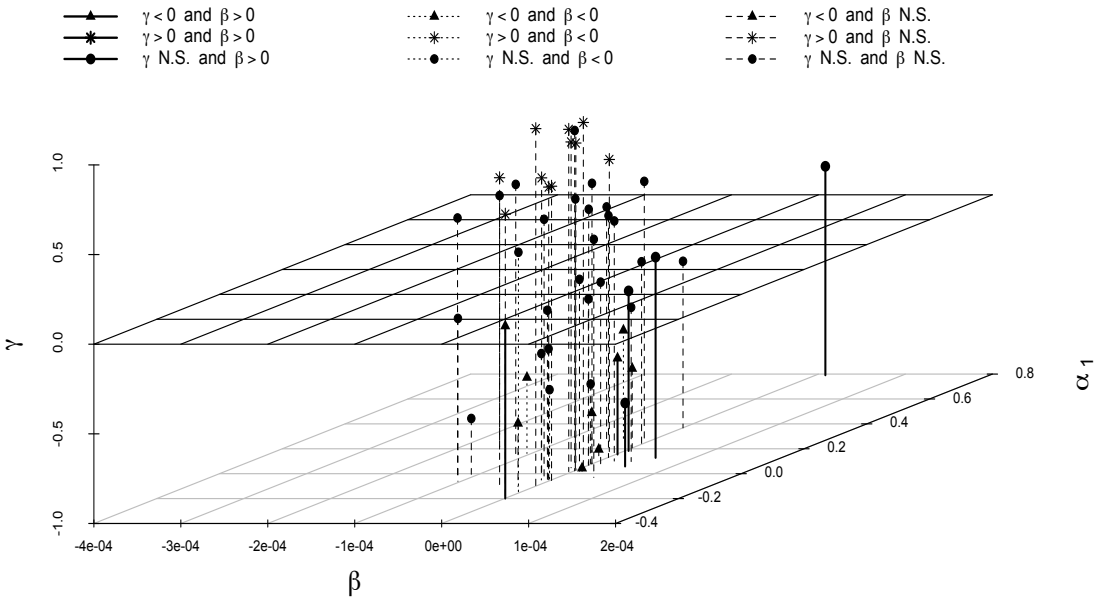
Note: In the x-axis we plot the trend (β), in the y-axis the persistence parameter (γ) and in the z-axis the clustering parameter (α_1) estimates of the GARCH with Trend Model. Asterisks pin heads denote positive and statistically significant persistence, triangles represent negative and statistically significant persistence, whereas circles show non- statistically significant persistence parameters. Solid dark lines represent negative and statistically significant trends, dotted lines denote negative and statistically significant trends, whereas dashed lines show non-statistically significant trends.

Figure 5: 1994-2005 Price Volatility Clustering, Trends and Persistence Parameters for International Prices



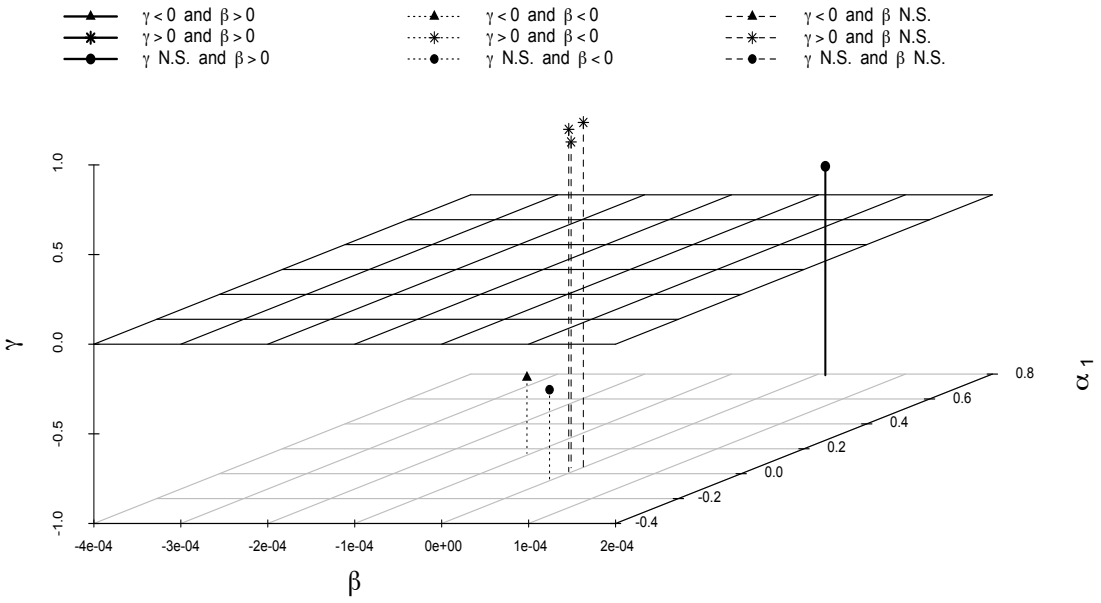
Note: In the x-axis we plot the trend (β), in the y-axis the persistence parameter (γ) and in the z-axis the clustering parameter (α_1) estimates of the GARCH with Trend Model. Asterisks pin heads denote positive and statistically significant persistence, triangles represent negative and statistically significant persistence, whereas circles show non- statistically significant persistence parameters. Solid dark lines represent negative and statistically significant trends, dotted lines denote negative and statistically significant trends, whereas dashed lines show non-statistically significant trends.

Figure 6: 2006-2014 Price Volatility Clustering, Trends and Persistence Parameters for Domestic Prices



Note: In the x-axis we plot the trend (β), in the y-axis the persistence parameter (γ) and in the z-axis the clustering parameter (α_1) estimates of the GARCH with Trend Model. Asterisks pin heads denote positive and statistically significant persistence, triangles represent negative and statistically significant persistence, whereas circles show non- statistically significant persistence parameters. Solid dark lines represent negative and statistically significant trends, dotted lines denote negative and statistically significant trends, whereas dashed lines show non-statistically significant trends.

Figure 7: 2006-2014 Price Volatility Clustering, Trends and Persistence Parameters for International Prices



Note: In the x-axis we plot the trend (β), in the y-axis the persistence parameter (γ) and in the z-axis the clustering parameter (α_1) estimates of the GARCH with Trend Model. Asterisks pin heads denote positive and statistically significant persistence, triangles represent negative and statistically significant persistence, whereas circles show non- statistically significant persistence parameters. Solid dark lines represent negative and statistically significant trends, dotted lines denote negative and statistically significant trends, whereas dashed lines show non-statistically significant trends.

Table 1: Volatility Parameters for Selected Agricultural Products (1987-1993)

Product	α_0	α_1	β	γ
Avocado	0.0129**	-0.1301***	-2.10E-05	-0.3356
	(0.0246)	(0.0000)	(0.1145)	(0.3108)
Beans	0.0011***	0.7997***	-1.37E-05***	-0.1492**
	(0.0000)	(0.0042)	(0.0000)	(0.0235)
Beef	9.05E-05	0.3657**	-1.16E-06	0.4203*
	(0.1435)	(0.0450)	(0.1464)	(0.0770)
Chicken	0.0006	0.2270*	-5.77E-06	0.6016**
	(0.2417)	(0.0905)	(0.3163)	(0.0350)
Dry Chile	0.0012*	0.3922**	-1.50E-05*	0.3153*
	(0.0543)	(0.0319)	(0.0545)	(0.0955)
Pork	6.44E-05	0.1743	-8.72E-07	0.6458**
	(0.3137)	(0.1336)	(0.3093)	(0.0443)
Potatoes and Other Tubers	0.0190***	0.1937	-0.0002***	-0.1848
	(0.0066)	(0.1093)	(0.0068)	(0.3048)
Agricultural and Livestock CPI	0.0002	0.3013*	-2.17E-06	0.3075
	(0.1059)	(0.0656)	(0.1894)	(0.1788)
Agricultural CPI	0.0024***	-0.2186***	-1.63E-05**	0.1440
	(0.0000)	(0.0000)	(0.0371)	(0.2125)
Livestock CPI	0.0001	0.5686**	1.19E-06	0.2604
	(0.1094)	(0.0149)	(0.1516)	(0.1225)
International Wheat	0.0016	0.0181	1.86E-06	0.1513
	(0.2346)	(0.4336)	(0.4508)	(0.4499)
International Maize	0.0031	-0.0339***	-2.03E-05	0.0421
	(0.1661)	(0.0011)	(0.1995)	(0.4836)
International Beef	0.0003*	-0.0686***	-1.07E-06	0.7909***
	(0.0892)	(0.0000)	(0.4229)	(0.0093)
International Swine	0.0079	-0.0922*	-8.99E-05	0.7839**
	(0.1378)	(0.0803)	(0.1442)	(0.0152)
International Poultry	0.0005***	0.2421*	1.06E-06	-0.6194***
	(0.0007)	(0.0695)	(0.3313)	(0.0000)
International Sugar	0.0105***	0.2106*	-5.20E-05	-0.3011
	(0.0066)	(0.0952)	(0.1316)	(0.1999)

Note: Estimates of the parameters of the GARCH with Trend model, where α_0 represents the constant, α_1 the clustering, β the trend and γ the persistence of the price volatility. *, **, *** statistically significant at 10%, 5% and 1% levels, respectively.

Table 2: Volatility Parameters for Selected Agricultural Products (1994-2005)

Product	α_0	α_1	β	γ
Avocado	0.0036***	0.2881**	2.05E-05	0.0109
	(0.0098)	(0.0228)	(0.1480)	(0.4791)
Beans	2.18E-05	0.0090	-2.29E-07	0.9613***
	(0.1923)	(0.3961)	(0.2776)	(0.0000)
Beef	5.83E-05	0.0518	-3.51E-07	0.5381
	(0.1808)	(0.2391)	(0.1891)	(0.1344)
Chicken	0.0005	0.0314	-2.57E-06	-0.1093
	(0.2057)	(0.3593)	(0.2134)	(0.4667)
Dry Chile	0.0003***	0.6996***	-1.79E-06***	-0.0160
	(0.0000)	(0.0013)	(0.0081)	(0.2591)
Pork	0.0001***	0.3852**	-7.07E-07**	-0.0173
	(0.0003)	(0.0120)	(0.0417)	(0.4412)
Potatoes and Other Tubers	0.0004	-0.0451	2.93E-05	0.1058
	(0.1614)	(0.1137)	(0.1830)	(0.4565)
Agricultural and Livestock	0.0001***	0.3091**	1.96E-06***	-0.1649***
	(0.0002)	(0.0120)	(0.0032)	(0.0002)
Agricultural	0.0001	0.6852***	1.15E-05***	-0.0186
	(0.1188)	(0.0005)	(0.0001)	(0.1233)
Livestock	0.0001	-0.0485***	-4.98E-07	0.1674
	(0.1148)	(0.0000)	(0.1364)	(0.4100)
International Wheat	0.0070***	-0.0254	-2.16E-05**	-0.9305***
	(0.0000)	(0.3145)	(0.0206)	(0.0006)
International Maize	0.0014	-0.0261	-2.48E-06	0.5170
	(0.2063)	(0.2316)	(0.3660)	(0.2211)
International Beef	0.0003	0.1914*	4.32E-07	0.5572*
	(0.2668)	(0.0605)	(0.4552)	(0.0734)
International Swine	0.0016	-0.0111	0.0001	0.6941
	(0.2295)	(0.4196)	(0.3868)	(0.2272)
International Poultry	0.0001*	0.2553**	-1.22E-06	0.3316
	(0.0888)	(0.0374)	(0.1053)	(0.1525)
International Sugar	0.0014	0.0526	3.74E-05	0.1341
	(0.2217)	(0.2954)	(0.2397)	(0.4502)

Note: Estimates of the parameters of the GARCH with Trend model, where α_0 represents the constant, α_1 the clustering, β the trend and γ the persistence of the price volatility. *, **, *** statistically significant at 10%, 5% and 1% levels, respectively.

Table 3: Volatility Parameters for Selected Agricultural Products (2006-2014)

Product	α_0	α_1	β	γ
Avocado	0.0006	0.1278	5.54E-05*	0.1190
	(0.2080)	(0.1318)	(0.0533)	(0.3918)
Beans	2.21E-05	0.1832	4.34E-06**	-0.1078
	(0.1585)	(0.1040)	(0.0213)	(0.3871)
Beef	3.54E-05***	0.2066**	4.36E-08	-0.5570***
	(0.0000)	(0.0493)	(0.3475)	(0.0025)
Chicken	0.0001**	0.0261	7.58E-06***	-0.9874**
	(0.0175)	(0.3189)	(0.0000)	(0.0302)
Dry Chile	4.43E-05**	-0.1547***	-2.85E-07*	0.3425
	(0.0428)	(0.0000)	(0.0756)	(0.2139)
Pork	2.11E-05	0.1300	3.96E-07	0.3488
	(0.1900)	(0.1679)	(0.2572)	(0.2859)
Potatoes and Other Tubers	0.0002***	-0.0963**	-1.47E-06	0.9915***
	(0.0090)	(0.0110)	(0.3938)	(0.0000)
Agricultural and Livestock CPI	0.0004	0.0726	-1.95E-06	-0.0757
	(0.1396)	(0.2640)	(0.1927)	(0.4670)
Agricultural CPI	0.0019**	0.0812	-2.50E-06	-0.5561
	(0.0221)	(0.2452)	(0.3596)	(0.1688)
Livestock CPI	3.59E-06	-0.0667**	1.27E-06	-0.0420
	(0.1053)	(0.0274)	(0.1580)	(0.4826)
International Wheat	0.0140***	0.1636*	-0.0001***	-0.5771***
	(0.0000)	(0.0960)	(0.0000)	(0.0000)
International Maize	0.0006	0.0170	-4.49E-06	0.9087***
	(0.2057)	(0.4225)	(0.3968)	(0.0010)
International Beef	0.0001	0.7914***	1.10E-05**	0.1648
	(0.1807)	(0.0039)	(0.0279)	(0.1296)
International Swine	0.0003	0.0536	-9.08E-07	0.9217***
	(0.2932)	(0.3206)	(0.4930)	(0.0097)
International Poultry	0.0001*	-0.0537	-9.71E-07*	-0.4941
	(0.0627)	(0.2277)	(0.0754)	(0.3054)
International Sugar	0.0014	0.0479	-1.29E-05	0.8171***
	(0.2440)	(0.3334)	(0.2520)	(0.0026)

Note: Estimates of the parameters of the GARCH with Trend model, where α_0 represents the constant, α_1 the clustering, β the trend and γ the persistence of the price volatility. *, **, *** statistically significant at 10%, 5% and 1% levels, respectively.

Table 4: Percentage of time series for which the Breusch–Pagan Tests H0 could not be rejected

	1987-1993	1994-2005	2006-2014
GARCH with Trend (1,1)	86.8	92	100
ARCH (1)	43.4	48	69.5
GARCH (1,1)	54.7	56	72.9

Note: Percentage of the number of products for which the Breusch–Pagan Tests null hypothesis of homoscedastic errors could not be rejected at the 10% significance level.