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Forward Guidance in an Advanced Small Open Economy in the Effective Lower Bound*

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Abstract: We examine forward guidance (with known and uncertain duration) in a New Keynesian model for an advanced small open economy, showing that the response of the economy to this policy depends, both quantitatively and qualitatively, on some structural features through calibrations for Sweden and Spain. In particular, an announcement of future expansionary policy is positively related to the exchange rate pass-through and is larger than in the closed economy counterpart because of a better inflation-output trade-off and the exchange rate channel. We also show that multiple equilibria could arise and that the real exchange rate is a key variable driving this result. In particular, the response of output and inflation is amplified when aggregate supply is negatively related to the real exchange rate. These results could not necessarily be extended to emerging market economies.
Keywords: Monetary policy, advanced small open economy, forward guidance.
JEL Classification: E31, E52

Resumen: Estudiamos la guía futura (con duración conocida e incierta) en un modelo Nuevo-Keynesiano para una economía avanzada pequeña y abierta, mostrando que la respuesta de la economía a esta política depende, de manera cuantitativa y cualitativa, de algunas características estructurales mediante calibraciones para Suecia y España. En particular, anunciar una política expansiva futura está relacionado positivamente con el traspaso del tipo de cambio a la inflación y es mayor que en una economía cerrada, debido a una mejor disyuntiva entre producto e inflación y al canal del tipo de cambio. Además, se demuestra que múltiples equilibrios pueden ocurrir y que el tipo de cambio real es una variable crucial para obtener tal resultado. En particular, la respuesta del producto y la inflación se amplifica cuando la oferta agregada está relacionada negativamente al tipo de cambio real. Estos resultados no necesariamente se podrían extrapolar a economías emergentes.
Palabras Clave: Política monetaria, economía avanzada pequeña y abierta, guía futura.

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1 Introduction

The global financial crisis has triggered a vivid interest in theoretical and empirical research on unconventional monetary policies that look for a substitute of the short-term nominal rate when the latter reaches the zero lower bound. A key example of such unconventional policies is given by forward guidance, through which policymakers announce a path of the nominal interest rate starting immediately or in the future for a particular duration. Through this policy, the central bank tries to manage expectations of the future policy rates once the zero lower bound is no longer binding in order to influence macroeconomic dynamics. In the basic New Keynesian DSGE model, an anticipated change in the real rate produces an effect on output which is independent of the duration and timing: this is the forward guidance puzzle. As a consequence, the effects of a temporary variation in the policy rate that takes place very far in the future is the same if the variation were to take place immediately or in the near future. This puzzle, discussed by Del Negro et al. (2012), Carlstrom et al. (2015) and McKay et al. (2016), derives from the fact that, in a baseline New Keynesian DSGE model, the dynamic IS relationship has no discounting of the expected output gap and, in turn, of future real interest rates. Consequently, the literature introduced some discounting mechanism in the Euler equation so that aggregate demand responds less than one-to-one to its future expected changes. Examples include an overlapping-generations structure *à la* Blanchard and Yaari in the demand side (Del Negro et al. (2012)), heterogeneous agents and incomplete markets (McKay et al. (2016)), sticky information (Carlstrom et al. (2015)), bounded rationality (Beqiraj et al. (2019)). McClung (2020) shows that a regime characterized by passive monetary policy and active fiscal policy does not imply forward guidance puzzle. With active fiscal policy, Ricardian equivalence does not hold and agents perceive government debt as net wealth. As a consequence, forward guidance announcements that lower the expectations of future interest rates produce negative wealth effects that counteract the monetary stimulus. Furthermore, Hagedorn et al. (2019) show that under commitment, forward guidance has only transitory effects on the economy and Nakata et al. (2019) study optimal forward guidance in a setup where forward guidance puzzle is attenuated.

In this paper we analyze the theoretical implications of forward guidance in an advanced small open economy, focusing on the international transmission of such a policy. To the best of our knowledge, Galí (2020) is the only theoretical contribution about forward gui-dance in open economy. He shows that if the home central bank announces an increase (decrease) of the nominal interest rate of T periods, with no reaction from the foreign central bank, the exchange rate appreciation (depreciation) at the time of announcement is proportional to the duration and the size of the interest rate change, but it is independent on the duration of the forward guidance. Therefore forward guidance puzzle arises also in a small-open economy model. We follow Galí and Monacelli (2005) and Leitemo and Söderström (2008) in modeling a small country that freely trades with the rest of the world, constituted of a continuum of foreign economies. Forward guidance will be analyzed in a deterministic scenario, where its duration is known with certainty, and in a stochastic setting, modeled along the lines of Bilbiie (2019). We evaluate the forward guidance policy in normal times and under a liquidity trap with the central bank committing to keep the interest rate fixed also when the economy is out of the liquidity trap.

Our main results are the following ones. First, we show that forward guidance may have distinct effects on inflation and output gap, both in terms of magnitude and directions, compared to the closed economy. A key determinant is the elasticity of inflation with respect to the real exchange rate which could be either positive or negative. In the latter case, in a liquidity trap forward guidance may lower the output gap and inflation, thus offsetting the stimulating intertemporal effects of expansionary forward guidance when the forward guidance policy is followed for a short-period of time[1]. Second, the real depreciation that occurs in presence of negative elasticity of inflation with respect to the real exchange rate drives the expansionary effects of forward guidance also in a stochastic setup, where the duration of the policy and the state of the economy ("normal times" versus liquidity trap) follow a Markov chain. We also show that keeping the interest rate fixed after the liquidity trap is over can generate an increase in output gap and inflation instead in an advanced small open economy. Finally, compared to the closed-economy counterpart, forward guidance tends to be more expansionary in open economy: this is due to the combination the role played by the real exchange rate and to the better trade off between output and inflation (because of a larger Phillips' curve slope). Such results might not necessarily be applied to emerging market economies.

The paper is organized as follows: Section 2 describes the reference model, while Section 3 studies forward guidance when the duration of the policy is known with certainty. In Section 4 we consider the same experiment with uncertainty about the duration of liquidity trap and forward guidance before concluding in Section 5.

[1]The transmission under positive pass-through proceeds inversely in the liquidity trap.

2 The Model

We shortly summarize, with some slight changes in notation, the small open economy model of Galí and Monacelli (2005) and Leitemo and Söderström (2008). With the objective of deriving analytical solutions, the only shock (defined later) is a preference shock that drives the economy into a liquidity trap.

The small domestic country freely trades with the rest of the world (foreign country), constituted of a continuum of foreign economies. We assume that foreign and domestic countries share preferences and technology. Domestic and foreign firms produce traded consumption goods, using labor as the sole input. Households derive their utility from consuming both domestic and foreign goods, and have a marginal decreasing disutility in labor supply to firms.

Denoting by e_t the log-linearized real exchange rate, we have by definition

$$e_t = s_t + p_t^f - p_t, \tag{1}$$

with s_t being the nominal exchange rate (units of domestic currency against one unit of foreign currency), p_t^f the price level of the goods produced in the foreign country and p_t the price level of domestically produced goods.

The real exchange rate is directly related to the inflation rate in the domestic goods sector, π_t, via the New Keynesian Phillips curve:[2]

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t - \phi e_t, \tag{2}$$

where x_t denotes the output gap, $0 < \beta < 1$ the discount factor, and E_t the rational expectations formed by private agents (conditional on information set available at time t). The composite parameter $\kappa = \hat{\kappa}(\eta + \sigma)$, with $\hat{\kappa} \equiv \frac{(1-\vartheta)(1-\vartheta\beta)}{\vartheta}$, is the output-gap elasticity of inflation and encompasses the effect of the output gap on inflation via real marginal costs. Phillips' curve slope depends on ϑ, the share of firms that do not optimally adjust but simply update in period t their previous price by the steady-state inflation rate, on η, which represents the steady-state Frisch elasticity of labor supply, and on $\sigma \equiv \frac{\hat{\sigma}}{1-\omega}$ with $\hat{\sigma}$ denoting the inverse of the elasticity of intertemporal substitution, and $0 \leq \omega \leq 1$ the share of foreign goods in domestic consumption. The real exchange rate enters the Phillips curve

[2]Differently from Galí and Monacelli (2005), Leitemo and Söderström (2008) derive a Phillips curve including the real exchange rate. For the microfoundations of the model, see Leitemo and Söderström (2008). Notice that π_t is different from the inflation rate of the consumer price index that also takes into account the inflation of foreign goods consumed by residents. In the closed economy, π_t represents both producer and consumer price inflation rates.

through the coefficient $\phi = \omega\hat{\kappa}[(2-\omega)\zeta\sigma - 1]$, where ζ stands for the elasticity of substitution across domestic and foreign goods. In general, there is not unanimous consensus about the sign of the relationship: differently from Leitemo and Söderström (2008), Walsh (1999) and Razin and Yuen (2002) obtain a positive relationship between these two variables in theoretical models. Therefore, in our analysis we will consider both signs in the relationship. The economic intuition behind the relationship between inflation and real exchange rate is the following: when households choose labor supply, they care about the purchasing power of their wage deflated by the consumer price index that also includes prices of imported goods, implying that the equilibrium wage and hence the real marginal costs depend on the real exchange rate. As highlighted in Leitemo and Söderström (2008), there are two competing effects shaping the relationship between exchange rate and inflation. On the one hand, an exchange rate depreciation (i.e. an increase in the exchange rate) increases consumer prices and therefore reduces households' purchasing power. The optimal labor supply choice will imply higher wages and, in turn, higher marginal costs and inflation. On the other hand, an exchange rate depreciation leads to a decrease in the demand for imports and therefore a reduction in aggregate consumption. The marginal rate of substitution then falls, leading to lower real wages and marginal cost. The composite parameter ϕ is positive as long as $(2-\omega)\zeta\sigma > 1$: this condition holds in Leitemo and Söderström (2008), determining a negative relationship between inflation and exchange rate for their model calibrated to Sweden. However, for economies whose main exports are based on price competitiveness, generally the first effect dominates and an exchange rate depreciation induces higher inflation, which reduces domestic consumption. For instance, Mihailov et al. (2011) show that for Spain a currency depreciation increases the possibility to export at the cost of a lower purchasing power for consumers.

The New Keynesian IS equation is given by

$$x_t = E_t x_{t+1} - \sigma^{-1}(r_t - E_t\pi_{t+1}) + \sigma^{-1}\rho_t - \delta\left(E_t e_{t+1} - e_t\right), \tag{3}$$

where r_t is the nominal short-term interest rate, ρ_t represents an exogenous disturbance that moves the natural interest rate, and δ a composite parameter defined by $\delta \equiv \frac{1}{\sigma}\left[\frac{\Omega}{(1-\omega)} - 1\right]$ with $\Omega \equiv (1-\omega)\left[(1-\omega) + (2-\omega)\omega\zeta\sigma\right]$. The composite parameter δ is the elasticity of the output gap with respect to the expected change in the real exchange rate, reflecting the substitution effect induced by such a change on the demand of domestically produced goods.[3] Also with respect to the output gap, there are two competing effects of exchange rate movement. On the one hand, an exchange rate depreciation increases consumer prices

[3]Note that Ω and δ are positive for $(2-\omega)\zeta\sigma > 1$.

and reduces expected inflation; the resulting increase in the real interest rate reduces consumption and the output gap, given the expected future exchange rate. On the other hand, the exchange rate depreciation increases export demand, and therefore output. As shown in Leitemo and Söderström (2008), the same condition shown above for ϕ determines the type of relationship between exchange rate and output gap indirectly through the Phillips curve; ϕ determines whether a country would export more following a depreciation of its national currency.

Finally, the real UIP condition relates the real interest rate differential with the expected rate of real depreciation:

$$r_t - E_t \pi_{t+1} = E_t e_{t+1} - e_t, \tag{4}$$

where foreign variables are set to zero for simplicity.

3 Forward Guidance with Known Duration

In this section, we consider the case in which the central bank announces that it will keep the nominal interest rate fixed for T periods. The central bank is assumed to conduct policy through a Taylor rule like the following one[4]

$$r_t = \rho_t + \phi_\pi \pi_t \quad \phi_\pi > 1 \tag{5}$$

Such a rule guarantees that ignoring the zero lower bound there is a unique equilibrium characterized by $\pi_t = x_t = e_t = 0$.

The experiment that we consider is the following. After a shock to the natural rate the economy enters in the zero lower bound for a period of time. The central bank responds by adopting forward guidance, ie announcing that the nominal interest rate will be constant to $\bar{r}$ for a number of quarters T that does not necessarily coincide with that of the liquidity trap. We characterize the dynamics of the economy under the forward guidance period knowing that, after the forward guidance regime, the Taylor rule will provide a unique equilibrium. In particular, at time T, inflation, output gap and the real exchange rate will

[4]The specification of the Taylor rule is such that in normal times equilibrium is unique. Alternative specifications, including a response to output gap and/or real exchange rate could be considered.

be equal to

$$
\begin{aligned}
\pi_T &= \left(\phi - \kappa\delta - \frac{\kappa}{\sigma}\right)\bar{r} \\
x_T &= -\left(\delta + \frac{1}{\sigma}\right)\bar{r} \\
e_T &= -\bar{r}
\end{aligned}
$$

To solve the model during forward guidance, we write the system (2) – (4) in matrix notation with $Z \equiv \left(\begin{array}{ccc} \pi & x & e \end{array}\right)'$. It is convenient to invert this system, running time backwards from the end of the period of fixed interest rates. That is, let Y_s denote the value of Z s periods before time T: $Y_s \equiv Z_{T+1-s}$. Changing the system in this way will allow us to reinterpret the final conditions on π, x and e as initial conditions in a system of difference equations. Therefore the system (2) – (4) is written as

$$
Y_s = AY_{s-1} + B\bar{r} \tag{6}
$$

where $A \equiv \begin{bmatrix} \beta + \frac{\kappa}{\sigma} + \frac{\kappa}{\delta} - \phi & \kappa & -\phi \\ \frac{1}{\sigma} + \delta & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$, $B \equiv \begin{bmatrix} -\kappa\left(\frac{1}{\sigma} + \delta\right) \\ -\frac{1}{\sigma} - \delta \\ -1 \end{bmatrix}$

The system (6) has a solution composed by a non-homogenous part and a homogeneous part, while the initial values will be given by the vector

$$
\begin{pmatrix} \phi - \frac{\kappa}{\sigma} - \frac{\kappa}{\delta} \\ -\frac{1}{\sigma} - \delta \\ -1 \end{pmatrix} \bar{r}
$$

In this model, it turns out that the solution is not unique, as the matrix $(\mathbf{I} - A)$ is not invertible and the system admits ∞^1 solutions. This result confirms the classical result of exchange-rate indeterminacy following exogenous interest-rate rules, even under the assumption that this policy has a finite duration, which has been extensively discussed in the literature, as in Kareken and Wallace (1981), Obstfeld et al. (1996) and Benigno et al. (2007). However, the system admits solutions with the following structure

$$
Y_T = Z_1 = A^T Y_0 + \sum_{s=0}^{T-1} A^{T-s-1} B\bar{r} \tag{7}
$$

The previous model can be solved analytically but the results are hard to interpret. Therefore we calibrate it and solve it numerically. As a baseline calibration, we follow the values that Corbo et al. (2020) use for Sweden. In particular, they assume that $\hat{\sigma} = 1$, as well as the elasticity of substitution between domestic and foreign goods ζ. The Frisch elasticity of labor supply $\eta = 3.65$, while the share of firms that do not optimally set their prices ϑ is 0.93 and the degree of openness $\omega = 0.19$. These values imply that the composite parameters are given by $\kappa = 0.0292$, $\sigma = 1.235$, $\phi = 0.0011$ and $\delta = 0.15$. Therefore, this calibration is related to a country where the real exchange rate is negatively related with inflation and output.

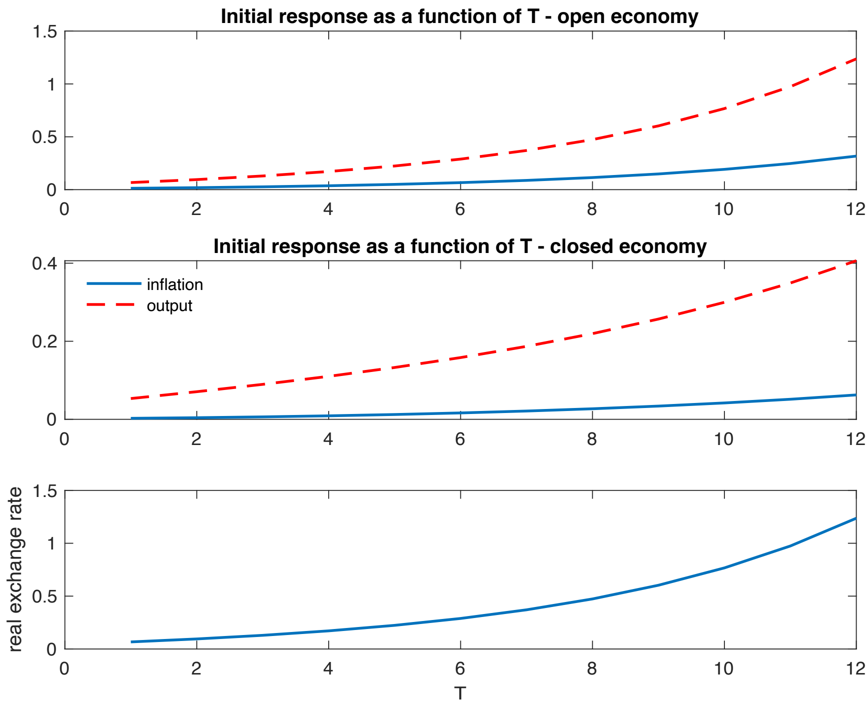


Figure 1: Initial response of the economy: closed versus open economy (authors' Matlab simulation)

Figure 1 compares the initial response of inflation and output gap during the forward guidance regime in open economy (top panel) and in closed economy (mid panel) as a function of forward guidance duration. The graph shows that the forward guidance puzzle arises both in closed and open economy and that the response of inflation and output gap is magnified in open economy[5] especially if we increase the time in which interest rate is fixed. We believe that two factors determine this higher response in open economy. The first one is the larger interest rate elasticity of the output gap which, in turn, improves the inflation-output trade-off in the Phillips' curve. The second factor is the exchange rate depreciation which boosts aggregate demand; in particular the exchange rate (bottom panel) moves exactly in the same way as the output gap[6] and this reinforces

[5]This is the reason why we put in two separate panels the variables in closed versus open economy.

[6]This is due to the fact that, as we showed before, in open economy, the system (2) – (4) admits infinite solutions so that output gap and exchange rate cannot be decoupled.

the expansionary effect of forward guidance.

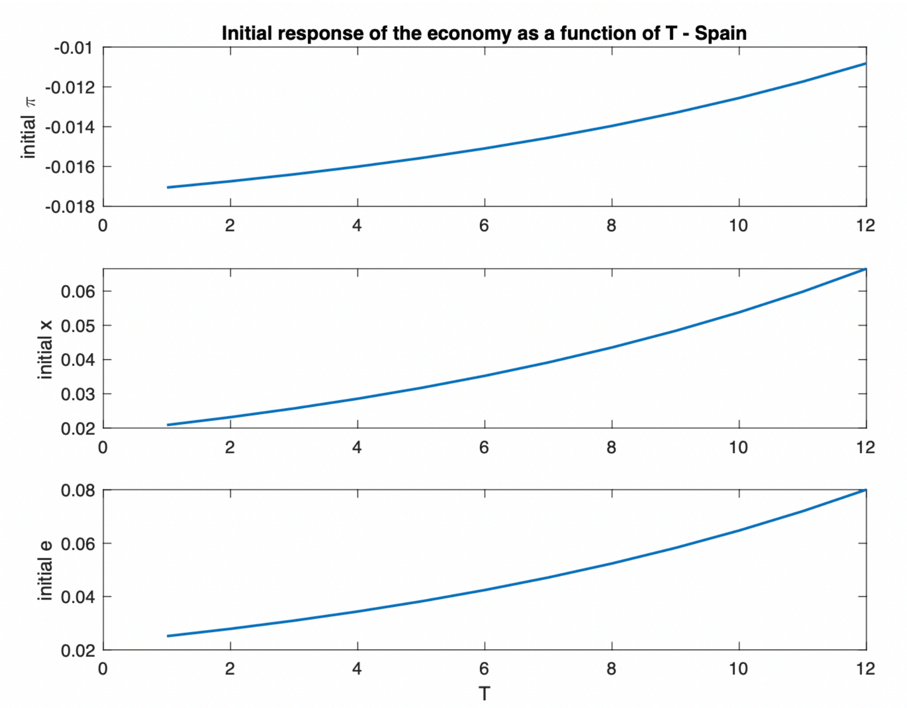


Figure 2: Initial response of the economy with negative ϕ and δ (authors' Matlab simulation)

As highlighted in section 2, the elasticity of inflation and output with respect to the real exchange rate can be either positive or negative. Therefore, we repeat the same analysis as before calibrating the model following the evidence Mihailov et al. (2011) for Spain. More in detail, the different values for such an economy are $\zeta = 0.25$, $\hat{\sigma} = 0.78$, $\vartheta = 0.85$ and $\omega = 0.25$, which, in turn, imply $\sigma = 1.045$, $\kappa = 0.1131$, $\phi = -0.0038$ and $\delta = -0.1299$. Figure 2 shows that, with negative passthrough, there are two stark differences compared to the previous analysis. First of all, it shows that the response of the economy is lower to both the cases described in Figure 1: in this case, due to the lower reactiveness of inflation and output gap to the exchange rate, inflation, output gap and the exchange rate react much less. Moreover, the initial level of inflation is negative and it is necessary to extend the duration of forward guidance (not shown here) to have positive inflation. This suggests that if a central bank wants to stimulate the economy through a forward guidance policy, it is necessary a commitment to maintain this regime for a long period when there is a low reactiveness to the exchange rate. Also in this case, under a theoretical viewpoint, there is forward guidance puzzle, albeit on a lower scale.

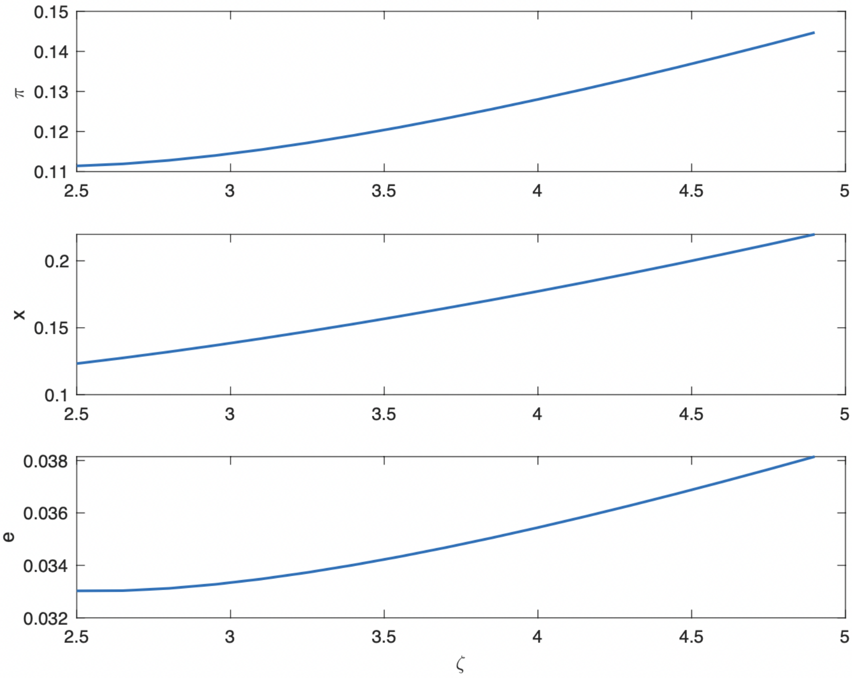


Figure 3: Response of the economy varying the elasticity of substitution between domestic and foreign goods. Inflation and output gap are expressed as a ratio over their closed-economy counterparts (authors' Matlab simulation).

The previous analysis has been conducted under the assumption of a unitary elasticity of substitution between domestic and foreign goods. However, over this parameter, there is not unanimous consensus in the literature. As emphasized by Di Giorgio and Nisticò (2013), when using aggregate data, the literature provides evidence of a value for the elasticity in the range of unity, as in Hooper et al. (2000), while this value is much above one when disaggregated data are used (see Obstfeld and Rogoff (2000) and McDaniel and Balistreri (2003)). More recently, Bajzik et al. (2020) collect 3524 reported estimates of the elasticity and investigate what drives the heterogeneity in the results. Taking into account also publication bias, the elasticity lies in the range 2.5-5.1 with a median of 3.8. Based on this empirical evidence, we repeat the analysis performed above varying ζ (and consequently ϕ and δ) in figure 3 for a forward guidance duration of $T = 12$. The figure shows that the initial response of the three endogenous variables are increasing in the elasticity of substitution: this occurs because the larger the elasticity of substitution, the larger the exchange rate pass-through to inflation and the output gap. As a result, the effects of a policy that keeps the policy rate fixed are amplified.

4 Forward Guidance with Stochastic Duration

Now we consider a version of the model where the duration of forward guidance is stochastic, focusing, as before, on the difference between closed and open economy, and also on the role played by exchange rate passthrough.

We assume that the central bank performs a fully credible forward guidance policy. As in Bilbiie (2019), we model forward guidance stochastically through a Markov chain as a state of the world with a probability distribution of p for the liquidity trap to happen. Consequently the expected stochastic duration of the liquidity trap is $T_L = (1-p)^{-1}$ which is the stopping time of the Markov chain. ρ_t follows a Markov chain of 3 states, one first state is the steady state S where $\rho_t = \rho$ and once reached, there is a probability 1 of staying there. The second state is the liquidity trap, being transitory, denoted by L where $r_t = 0$ and $\rho_t = \rho_L < 0$ with persistence probability p. After this time T_L, the CB sets $r_t = 0$ while $\rho_t = \rho > 0$, with probability q. The probability to move back to steady state from F is $1-q$. We denote this state F, with expected duration $T_F = (1-q)^{-1}$. The state F can be interpreted as CB's commitment to maintain a low policy rate low despite the fact the economy is out of the liquidity trap. Therefore, we have the following three states of the world:

1. Liquidity trap L, with $r_t = 0$ and $\rho_t = \rho_L$. The economy remains in this state with probability p and arrives to the state of forward guidance with probability $(1-p)q$.

2. Forward guidance F, with $r_t = 0$ and $\rho_t = \rho$. The economy is in this state with probability q and goes back to the steady state with probability $1-q$.

3. Steady state S with $r_t = \rho_t = \rho$ (absorbing state).

Given these assumptions, we can write the following expectations for our endogenous variables:

$$
\begin{aligned}
E_t x_{t+1} &= p x_L + (1-p) q x_F, \\
E_t \pi_{t+1} &= p \pi_L + (1-p) q \pi_F \\
E_t e_{t+1} &= p e_L + (1-p) q e_F
\end{aligned}
$$

4.1 Closed versus Open Economy

A natural question that arises is if forward guidance is more or less effective in open economy. Figure 4 shows the value of inflation and output gap, both in state F and in state L, for the open-economy case (with $\phi = 0.0011$, as for Sweden, solid lines) and for the closed-economy case (circled line), varying the probability of forward guidance q.

In state F, we can observe that the variables barely move when q is lower than 0.2 while they respond much more for large value of q (specifically, for $q > 0.8$). In terms of output, we observe the same peak in closed and in open economy, even if in open economy it is necessary to follow a fixed-interest rate policy for a longer period to arrive at this point. Interestingly, in state F forward guidance is not monotonically expansionary, in fact for $q > 0.8$ in open economy and for $q > 0.6$, we observe a deflation associated with an appreciation and a recession.

In state L, forward guidance is more expansionary in open economy: this is due to larger Phillips' curve slope κ and interest rate elasticity of the output gap σ, compared to closed economy. The other factor explaining the more expansionary effect in open economy is the exchange rate depreciation which boosts aggregate demand without the same increase in inflation observed in closed economy, due to the possibility of substituting domestic goods with foreign ones. Moreover, in closed economy a shorter period of forward guidance ($0 < q < 0.5$) is sufficient for the economy to reach the largest expansion for output in normal times due to direct transmission channels between inflation and output gap. Being in a small-open economy, the dynamics of the output gap follow closely those of the exchange rate, as highlighted by the literature, eg Galí and Monacelli (2005).

In state L, the path followed by output gap and inflation presents more differences across open and closed economy. While in closed economy the effect is almost muted up to approximately before $q = 0.5$ and then we observe a through followed by a peak, in open economy the effect is globally more expansionary (as already discussed for state F) and the troughs are sensibly lower. More in detail, the economy experiences a peak for $q = 0.65$, then we observe a decrease with inflation and output gap going into negative territory. Again, there is a key contribution of the real exchange rate: when it appreciates the economy enters in a deflation and a recession.

4.2 Comparison of Different Open Economies

We now compare the effects of forward guidance duration for two different open economies using different calibrations for Sweden and Spain respectively (see Table 1).

In the forward guidance experiment shown in Figure 5, the variables evolve in the same direction whether the economy has a positive or a negative exchange rate pass-through. In state F, the largest exchange rate depreciation, obtained between $0.6 \leq q <$

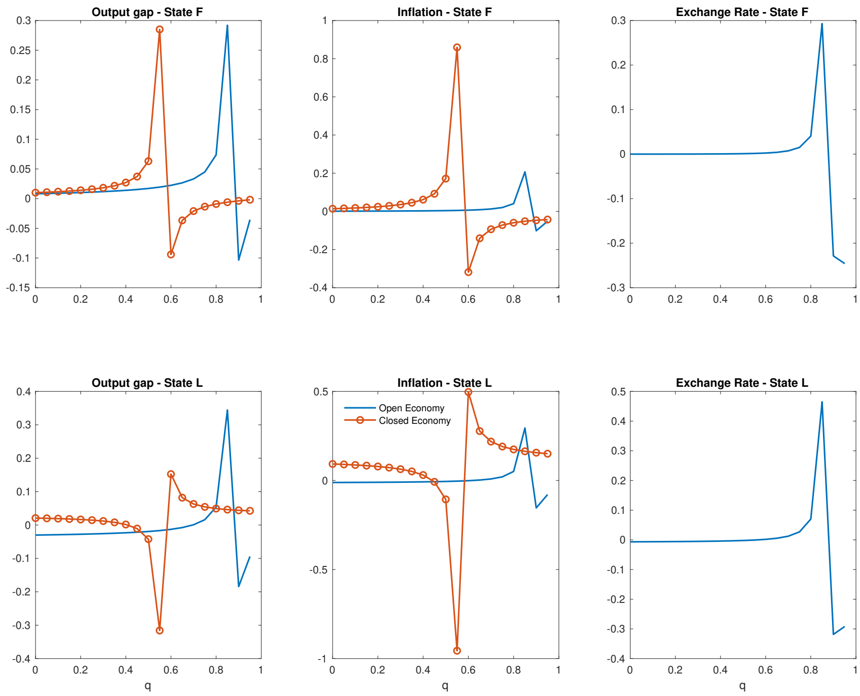


Figure 4: Comparison of forward guidance between a closed and open economy (authors' Matlab simulation)

	Parameters Sweden	Source	Parameters Spain	Source
β	0.99	Corbo et al. (2020)	0.99	Mihailov et al. (2011)
$\hat{\sigma}$	1	Corbo et al. (2020)	0.78	Mihailov et al. (2011)
ζ	0.87	Corbo et al. (2020)	0.25	Mihailov et al. (2011)
ω	0.19	Corbo et al. (2020)	0.25	Mihailov et al. (2011)
ϑ	0.93	Corbo et al. (2020)	0.85	Mihailov et al. (2011)
η	3.65	Corbo et al. (2020)	3	Mihailov et al. (2011)
σ	1.2346	Author's calculation	1.04	Author's calculation
ϕ	0.011	Author's calculation	-0.0038	Author's calculation
κ	0.029	Author's calculation	0.1130	Author's calculation

Table 1: Calibrations for Sweden and Spain

0.7 goes hand-in-hand with a peak in inflation and output gap for the case of a positive exchange rate pass-through (dashed line). For the case of Spain, i.e., negative exchange rate pass-through, we observe that we should at least engineer a forward guidance duration of 2.5 quarters to obtain a response of the exchange rate that depreciates in a small interval between $0.6 < q < 0.7$. Also in this case, the depreciation is associated with an expansion of output gap and inflation. Overall, the effect of forward guidance policy for Spain are lower compared to Sweden because of the much lower exchange rate pass-through[7].

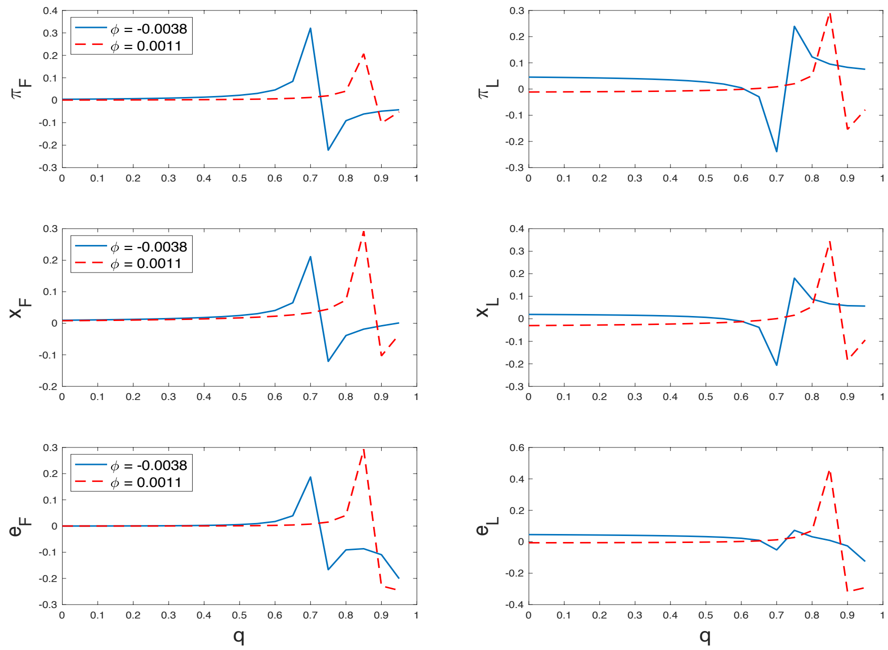


Figure 5: Inflation level, output gap and exchange rate for different ϕ corresponding to distinct economies: Spain ($\phi = -0.0038$, solid line), and Sweden ($\phi = 0.0011$, dashed line). (Authors' Matlab simulation)

As to the effects of forward guidance in a liquidity trap, as in the previous case, the response of the three variables becomes sizable for values of $q \geq 0.6$. However, under liquidity trap, inflation in Sweden is characterized by a peak and then it does not move substantially, while for Spain we observe that there is a trough (more or less when there is the peak for Sweden), followed by a peak and then inflation remains positive. Output gap moves in the same direction for both countries, while the real exchange rate depreciates significantly but temporarily only for the case of a larger exchange rate pass-trough. Overall, these results confirm that the exchange rate pass-through is a key variable.

4.3 Marginal Effects of the Exchange-Rate Pass Through and Forward Guidance Duration

Here we study the effects of the exchange-rate pass-through as well as forward guidance duration on the endogenous variables. Since analytical solutions become cumbersome,

[7]Moreover, if we calibrate the economy using the data for for Spain, we could show that forward guidance turns out to be more expansionary in closed economy.

for tractability, we first study closed-form solutions only for a version of the model with contemporaneous Phillips curve (as in Bilbiie (2019)), then we examine a more general version through numerical simulations.

4.3.1 Marginal Effects for the Contemporaneous Phillips Curve in State F

Following the methodology in Bilbiie (2019), and taking into account the expected values for output gap, inflation and exchange rate, we first solve for the state F and then for the state L. In state F we must solve the following system in x_F, π_F and e_F, (for $\beta = 0$, meaning that the Phillips curve is contemporaneous):

$$\begin{aligned}
\pi_F &= \kappa x_F - \phi e_F, \\
x_F &= (1-p)qx_F - \sigma^{-1}(\rho_t - (1-p)q\pi_F) + \sigma^{-1}\rho_t - \delta\left((1-p)qe_F - e_F\right) \\
\rho_t - (1-p)q\pi_F &= (1-p)qe_F - e_F
\end{aligned}$$

Solving the previous expression and remembering the relationship with π_F, we get the following pair of values for the state F:

$$\begin{aligned}
\pi_F &= \frac{-(\kappa\delta - \phi)}{\{1 - q[\kappa(\delta + \sigma^{-1}) - \phi + 1]\}}\rho_t \\
x_F &= \frac{-\left[\delta(1-q) - \phi\sigma^{-1}q\right]}{(1-q)\{1 - q[\kappa(\delta + \sigma^{-1}) - \phi + 1]\}}\rho_t \\
e_F &= \frac{-\left[1 - q - q\kappa\sigma^{-1}\right]}{(1-q)\{1 - q[\kappa(\delta + \sigma^{-1}) - \phi + 1]\}}\rho_t
\end{aligned}$$

We now expose the solutions obtained for state L using values given in state F:

$$\begin{aligned}
\pi_L = \; & \kappa\left\{\frac{q(1-p)\left(1+\kappa\sigma^{-1}\right)}{(1-p-\sigma^{-1}p\kappa)} + \frac{\phi\,[p + (1-p)q]}{[\phi p + (1-p)]}\right\}x_F \\
& -q(1-p)\left\{\frac{\kappa\left(\sigma^{-1}\phi + \delta\right)}{(1-p-\sigma^{-1}p\kappa)} + \frac{\phi\,(1+\phi)}{[\phi p + (1-p)]}\right\}e_F \\
& -\frac{\phi}{[\phi p + (1-p)]}\rho_L + \frac{\kappa\left[\delta(1-p) - \sigma^{-1}\phi p\right]}{(1-p-\sigma^{-1}p\kappa)}e_L
\end{aligned}$$

$$x_L = \frac{\left(1-p-\sigma^{-1}p\kappa\right)}{q(1-p)(1+\kappa\sigma^{-1})}x_L - \frac{\left[\delta(1-p) - \phi\sigma^{-1}\sigma^{-1}p\right]}{q(1-p)(1+\kappa\sigma^{-1})}e_L + \frac{\left(\sigma^{-1}\phi + \delta\right)}{(1+\kappa\sigma^{-1})}e_F$$

$$
\begin{aligned}
e_L &= \frac{[\phi p+(1-p)]q(1-p)\left(1+\kappa\sigma^{-1}\right)}{[\phi p+(1-p)]A}\rho_L \\
&+q(1-p)\frac{q(1-p)\left[(1+\phi)\left(1+\kappa\sigma^{-1}\right)-\left(\sigma^{-1}\phi+\delta\right)\right]-p\left(\sigma^{-1}\phi+\delta\right)}{A}e_F \\
&-\frac{\kappa[p+(1-p)q]\left(1-p-\sigma^{-1}p\kappa\right)}{A}x_L
\end{aligned}
$$

where $A=\left\{[\phi p+(1-p)]q(1-p)\left(1+\kappa\sigma^{-1}\right)-\kappa[p+(1-p)q]\left[\delta(1-p)-\phi\sigma^{-1}p\right]\right\}$.

With a contemporaneous Phillips curve, the effect of forward guidance duration on output gap is positive for a critical value of q:

$$
\begin{aligned}
\frac{\partial\pi_F}{\partial q} &= \frac{q(\kappa\delta-\phi)\left[\kappa\left(\delta+\sigma^{-1}\right)+1-\phi\right]}{\left\{1-q[\kappa(\delta+\sigma^{-1})-\phi+1]\right\}^2}\rho_t>0 \\
\frac{\partial x_F}{\partial q} &= \frac{\phi\sigma^{-1}-\left[\kappa\left(\delta+\sigma^{-1}\right)+1-\phi\right]\left[\delta(1-q)^2+\phi\sigma^{-1}q^2\right]}{(1-q)^2\left\{1-q[\kappa(\delta+\sigma^{-1})-\phi+1]\right\}^2}\rho_t>0, \\
\frac{\partial e_F}{\partial q} &= \frac{\kappa\sigma^{-1}\left\{1-q^2\left[\kappa\left(\delta+\sigma^{-1}\right)+1-\phi\right]\right\}-(1-q)^2\left[\kappa\left(\delta+\sigma^{-1}\right)+1-\phi\right]}{(1-q)^2\left\{1-q[\kappa(\delta+\sigma^{-1})-\phi+1]\right\}^2}\rho_t, \\
\frac{\partial\pi_F}{\partial\phi} &= \frac{2\phi-\kappa\delta-q\phi\left[\kappa\left(\delta+\sigma^{-1}\right)+1-\phi\right]}{\left\{1-q[\kappa(\delta+\sigma^{-1})-\phi+1]\right\}^2}\rho_t>0, \\
\frac{\partial x_F}{\partial\phi} &= q\frac{\sigma^{-1}\left\{1-q\left[\kappa\left(\delta+\sigma^{-1}\right)+1\right]\right\}+\delta(1-q)}{(1-q)\left\{1-q[\kappa(\delta+\sigma^{-1})-\phi+1]\right\}^2}\rho_t>0, \\
\frac{\partial e_F}{\partial\phi} &= \frac{-q}{\left\{1-q[\kappa(\delta+\sigma^{-1})-\phi+1]\right\}^2}\rho_t<0.
\end{aligned}
$$

Under the condition $(\kappa\delta-\phi)>0$ which generally holds in our simulations with $\beta=0$, any increase of q generates positive effects in state F on inflation and output gap; the effect is ambiguous on the exchange rate. Taking into account the relationship of π_F with x_F, we can conclude that forward guidance determines output and inflation expansion, together with real depreciation.

Furthermore, any increase in ϕ generates positive response in state F of inflation, the output gap but always a negative one of the exchange rate. This is straightforward from observing equations (1)-(4).

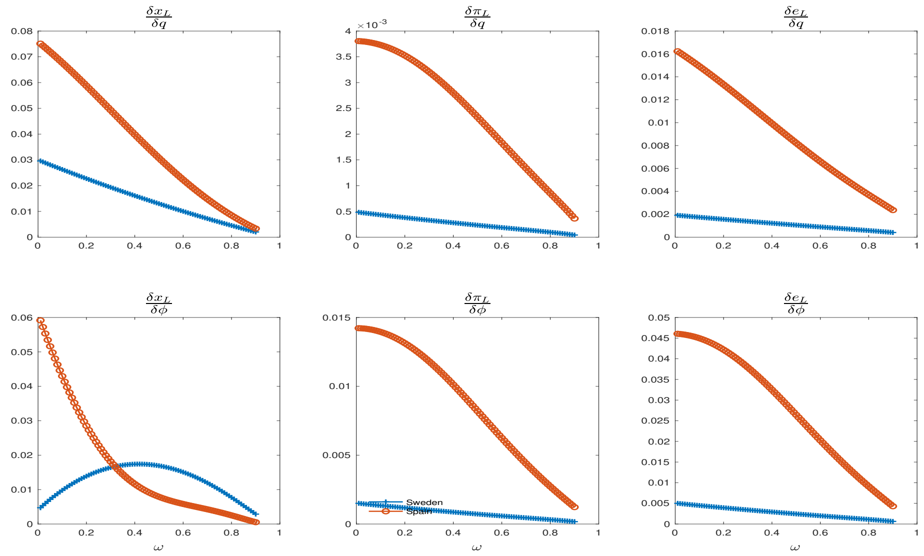
4.3.2 Marginal Effects in a Liquidity Trap

We now turn to the initial model where the Phillips curve corresponds to equation (2). Here we consider the model above described using the calibrations for two economies: Sweden and Spain. We solve numerically the model and then compute the derivatives of inflation, output gap and exchange rate with respect to q and ϕ respectively. This will allow us to characterize analytically how exchange rate passthrough and duration of the policy affect the transmission of forward guidance.

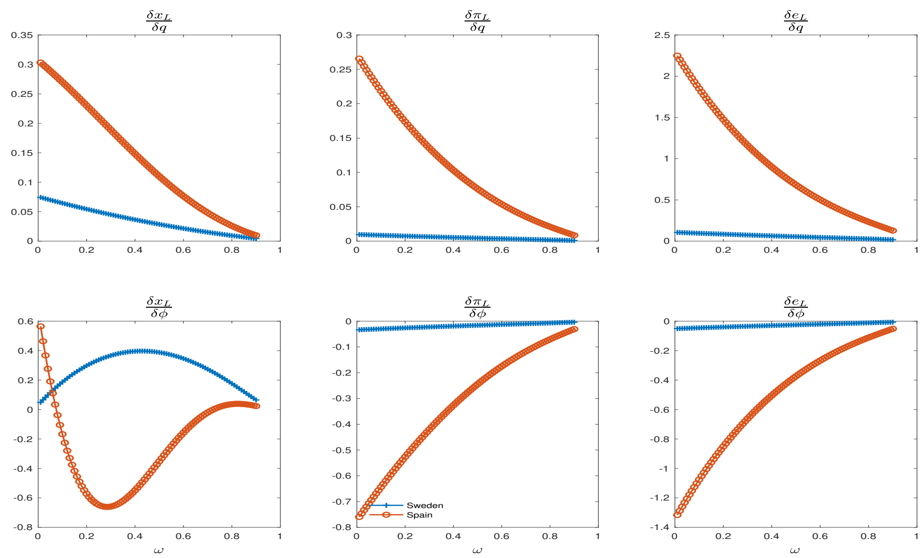
Marginal effect of forward guidance duration Here we study the effect of an increase in the duration of forward guidance episod in state L depending on the openness of the economy. We show numerically that, in Spain and Sweden, an increase in the duration of the forward guidance regime, always yields higher inflation, a higher output gap and the depreciation of the exchange rate, everything else being equal for any value of ω (Figures 6a-6b) and q (Figure 7) in state L. For both economies, this effect is stronger the longer the forward guidance, but an economy that is quite open (high ω) will experience lower marginal effects. If we consider Spain for instance, when comparing the response when the economy is closed ($\omega = 0$) and $q = 0.2$ corresponding to Figure 6a, with the case where the economy is closed and $q = 0.8$ corresponding to Figure 6b, the value is almost 4 times bigger when the duration of forward guidance increases from 1 to 5 quarters. This may suggest that there exists an adequate duration of the policy depending on the size of the stimulus the policy maker is aiming at.

Marginal effect of pass-through of exchange rate to inflation Comparing both countries, we can see that the variations of the exchange-rate pass-through produce different effects depending on the duration of forward guidance. With $q = 0.2$ (figure 6a) output, inflation and exchange rate in Spain are decreasing in ϕ. With $q = 0.8$ (figure 6b), inflation and exchange rate are increasing, while output first decreases reaching a minimum then it bounces back. On the other hand, we observe that in Sweden the three variables are much less sensitive. Therefore, in state L, the marginal effect of the pass-through depends strongly on the features of the economy, in particular on the elasticity of the interest rate to the output gap in the IS curve, σ, which is higher in Spain compared to Sweden (see Table 1).

From Figure 7, it turns out that in Spain whenever $q > 0.3$, a higher pass-through reduces output, while a threshold value of 0.4 and 0.55 are those that determine a negative response of inflation and the exchange rate respectively. A lower duration of forward guidance do not affect the transmission of exchange rate to the economy. As to Sweden, as already highlighted above, the effects are sensibly lower. The top panel of Figure 7 confirms that the expansionary effects of forward guidance increase in its duration.



(a) Derivatives in state L for $q = 0.2$



(b) Derivatives in state L for $q = 0.8$

Figure 6: Derivatives in state L for different openness of each economy for a given duration of forward guidance regime.

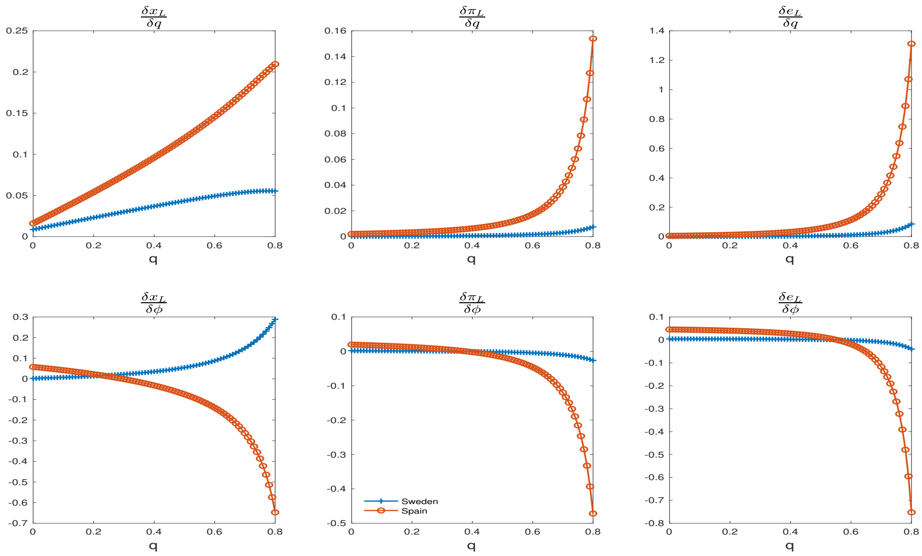


Figure 7: Derivatives in state L for different q with calibrations from Table 1 (authors' Matlab simulation).

5 Conclusion

This paper studies forward guidance in a theoretical DSGE for an advanced small open economy. We show that the elasticity of inflation to the real exchange rate is a key variable in determining the qualitative and quantitative response of the economy to a policy of fixed interest rates. The expansionary effect of the policy is positively related to the exchange rate pass-through and larger than in the closed economy counterpart because of a better inflation-output trade-off and the exchange rate channel. These findings generally hold also in the case in which forward guidance is implemented during a liquidity trap.

Our analysis suggests that when considering the possibility to adopt forward guidance, central banks should take into account how the exchange rate channel impacts in the policy transmission, as multiple equilibria and different responses could arise depending on the structure of the economy. These results might not necessarily be extended to emerging market economies. Several extensions to our setup can be considered. First, we do not analyze optimal forward guidance and in particular how it is related to the open economy dimension. Second, with incomplete information set available to the central bank, there might be an attenuation of the forward guidance puzzle. Third, forward guidance could produce heterogeneous effects as shown by Ferrante and Paustian (2019) in a HANK model. Finally, we have abstracted from fiscal shocks and on how the interaction between monetary and fiscal policy modifies the transmission of forward guidance.

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