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Alfredo Salgado
Banco de México

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Alfredo Salgado†
Banco de México

Abstract: This paper theoretically analyzes the role of reference prices on competition and welfare in a context of a circular city model with free entry and reference prices, in which paying market prices above a reference negatively affects the utility of consumers. Agents interact in a three-stage game: 1. A policymaker chooses a reference price to maximize consumer welfare. 2. Firms make their entry decision. 3. Firms compete in prices and consumers make their consumption decisions. We find that in equilibrium the market price and the optimal reference price chosen by the policymaker always coincide. In addition, it is shown that the use of reference prices reduces market equilibrium prices compared with the case without reference prices, which implies a net welfare gain for consumers. These gains could be higher in less competitive environments and lower in the presence of higher marginal costs.
Keywords: Regulation, Reference Prices, Welfare Gains.
JEL Classification: C7, D4, D9, L1, L2, L5.

Resumen: Este documento analiza teóricamente el papel de los precios de referencia en la competencia y el bienestar en un modelo de ciudad circular con libre entrada de empresas en el que pagar precios de mercado mayores a una referencia afecta negativamente la utilidad de los consumidores. Los agentes interactúan en un juego de tres etapas: 1. Un hacedor de política elige un precio de referencia con el fin de maximizar el bienestar de los consumidores. 2. Las empresas deciden si entrar o no al mercado. 3. Las empresas compiten en precios y los consumidores toman sus decisiones de gasto. Se encuentra que en equilibrio el precio de mercado y el precio de referencia óptimo elegido por el hacedor de política siempre coinciden. Adicionalmente, se muestra que el uso de precios de referencia reduce los precios de equilibrio de mercado en comparación con el caso sin precios de referencia, lo cual implica una ganancia neta de bienestar para los consumidores. Estas ganancias podrían ser mayores en entornos menos competitivos y menores en presencia de costos marginales más elevados.
Palabras Clave: Regulación, Precios de Referencia, Ganancias de Bienestar.

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† Dirección General de Investigación Económica. Email: asalgadot@banxico.org.mx.

1 Introduction

In this paper, we analyze the theoretical role of reference prices on competition and welfare in a context of a circular city model with free entry. We consider an environment in which the reference price is a decision of a policymaker. We use a simple environment where consumers are uniformly distributed along a unit circle and pay a transportation cost in order to buy one unit of an homogeneous good. Firms enter into the market at a fixed cost and pay a constant marginal cost of production (Salop (1979)). Based on previous literature, we introduce reference price effects on consumer preferences as a given parameter such that a consumer experiments a disutility from paying market prices above the reference price. However, unlike the previous literature where reference prices implemented by the policymaker are determined based on different exogenous rules[1] in our setting reference prices are optimally chosen by a policymaker who seeks to maximize consumer welfare. Although this scheme is interesting and differs from previous settings since it allows the optimal reference prices to be determined endogenously as a function of the fundamental parameters of preferences and technology, it is restrictive for several reasons. For example, the reference price effect parameter on consumer preferences is taken as given; it also is assumed that the policymaker is able of performing necessary calculations to determine the optimal reference prices; and it is also considered that the policymaker knows consumer preferences and firms technology.

In this setting, we analyze the effect of introducing reference prices on equilibrium market prices, competition and welfare. Three types of agents interact: a policymaker, firms and consumers. The policymaker takes consumer preferences and firms technology as given. Under these conditions, agents play a game in three stages. In the first stage, a policymaker chooses a reference price in order to maximize the consumer surplus. In the second stage, firms freely choose whether to enter or not into the market and where to locate. In the third stage, firms compete in prices taking as given the reference price and consumer demand an consumers make consumption decisions. We characterize the equilibrium of the reference price game, which is determined by an optimal reference price and equilibrium market prices. Our main result shows that in equilibrium the optimal reference price and the equilibrium market prices always coincide, intuitively reference prices play as a focal point that agents use to coordinate their strategic choices. In addition, we also show that the optimal reference price depends negatively on the intensity of the reference price effect on consumer preferences and positively on the marginal cost, on the extent of product differentiation determined by

[1]For instance, as the price of a product that is already offered in the market (national or international), as a linear combination of the prices of various products, or a linear combinations of past prices,etc.

transportation costs and on the cost of entry into the market.

Our characterization of the equilibrium allow us to perform welfare comparisons between a setting without references prices and our model. In a comparison with a standard Salop model, we show that in equilibrium market prices reduce in the face of reference price effects. Intuitively, this reduction in market prices leads to a reduction in firms revenue, leading to lower profits that finally translate into a lower entry of firms into the market. In the face of a smaller number of firms, consumers are forced to pay higher transportation costs to satisfy their demand. These two results have opposite effects on consumer welfare, on the one hand, the price reduction increases their utility, while the increase in transportation costs reduces it. According to the previous argument, a natural question regards the sign of the net effect of the reference price effects on consumer welfare (which in equilibrium coincides with social welfare). Our results show that in equilibrium positive welfare gains associated with the reference price effects are achieved, this implies that the increase in transportation costs is compensated by the decrease in prices, leading to a net positive effect on consumer utility. In addition to the previous analytical result, numerical solutions of the model suggest that those welfare gains could be even greater in markets which are naturally less competitive, i.e. in a context with either high entry cost or high transportation costs. Likewise, it also suggested to have small welfare gains in markets with high inefficiencies in production, i.e. markets with high marginal costs.

Although the model is limited by its simplicity the theoretical results suggest that in a context in which a policymaker seeks to strengthen competition through free entry, a regulatory policy through reference prices seems to be effective for improving social welfare whenever consumer preferences are characterized by having high enough reference price effects. In addition to pharmaceutical markets, the analysis of reference prices as a regulatory tool could be interesting in certain retail energy markets such as the gasoline market and the gas for domestic consumption. However, even when according to our results, a policymaker could encourage a reduction of market prices and an increase in consumer welfare through the use of reference prices, determining the appropriate regulatory policy in this kind of complex markets is not straightforward and requires a deep empirical analysis.

On the one hand, a deep analysis of consumer preferences is necessary because the results of our model strongly depend on the existence of reference price effects, which is a strong assumption about consumer utility functions. On the other hand, it is also assumed that the policymaker has complete information, i.e. he knows consumer preferences, the degree of competition and firms technology. Likewise, very particular preferences of the policymaker are assumed whose objective is to maximize consumer welfare (which coincides with social welfare only in equilibrium). In addition, other potential issues of real-life markets must be considered. For example, there could be complex barriers to entry in each link of the production chain. Entry could depend on the allocation of permits, concessions, or any other kind of complex property rights that would hinder the free entry of firms in the market. In addition, it is also important to consider the degree of dependence on foreign markets, for example, in the gas and gasoline markets, movements in retail prices strongly depend on international prices due to the important share of imports in total consumption. Also the presence of taxes and subsidies that distort market prices could make difficult to implement a reference price policy. Finally, it is also necessary to consider that regulation through reference prices could generate externalities on the whole production chain depending on the link in which it is implemented.

The rest of the paper is organized as follows. In section 2, we present a brief literature review. In section 3, we introduce the model and main definitions. In section 4, we characterize the equilibrium of the reference price game played by the policymaker, firms, and consumers. In section 5, we develop several comparative static exercises in order to understand the effect of changes in crucial parameters of the model on the equilibrium reference price. In section 6, we analyze the welfare implications of reference prices as well as the effect of changes of parameters of the model on welfare. Finally, in section 7, we present some general conclusions. All proofs can be found in the Appendix.

2 Related Literature

Reference prices have been analyzed in the literature as a regulatory tool. In general, the implementation of this type of policy seeks to increase consumer welfare and, under certain conditions, social welfare by reducing market prices paid by consumers and market power of firms. The related literature has analyzed different rules to determine reference prices such as an international price, the average price of a bundle of goods offered in the market or an average of prices observed in the past. However, the possibility of determining the reference price directly as the result of maximizing consumer welfare has not been analyzed. This analysis is interesting because, as we will show later, it is possible to determine endogenously an optimal reference price as a function of the parameters of interest of the model, to analyze its relationship with market equilibrium prices, and to establish some comparative statics exercises with respect to those parameters. Thus, in this paper, we add to the existing literature by analyzing the case in which the reference price is a policy decision, such that reference prices are chosen by a policymaker with the objective of maximizing consumer surplus in a simple static circular city model (Salop (1979)) with reference price effects.

Several theoretical and empirical papers have analyzed the role of price perception in models of consumer demand (Koschate-Fischer and Wüllner (2017) and Putler (1992)). These models have been developed in both dynamic and static schemes. In both settings, it is assumed that a consumer buys a product whenever its current price level is below a reference price. In a dynamic framework, it is usual to assume that consumers can use past prices and other setting variables to form a subjective reference price to take a consumption decision (Chenavaz (2016) and Popescu and Wu (2007)), because we focus on a static environment in our model, it is not possible to consider past prices or products purchased in the past as inputs to determine reference prices. In a static setting, the presence of reference price effects has been analyzed in the context of pharmaceutical markets where the reference price can be modeled as a price of an international product or as a price of a generic or as a price of a bundle of several products (See for instance, Brekke et al. (2016) and Brekke et al. (2009), Brekke et al. (2011), Kaiser et al. (2014). In a more general static setting, Zhou (2011) examines the impact of consumer reference dependence on market competition when consumers take some actual product in the market as the reference point. He shows that the prominent firm whose product is more likely to be taken as the reference point has an incentive to randomize between a high and a low price. Hence, reference dependence can cause price variation in the market.

Reference price effect literature has also analyzed different issues related to the implementation of reference prices as a tool for regulation. For instance, Brekke et al. (2009) compare the effects of regulating pharmaceutical prices through either reference prices or price caps. They find that reference prices seem to be more effective than price caps for lowering consumer drug prices. These results suggest the possibility of attaining welfare gains for consumers by implementing reference prices as a tool for regulators. In a similar paper, Brekke et al. (2011) also show that reference prices result in significantly lower brand-name market shares. In a setting that considers the role of external sector, Kaiser et al. (2014) study the result of a change from international to local reference pricing in Denmark in 2005 and find that this policy change yields to substantial reductions in retail prices, reference prices and patient co-payments as well as a decrease in overall producer revenues and health care expenditures. In a recent paper Brekke et al. (2016) develop a model where a brand-name producer competes in prices with several generic producers. They find that reference prices discourage generic producers to enter into the market, whereas the net effect of reference pricing on drug prices is ambiguous, implying that in some settings reference pricing can be counterproductive in reducing expenditures. However, they also show that reference pricing may be welfare improving when accounting for brand preferences instead of its effects on entry and prices.Unlike this literature, in our model we only compare the results in terms of prices, competition and welfare between a regulated environment with reference prices and another without regulation. Under the conditions of the model, we find that the reference prices allow for lower prices, welfare gains and less entry of firms, which brings the model closer to its first best result.

Another important contributions to reference price regulation literature include Miraldo (2009), who analyzed different reference price rules: (i) reference price as the minimum of the observed prices in the market, (ii) reference price as a linear combination of firm prices. Her results show that under the minimum policy firms are not able to coordinate on higher prices while a linear policy, implicitly, provides a coordination device. With quality differentiation both the "minimum" and "linear" policies unambiguously lead to higher prices. Birg (2015) studies the effect of two regulatory instruments (a price cap and a reference price system), a mandatory substitution rule, and the combination of both on generic competition in a Salop-type model in a pharmaceutical industry. Among other results, she shows that the two regulatory instruments reduce the brand-name drug price. In addition, the reference price system reduces generic prices and the price cap only if applied in combination with the mandatory substitution rule. On the downside, both regulatory instruments reduce the generic market share and the number of generic competitors. This suggests that there may be a conflict between price reductions and generic competition. In contrast to this literature, in our model reference prices are determined endogenously, as the result of a maximization problem for the policymaker whose preferences depend on consumer surplus. Thus, the optimal reference prices depend on the fundamental parameters of the model such as consumer preferences and firms technology.

3 The Model

There are three types of agents; a policymaker who chooses a reference price in order to maximize consumer welfare, a set of consumers who take as given the reference and market prices and seek to maximize their utility, and firms that compete in prices in order to maximize profits taking as given the reference price and consumer preferences.

Consumers are uniformly distributed on the unit circle. A consumer demands one unit of an indivisible good resulting in a gross utility $U > 0$. In order to buy a unit of the good from a firm i, a consumer must pay a price p_i and a transportation cost $t\,|d|$ where $|d|$ is the linear distance between the consumer and the firm i and t is the transportation cost per unit of distance. Unlike the classical Salop (1979) model, we include a reference price effect in consumer preferences. In particular, we assume that consumers suffer an utility loss from paying prices above a reference price that is published by a policymaker. In line with the price effect literature, we assume that such effect is asymmetric for consumers, i.e. given a reference price r whenever $p_i > r$ we assume that consumers incur into a disutility proportional to a margin above the reference price, otherwise consumers do not suffer an utility loss (Putler (1992)). In general, a consumer who buys one unit of the good from the firm i obtains a net utility given by the following expression.

$$U - \theta max\left\{\frac{p_i - r}{r}, 0\right\} - p_i - t\,|d| \tag{1}$$

Where $\frac{p_i - r}{r}$ is the margin over the reference price that is observed by consumers and $\theta > 0$ is a parameter that measures the intensity of reference price effect in consumer preferences of the model. Note that in the case where $\theta = 0$ the model collapses to the classical Salop (1979) model. Firms pay a fixed cost of entry and a variable cost with constant marginal cost.

$$C(q) = cq + F \tag{2}$$

There is free entry, i.e. firms enter into the market until they obtain profits equal to zero. Agents interact in a game in extensive form that runs in three stages as following:

1. The policymaker chooses a reference price r^{***} in order to maximize the consumer surplus.

2. Given the optimal reference price r^{***} and consumer preferences, firms freely choose whether to enter or not into the market and where to locate.

3. Firms that decided to enter into the market compete in prices in order to maximize their profits.

The equilibrium of the model is characterized by backward induction. At the third stage of the game, $N > 0$ firms compete in prices taking as given the reference price chosen by the policymaker and consumer preferences. At this stage, equilibrium market prices are characterized as a function of the reference price r, the parameters of consumer preferences, t and θ; the marginal cost c and the fixed number of firms $N > 0$. At the second stage of the game, firms make their entry decisions and the number of firms that enter the market is determined by a zero-profit condition. At this stage, equilibrium market prices and the number of firms are determined as functions of the reference price r and the relevant parameters of the model c, t, F and θ. At the first stage, the policymaker chooses an optimal reference price r^{***} taking as given the optimal behavior of firms and consumers in the next two stages. Basically, the policymaker calculates the consumer surplus, which by the free entry of firms coincides with the social surplus of the market, and chooses a reference price that maximizes the consumer welfare. Hence, the optimal reference price r^{***} must depend on the parameters c, t, F and θ.

4 Characterization of Equilibrium

4.1 Price Competition

At the last stage of the reference price game, $N \geq 0$ firms compete in prices by taking as given the references price already chosen by the policymaker and consumer preferences. We know that a consumer, who has bought one unit of the homogeneous good from a firm i, obtains a net utility given by the equation (1). Without loss of generality, we assume that firms locate along the unit circle market according to the principle of maximal differentiation, i.e. N firms will be located equidistant each other along the unit circle so that the distance between any two firms will be equal to $\frac{1}{N}$. We characterize a symmetric equilibrium of the game played by N firms, in which all firms will choose the same equilibrium price, i.e. $p^* = p_1 = p_2 = ... = p_N$. In order to characterize this equilibrium, we will consider the optimization problem of one firm i that takes as given the symmetric strategy followed by the rest of competitors, i.e. the firm i will chose a price p_i by considering that the rest of firms play the same strategy p. In this situation, an indifferent consumer located at the position x must obtain the same net utility from consuming one unit of the good from either firm i or the other closest available firm. Formally,

$$U - \theta max\left\{\frac{p_i - r}{r}, 0\right\} - p_i - tx = U - \theta max\left\{\frac{p - r}{r}, 0\right\} - p - t\left(\frac{1}{N} - x\right) \quad (3)$$

By solving the previous equation for x, we find that the indifferent consumer must satisfy the following condition:

$$x = \frac{1}{2N} + \frac{p - p_i}{2t} + \frac{\theta}{2tr}\left[max\left\{\frac{p-r}{r}, 0\right\} - max\left\{\frac{p_i - r}{r}, 0\right\}\right] \tag{4}$$

Since firms are equidistant to each other and all other firms but i are following a symmetric strategy, we know that the demand of firm i is equal to $d_i = 2x$. Note that as expected, the inclusion of a reference price in the circular city model changes the intercept and slope of the demand curve faced by any firm in the problem. The optimization problem of any typical firm i is given by the following program,

$$max_{p_i}\,(p_i - c)\left(\frac{1}{N} + \frac{p - p_i}{t} + \frac{\theta}{tr}\left[max\left\{p - r, 0\right\} - max\left\{p_i - r, 0\right\}\right]\right) \tag{5}$$

Before establishing our first result, we introduce the following Lemma that will be very useful to characterize the equilibrium market price of the model.

Lemma 1 *The function*

$$g(r) = c + \frac{tr}{N(r+\theta)} \tag{6}$$

has a unique fixed point r^* such that $g(r^*) = r^*$ and $c < r^* < c + \frac{t}{N}$.

Note that the profit function of the firm i given by the program 5 is continuous in p_i. However, the derivative of this profit function may have a discontinuity at $p_i = r$. Hence, the first order condition (FOC) of the maximization problem of the firm i will be useful to characterize a symmetric equilibrium only when the equilibrium price satisfies either $p^* > r$ or $p^* < r$, otherwise the best response function of the firm i is not well-defined. Given this observation, we are able to establish our first result.

Proposition 1 *The price strategy given by the function:*

$$p^*(r) = \begin{cases} c + \frac{tr}{N(r+\theta)} & if \quad 0 \le r < r^* \\ r & if \quad r^* \le r \le c + \frac{t}{N} \\ c + \frac{t}{N} & if \quad c + \frac{t}{N} < r \end{cases} \tag{7}$$

is a symmetric equilibrium of the price game for any given number of firms $N > 0$, where $r^* > 0$ is the unique reference price that satisfies the condition: $r^* = c + \frac{tr^*}{N(r^*+\theta)}$.

The equilibrium market price of the model with reference prices is an increasing and continuous function of the reference price of the market. Intuitively, consumers may delay their consumption plans in the presence of high market prices relative to the reference price, since they would incur an utility cost by paying a market price above the reference price that is provided by the policymaker. Hence, a high reference price allows firms to charge higher market prices, avoiding the reaction of consumers over market demand, since they also observe high reference prices. Another interesting implication of Proposition 1 is that in the presence of reference price effects consumers will, in general, pay a lower market price than in a situation with no reference prices. As we mentioned before, in the presence of reference prices consumers may delay their consumption plans, since a reference price allows consumers to calculate whether they are paying an over-price for consuming a good. Hence, in order to avoid a lower demand that may significantly reduce profits, firms react by charging lower prices than in a market without reference prices.

4.2 Free Entry of Firms

The equilibrium price characterized in the previous section depends on a fixed number of firms $N > 0$. In this section, we analyze the second part of the game by considering that in equilibrium firms enter into the market until a zero-profit condition is satisfied. Profits will depend on the demand and equilibrium prices faced by firms, i.e. $\Pi(r) = (p^*(r) - c)\frac{1}{N} - F$, since in a symmetric equilibrium with $N > 0$ firms, every firm will have a demand of $\frac{1}{N}$. Hence, firms' profits in a symmetric equilibrium would satisfy,

$$\Pi(r) = \begin{cases} \frac{tr}{N^2(r+\theta)} - F & if \quad 0 \leq r < r^* \\ \frac{r-c}{N} - F & if \quad r^* \leq r \leq c + \frac{t}{N} \\ \frac{t}{N^2} - F & if \quad c + \frac{t}{N} < r \end{cases} \tag{8}$$

The zero profit condition implies that the number of firms in the market will be a function of the reference price and other parameters of the model in the following general form,

$$N(r) = \begin{cases} \sqrt{\frac{r}{r+\theta}}\sqrt{\frac{t}{F}} & if \quad 0 \leq r < r^{**} \\ \frac{r-c}{F} & if \quad r^{**} \leq r \leq c + \frac{t}{N(r)} \\ \sqrt{\frac{t}{F}} & if \quad c + \frac{t}{N(r)} < r \end{cases} \tag{9}$$

Where r^{**} is the unique fixed point of the function $g(r) = c + \frac{tr}{N(r+\theta)}$ that is consistent with the free entry condition of the market, i.e. r^{**} is the unique reference price that satisfies $r^{**} = c + \sqrt{\frac{r^{**}}{r^{**}+\theta}}\sqrt{tF}$.[2] Hence, by substituting the number of firms that satisfy the zero

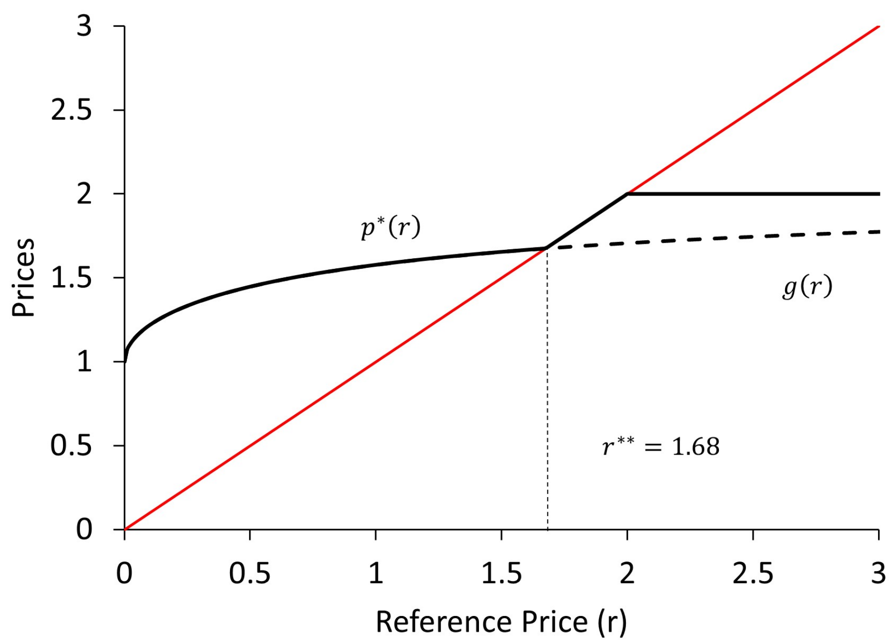
[2] As in the case of Lemma 1, it is easy to show that the function $\hat{g}(r) = c + \sqrt{\frac{r}{r+\theta}}\sqrt{tF}$ satisfies the conditions of having a unique fixed point, such that $\hat{g}(r^{**}) = r^{**}$ and $c < r^{**} < c + \sqrt{tF}$.

profit condition into the equilibrium market price function, we find that,

$$p^{**}(r) = \begin{cases} c + \sqrt{\frac{r}{r+\theta}}\sqrt{tF} & if \quad 0 \leq r < r^{**} \\ r & if \quad r^{**} \leq r \leq c + \sqrt{tF} \\ c + \sqrt{tF} & if \quad c + \sqrt{tF} < r \end{cases} \tag{10}$$

The previous result shows that in equilibrium both the number of firms that enter into the market and equilibrium market prices are increasing in the reference price of the market. Intuitively, higher reference prices allow firms to charge higher market prices, and this positive relation implies that in the presence of higher market prices more firms are willing to enter the market.

Figure 1: Equilibrium market price



Source: Based on equilibrium numerical solutions of the market price with parameters: $U = 10$, $c = 1$, $F = 1$, $t = 1$ and $\theta = 2$.

As can be observed in Figure 1, our characterization of the equilibrium market price allows us to distinguish three different areas depending on the value of the reference price. For reference prices below r^{**}, the market price is greater that the reference price and lies between the marginal cost c and r^{**}. For reference prices between r^{**} and $c + \sqrt{tF}$, the latter being the equilibrium price in the model without reference prices, the equilibrium market price is equal to the reference price. Finally, for reference prices above $c + \sqrt{tF}$ the market price is constant and equal to $c + \sqrt{tF}$. According to the previous argument, as a policy instrument, reference prices cannot induce inconsistent market prices in the sense that we will never observe either market prices below the marginal cost or above the equilibrium price without reference price effects. So optimal reference prices, if they exist, must lie between those well-defined limits.

In the next section, we will show that there exists a unique optimal reference price that maximizes consumer surplus. This result implies that there is a well-defined optimal policy that can be implemented by the policymaker. In addition, we will show that this optimal reference price has several interesting properties and it is related in a very particular way with equilibrium market prices.

4.3 Optimal Reference Price

A policymaker chooses a reference price r in order to maximizes consumer surplus. Note that in the symmetric equilibrium with free entry, consumer surplus is equivalent to the social welfare of the model since all firms make zero profits.

The consumer surplus for a given number of firms is equal to

$$CS = 2N \int_0^{\frac{1}{2N}} \left(U - \theta max \left\{ \frac{p-r}{r}, 0 \right\} - p - tx \right) dx \tag{11}$$

Which is equivalent to

$$CS = U - \theta max \left\{ \frac{p-r}{r}, 0 \right\} - p - \frac{t}{4N} \tag{12}$$

By substituting the number of firms consistent with the free entry condition and the equilibrium market price that is consistent with this number of firms, we can show that the consumer surplus satisfies the following expression,

$$CS(r) = \begin{cases} \begin{array}{l} U - \theta max \left\{ \frac{c + \sqrt{\frac{r}{r+\theta}}\sqrt{tF} - r}{r}, 0 \right\} - ... \\ ... - c - \sqrt{\frac{r}{r+\theta}}\sqrt{tF} - \frac{1}{4}\sqrt{\frac{r+\theta}{r}}\sqrt{tF} \end{array} & if \quad 0 < r < r^{**} \\ U - r - \frac{1}{4}\left(\frac{tF}{r-c}\right) & if \quad r^{**} \leq r \leq c + \sqrt{tF} \\ U - c - \sqrt{tF} - \frac{1}{4}\sqrt{tF} & if \quad c + \sqrt{tF} < r \end{cases} \tag{13}$$

The function $CS(r)$ is well-defined, continuous and differentiable almost everywhere.

Furthermore, note that for a sufficiently large r, reference price effects become negligible for consumer surplus (or social welfare), obtaining the same consumer welfare as in the model without reference price effects. This is consistent with the observation that market prices are increasing in r and lie between the limits c and $c + \sqrt{tF}$.

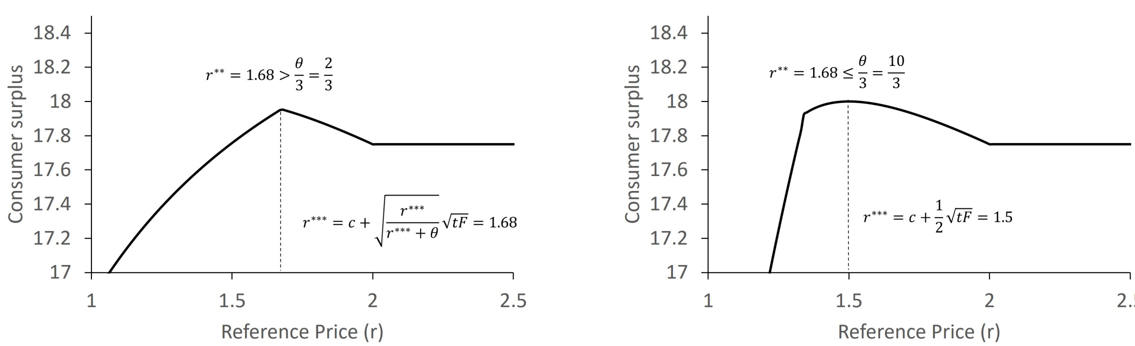
The problem of the policymaker consists in maximizing the consumer surplus function $CS\left(r\right)$ with respect to r. Note that the existence of an argmax of $CS(r)$ is not trivial, since $CS(r)$ is not differentiable at several points of the domain and is not a strictly concave function. Figure 2 illustrates the problems involved in the policymaker maximization problem. Fortunately, the following result shows that there is a well-defined and unique optimal reference price that globally maximizes the consumer surplus.

Proposition 2 *The reference price function*

$$r^{***} = \begin{cases} r^{**} & if \quad r^{**} > \frac{\theta}{3} \\ c + \frac{1}{2}\sqrt{tF} & if \quad r^{**} \leq \frac{\theta}{3} \end{cases} \tag{14}$$

is an optimal reference price for the policymaker, where r^{**} is the unique reference price that satisfies $r^{**} = c + \sqrt{\frac{r^{**}}{r^{**}+\theta}}\sqrt{tF}$.

Figure 2: Consumer Surplus Function



Source: Based on equilibrium numerical solutions of the policymaker problem with parameters: 1. Left: $U = 10$, $c = 1$, $F = 1$, $t = 1$ and $\theta = 2$ and 2. Right: $U = 10$, $c = 1$, $F = 1$, $t = 1$ and $\theta = 10$.

The optimal reference price chosen by the policymaker satisfies the condition $r^{**} \leq r^{***} < c + \sqrt{tF}$ for any value of the parameters of the model, including θ. Hence, a corollary of the previous result is that in equilibrium the optimal reference price chosen by the policymaker and the symmetric equilibrium market price always coincide (See Equation 10), formally,

Corollary 1 *The equilibrium market price of the model satisfies that $p^{**}(r^{***}) = r^{***}$ for any values of the parameters of the model, including θ.*

5 Comparative Statics

In equilibrium the optimal reference price chosen by the policymaker and the equilibrium market price attained through firm competition always coincide. This implies that comparative statics analyses of the optimal reference price is sufficient to understand how changes in underlying parameters of the model affect the equilibrium price of the market and welfare. There are four parameter of interest in the model that affect the equilibrium price:

1. Marginal cost, c;
2. Transportation cost, t;
3. Entry cost, F; and
4. Reference price effects on consumer preferences, θ.

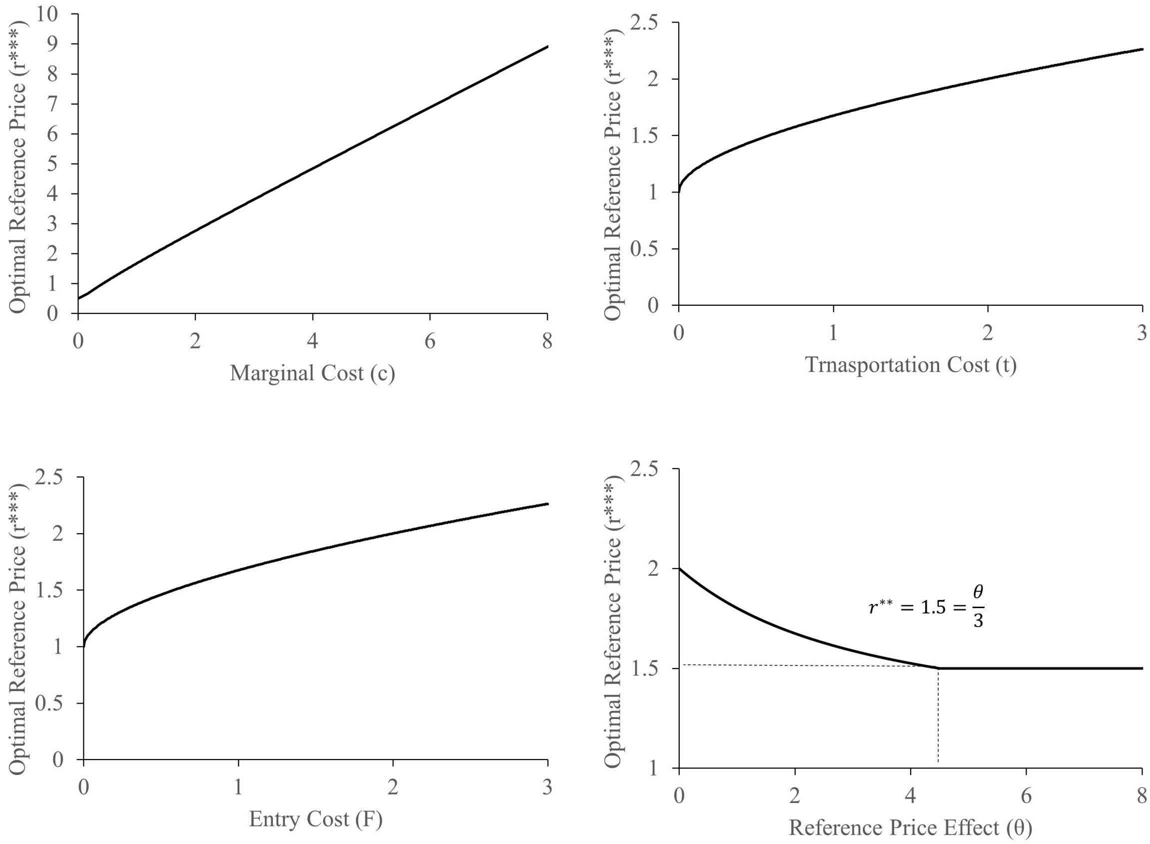
Given the characterization of equilibrium presented in the previous section, we can establish the following result.

Proposition 3 *The optimal reference price r^{***} satisfies the following properties:*

1. $\frac{\partial r^{***}}{\partial c} > 0$;
2. $\frac{\partial r^{***}}{\partial t} > 0$;
3. $\frac{\partial r^{***}}{\partial F} > 0$; *and*
4. $\frac{\partial r^{***}}{\partial \theta} < 0$ *whenever* $r^{**} > \frac{\theta}{3}$ *and* $\frac{\partial r^{***}}{\partial \theta} = 0$ *whenever* $r^{**} \leq \frac{\theta}{3}$.

Figure 3 shows that markets characterized by higher production costs, greater product differentiation or higher barriers to entry are characterized by higher reference prices. On the one hand, it is straightforward that higher marginal costs lead to higher reference prices. On the other hand, by having higher market entry costs or a greater differentiation between products through transportation costs, intuitively, we are in the presence of greater market power of firms due to which reference prices are higher. In contrast, when reference price effects are high, reference prices decrease. Intuitively, this greater sensitivity of consumers

Figure 3: Comparative Statics of the Optimal Reference Price r^{***}



Source: Based on equilibrium numerical solutions of the optimal reference price. The benchmark parameters of the model are: $U = 10$, $c = 1$, $F = 1$, $t = 1$ and $\theta = 2$.

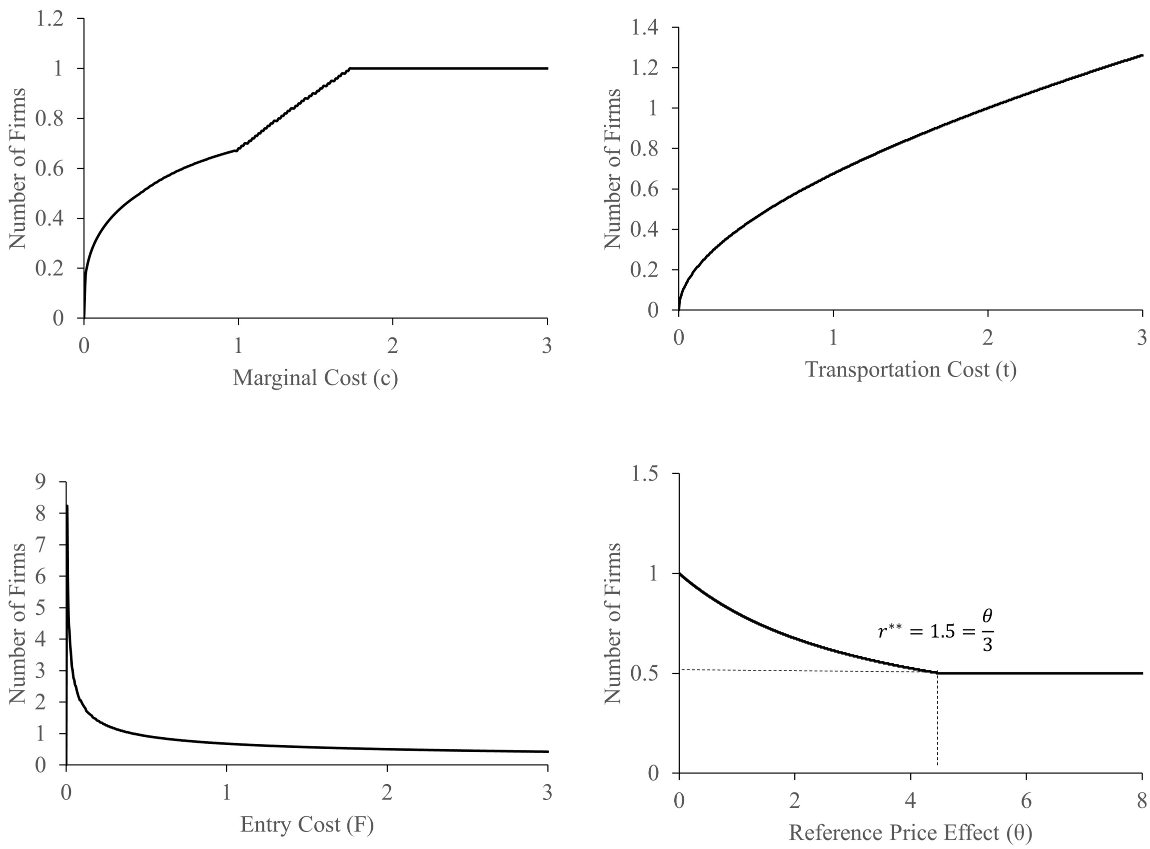
to reference prices reduces the market power of firms, contributing to have lower optimal reference prices.

Given that the equilibrium market price satisfies the condition $p^{**}\left(r^{***}\right) = r^{***}$, we know that all comparative static properties of the previous result extend directly as a corollary to the comparative statics analysis of the equilibrium market price. In addition, it is also possible to use the previous result to analyze what is the effect of movements in the underlying parameters of the model on the equilibrium number of firms that enter into the market and total transportation cost. In order to establish these results, we consider that the optimal number of firms satisfies the equation $NF\left(r\right) = \frac{r-c}{F}$ for $r^{**} \leq r < c + \sqrt{tF}$ while the optimal transportation cost satisfies the equation $TC\left(r\right) = \frac{1}{4}\left(\frac{tF}{r-c}\right)$ for $r^{**} \leq r < c + \sqrt{tF}$. Hence, we can establish the following results,

Proposition 4 *The optimal number of firms $NF(r)$ satisfies the following properties at the equilibrium market price $p^{**}(r^{***}) = r^{***}$:*

1. $\frac{\partial NF(r^{***})}{\partial c} > 0$ *whenever* $r^{**} > \frac{\theta}{3}$ *and* $\frac{\partial NF(r^{***})}{\partial c} = 0$ *whenever* $r^{**} \leq \frac{\theta}{3}$*;*
2. $\frac{\partial NF(r^{***})}{\partial t} > 0$*;*
3. $\frac{\partial NF(r^{***})}{\partial F} < 0$ *and*
4. $\frac{\partial NF(r^{***})}{\partial \theta} < 0$ *whenever* $r^{**} > \frac{\theta}{3}$ *and* $\frac{\partial NF(r^{***})}{\partial \theta} = 0$ *whenever* $r^{**} \leq \frac{\theta}{3}$.

Figure 4: Comparative statics over the equilibrium number of firms $NF(r^{***})$



Source: Based on equilibrium numerical solutions of the optimal reference price. The benchmark parameters of the model are: $U = 10$, $c = 1$, $F = 1$, $t = 1$ and $\theta = 2$.

Proposition 5 *The optimal transportation cost $TC(r)$ satisfies the following properties at the optimal reference price r^{***}:*

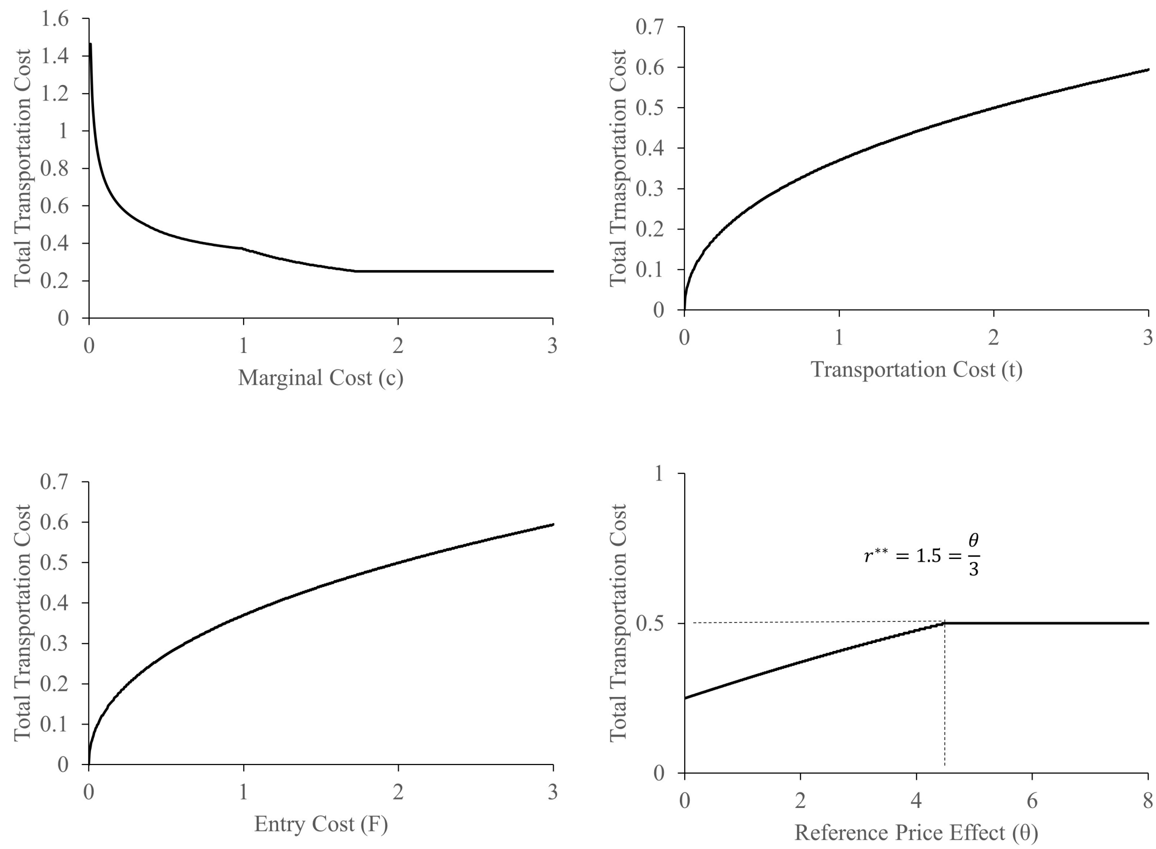
1. $\frac{\partial TC(r^{***})}{\partial c} < 0$ *whenever* $r^{**} > \frac{\theta}{3}$ *and* $\frac{\partial TC(r^{***})}{\partial c} = 0$ *whenever* $r^{**} \leq \frac{\theta}{3}$*;*

2. $\frac{\partial TC(r^{***})}{\partial t} > 0$*;*

3. $\frac{\partial TC(r^{***})}{\partial F} > 0$ *and*

4. $\frac{\partial TC(r^{***})}{\partial \theta} > 0$ *whenever* $r^{**} > \frac{\theta}{3}$ *and* $\frac{\partial TC(r^{***})}{\partial \theta} = 0$ *whenever* $r^{**} \leq \frac{\theta}{3}$.

Figure 5: Comparative Statics over the Equilibrium Transportation Cost $TC\left(r^{***}\right)$



Source: Based on equilibrium numerical solutions of the optimal reference price. The benchmark parameters of the model are: $U = 10$, $c = 1$, $F = 1$, $t = 1$ and $\theta = 2$.

Figures 4 and 5 illustrate an interesting effect of reference price effects on the optimal number of firms and the transportation costs paid by consumers in equilibrium. In fact, in equilibrium with a higher parameter of reference price effects, the optimal number of firms decreases while transportation costs increase, which implies that the degree of differentiation of products in the market decreases. This result leads to a reduction in the utility obtained from consumers, in turn we also show that an increase in the same parameter leads to a reduction in the market prices paid by consumers. These two results have opposite effects on consumer preferences and leave open the question regarding the sign of the net effect of the reference price effects on consumer welfare. In the next section, we explore this question and show that reference price effects always lead to a positive utility gain for consumers.

6 Welfare Comparisons

In the previous section, we showed that the optimal reference price increases as the parameters c, t and F increases. Since the equilibrium market price and the optimal reference price coincide, this result is consistent with a model with no reference price effects and it basically says that equilibrium market price must increase in the presence of greater inefficiencies in production (associated to a high marginal cost), higher horizontal differentiation (associated to a high transportation cost) and high entry cost for firms (high F). We also analyzed the case of an increase in the parameter θ, which is associated to a greater sensitivity of consumers to the difference between the market price and the reference price in the model. In this case, we also showed that an increase in the parameter θ is associated to a decrease in the equilibrium price whenever the parameter θ is sufficiently low, i.e. whenever $r^{**} > \frac{\theta}{3}$, otherwise an increase in the parameter θ has no effect on the equilibrium price.

We also show the results of the comparative static analysis on the equilibrium number of firms and transportation cost. The results of the previous section have several implications to understand the impact of an increase in the underlying parameters of the model on social welfare (i.e. consumer surplus in equilibrium) with free entry of firms. However, it is difficult to establish a direct conclusion about how consumer surplus is affected in the face of an increase in the parameters of the model. For instance, we showed that an increase in the parameter θ, associated to a greater reference price effect, decreases the equilibrium market price and increases the equilibrium total transportation cost. Hence, in principle the effect on social welfare may be ambiguous, since a decrease in prices must increase consumer surplus, while an increase in transportation costs must decrease it.

In addition, for a sufficiently low value of the parameter θ, i.e. $r^{**} > \frac{\theta}{3}$, the consumer surplus function is not differentiable at the argmax of the policymaker problem, then a direct application of the envelope theorem is not possible. Fortunately, right-hand and left-hand side derivatives of the consumer surplus exist at the argmax of this function, and this makes it possible to determine the upper and lower bounds of the variation of the value function of the policymaker, and determine the effects of a variation in parameters over the equilibrium consumer surplus of the problem. According to this argument, we establish the following result.

Proposition 6 *If $r^{**} > \frac{\theta}{3}$, then the social welfare function $CS(r)$ satisfies the following properties at r^{***}:*

1. $\frac{\partial CS(r^{***},c,t,F)}{\partial c} \in \left[-\frac{r^{**}+\theta}{r^{**}}, -\frac{r^{**}+\theta}{4r^{**}}\right]$,
2. $\frac{\partial CS(r^{***},c,t,F)}{\partial t} \in \left[-\frac{5}{8}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{\frac{F}{t}}, -\frac{1}{4}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{\frac{F}{t}}\right]$,
3. $\frac{\partial CS(r^{***},c,t,F)}{\partial F} \in \left[-\frac{5}{8}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{\frac{t}{F}}, -\frac{1}{4}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{\frac{t}{F}}\right]$, *and*
4. $\frac{\partial CS(r^{***},c,t,F)}{\partial \theta} \in \left[0, \frac{1}{2}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{tF}\left(\frac{1}{r^{**}+\theta} - \frac{1}{4r^{**}}\right)\right]$.

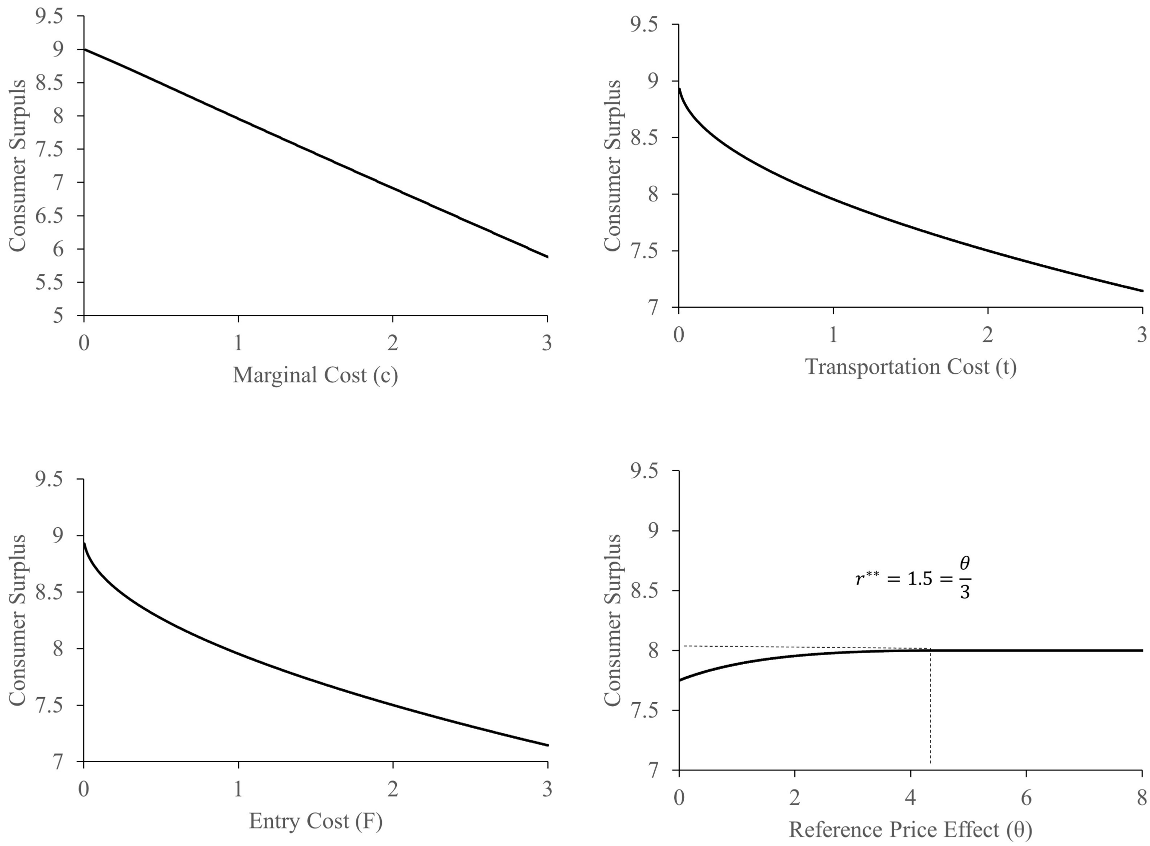
Otherwise, $CS(r)$ satisfies:

1. $\frac{\partial CS(r^{***},c,t,F)}{\partial c} = -1$,
2. $\frac{\partial CS(r^{***},c,t,F)}{\partial t} = -\frac{1}{2}\sqrt{\frac{F}{t}}$,
3. $\frac{\partial CS(r^{***},c,t,F)}{\partial F} = -\frac{1}{2}\sqrt{\frac{t}{F}}$, *and*
4. $\frac{\partial CS(r^{***},c,t,F)}{\partial \theta} = 0$.

According to the previous result, we can expect a decrease in social welfare in the face of greater inefficiencies in production (an increase in marginal cost, c), high entry cost (high F) that reduces the number of firms in the market or greater horizontal differentiation (higher t) that increases transportation cost. Our model also shows that a greater reference price effect (high θ) can be associated to an increase in consumer surplus which implies that the decrease in equilibrium market prices compensates the increase of transportation cost associated to a lower number of firms in the market (See Figure 6).

In addition to the comparative static analysis of equilibrium social welfare, a natural question arises regarding the comparison of social welfare of both models with and without reference price effects, in order to understand whether reference prices as policy tools are effective to obtain welfare gains in comparison to a standard Salop model. Let's define the following function $\Delta CS(r,c,t,F,\theta) = CS(r,c,t,F,\theta) - \left(U - c - \sqrt{tF} - \frac{1}{4}\sqrt{tF}\right)$ as the welfare difference between the equilibrium of the model with and without reference price effects. The following result may be established,

Figure 6: Comparative Statics over the Equilibrium Social Welfare $CS\left(r^{***}\right)$



Source: Based on equilibrium numerical solutions of the optimal reference price. The benchmark parameters of the model are: $U = 10$, $c = 1$, $F = 1$, $t = 1$ and $\theta = 2$.

Proposition 7 *The difference in social welfare* $\Delta CS\left(r, c, t, F, \theta\right)$ *is always positive at the optimal reference price* r^{***}

Intuitively, reference price effects entail three effects that result in greater social welfare with respect to a base model without reference prices. First, in equilibrium, market prices are always lower than those of a model without reference prices. Second, the equilibrium number of firms also is lower which makes the model closer to the one with an optimal number of firms, i.e. a model in which the sum of production and transportation cost is minimized. Third, even when a lower number of firms implies higher transportation cost, it is possible to show that in equilibrium the reduction in market prices compensate the increase in transportation cost leading to a higher social welfare in the presence of reference prices.

A second interesting question regards the comparative statics analysis of the difference in social welfare $\Delta CS\left(r,c,t,F,\theta\right)$. The previous result shows that the use of reference prices results in welfare gains in markets with horizontal differentiation where firms enter freely into the market and compete in prices. However, it would be interesting to have some conclusions about the size of welfare gains in the presence of reference prices in the face of greater inefficiencies, either high entry costs or high horizontal differentiation and a greater consumer sensitivity to reference prices. The following result is not entirely conclusive, since for sufficiently low values of the parameter θ the sign of the variation of the difference in social welfare is ambiguous. However, in the case of a sufficiently high value of the parameter θ, we can establish several conclusions. Formally, a Corollary of Proposition 6 is the following.

Corollary 2 *The difference in social welfare $\Delta CS\left(r,c,t,F,\theta\right)$ satisfies the following properties at r^{***}:*

1. $\frac{\partial \Delta CS(r^{***},c,t,F,\theta)}{\partial c} \in \left[-\frac{\theta}{r^{**}}, 1-\frac{r^{**}+\theta}{4r^{**}}\right]$ *if* $r^{**} > \frac{\theta}{3}$ *otherwise* $\frac{\partial \Delta CS(r^{***},c,t,F,\theta)}{\partial c} = 0.$

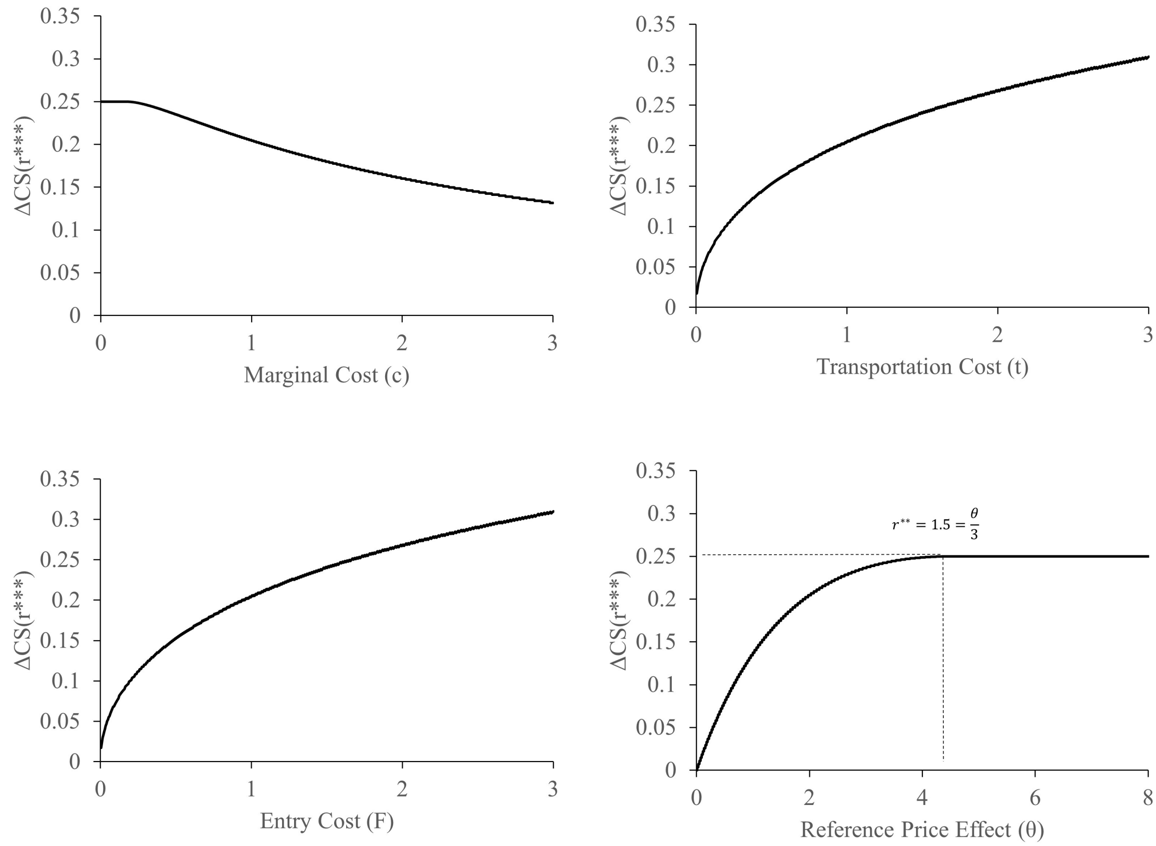
2. $\frac{\partial \Delta CS(r^{***},c,t,F,\theta)}{\partial t} \in \left[\frac{5}{8}\left(1-\sqrt{\frac{r^{**}+\theta}{r^{**}}}\right)\sqrt{\frac{F}{t}}, \frac{1}{4}\left(\frac{5}{2}-\sqrt{\frac{r^{**}+\theta}{r^{**}}}\right)\sqrt{\frac{F}{t}}\right]$ *if* $r^{**} > \frac{\theta}{3}$ *otherwise* $\frac{\partial \Delta CS(r^{***},c,t,F,\theta)}{\partial t} = \frac{1}{8}\sqrt{\frac{F}{t}}.$

3. $\frac{\partial \Delta CS(r^{***},c,t,F,\theta)}{\partial F} \in \left[\frac{5}{8}\left(1-\sqrt{\frac{r^{**}+\theta}{r^{**}}}\right)\sqrt{\frac{t}{F}}, \frac{1}{4}\left(\frac{5}{2}-\sqrt{\frac{r^{**}+\theta}{r^{**}}}\right)\sqrt{\frac{t}{F}}\right]$ *if* $r^{**} > \frac{\theta}{3}$ *otherwise* $\frac{\partial \Delta CS(r^{***},c,t,F,\theta)}{\partial F} = \frac{1}{8}\sqrt{\frac{t}{F}}.$

4. $\frac{\partial \Delta CS(r^{***},c,t,F,\theta)}{\partial \theta} \in \left[0, \frac{1}{2}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\left(\frac{1}{r^{**}+\theta}-\frac{1}{4r^{**}}\right)\sqrt{tF}\right]$ *if* $r^{**} > \frac{\theta}{3}$ *otherwise* $\frac{\partial \Delta CS(r^{***},c,t,F,\theta)}{\partial \theta} = 0.$

The previous result shows that the variation of welfare gains from the use of references prices as a regulation instrument is ambiguous in the face of an increase in underlying parameters of the model, since those welfare gains may be either positive or negative in the face of an increase in either c, t or F. Only in the case of the reference price effect parameter θ can be concluded that an increase of this parameter leads to an unambiguous increase in welfare gains. Only for sufficiently large values of θ, the variation of welfare gains is not ambiguous having positive variation for either t or F and no variation for either c or θ. Equilibrium numerical solutions of the model for several specifications of parameters show that, in general, we can expect a negative variation of welfare gains in the face of an increase of marginal cost. In the case of an increase of either horizontal differentiation t or entry cost F, it seems reasonable to expect an increase in welfare gains (See Figure 7).

Figure 7: Comparative Statics over the Difference in Equilibrium Social Welfare $\Delta CS\,(r^{***})$



Source: Based on equilibrium numerical solutions of the optimal reference price. The benchmark parameters of the model are: $U = 10$, $c = 1$, $F = 1$, $t = 1$ and $\theta = 2$.

Intuitively, our results show that the use of reference prices as a regulation tool not only leads to positive welfare gains with respect to a market with no reference prices, but also that these gains are even higher in less competitive markets with either a high entry or a high transportation costs. Likewise, it seems reasonable to expect small welfare gains in markets with a high marginal cost of production. Finally, even when a high reference price effect (higher values of θ) leads to greater welfare gains, we show that this effect is bounded above by the first best solution of the circular city model. For high values of θ equilibrium market prices stop declining in the face of a greater sensitivity of consumers to reference prices.

7 Conclusions

We analyze a circular city model with free entry and reference price effects, i.e. consumers suffer an utility loss from paying market prices above a reference price. Unlike the previous literature, we consider a reference price model where the reference price is explicitly modeled as a decision variable of a policymaker that seeks to maximize the consumer welfare of the market. In this setting, agents play a game in three stages. First, a policymaker chooses a reference price in order to maximize the consumer surplus. Second, firms freely choose whether to enter or not into the market and where to locate. Third, firms compete in prices taking as given the reference price chosen by the policymaker and consumer preferences. We characterize the equilibrium of the reference price game of the model by backward induction. The equilibrium of this model is determined by an optimal reference price and an equilibrium market price.

Our main result shows that in equilibrium the optimal reference price and the equilibrium market price always coincide. This result implies that the strategic interaction of the policymaker, firms and consumers is internally consistent. A comparative static analysis of the model shows that the optimal reference price (equilibrium market price) depends negatively on the intensity of the reference price effect of consumer preferences and positively on the marginal cost, the extent of product differentiation and the cost of market entry.

Our characterization of the equilibrium allow us to show that in equilibrium market prices reduce in a comparison with a standard Salop model with no reference price effects. Intuitively, this reduction in market prices leads to lower profits that finally translate into a lower entry of firms. In the face of a smaller number of firms, consumers pay higher transportation costs. Our results show that in equilibrium positive welfare gains associated with the reference price effects are achieved, this implies that the increase in transportation costs is compensated by the decrease in prices. Furthermore, numerical solutions of the model show evidence that those welfare gains are even higher in markets which are less competitive with either higher entry or transportation costs. Likewise, it seems reasonable to expect small welfare gains in markets with higher inefficiencies in production.

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Appendix: Proofs

Proof of Lemma 1

Proof. The function $g(r)$ is continuous and strictly increasing in the interval $[0, \infty)$, its derivative satisfies $g'(r) = \frac{t\theta}{N(r+\theta)^2} > 0$ for all $r \geq 0$. In addition, it is easy to show that $g(0) = c$ and $\lim g(r) = c + \frac{t}{N}$ as $r \to \infty$. Then, there must exists a unique r^* such that $g(r^*) = r^*$ that satisfies $c < r^* < c + \frac{t}{N}$ ■

Proof of Proposition 1

Proof. For a symmetric equilibrium of the model, there are three relevant cases for equilibrium prices for any given reference price r, i.e. $p^* > r$, $p^* < r$ and $p^* = r$.

The FOC of this maximization problem satisfies the following expression,

$$(p_i - c)\frac{\partial d_i}{\partial p_i} + \left(\frac{1}{N} + \frac{p - p_i}{t} + \frac{\theta}{tr}\left[max\left\{p - r, 0\right\} - max\left\{p_i - r, 0\right\}\right]\right) = 0 \tag{15}$$

where $\frac{\partial d_i}{\partial p_i} = -\left[\frac{1}{t} + \frac{\theta}{tr}\left(\frac{1+sign(p_i - r)}{2}\right)\right]$ and $sign(x) = \begin{cases} 1 & if \quad x > 0 \\ -1 & if \quad x < 0 \end{cases}$. Then the best response function of firm i satisfies the following equation,

$$BR_i(p) = \begin{cases} \frac{c}{2} + \frac{tr}{2N(r+\theta)} + \frac{pr}{2(r+\theta)} + \frac{\theta}{2(r+\theta)}\left(max\left\{p - r, 0\right\} + r\right) & if \quad p_i > r \\ \frac{c}{2} + \frac{t}{2N} + \frac{p}{2} + \frac{\theta}{2r}max\left\{p - r, 0\right\} & if \quad p_i < r \end{cases} \tag{16}$$

Hence, the FOC is useful to characterize the first two relevant cases for the equilibrium price. We know that in a symmetric equilibrium $p_i = p = p^*$. Based on the best response function of firms, it is easy to show that in the first relevant case of the proof, the equilibrium price must satisfy $p^* = c + \frac{tr}{N(r+\theta)}$ whenever $p^* > r$. According to Lemma 1 this condition holds whenever $r^* > r$, where r^* is the unique reference price that satisfies $r^* = c + \frac{tr^*}{N(r^*+\theta)}$. For the second case, the equilibrium price satisfies $p^* = c + \frac{t}{N}$ whenever $c + \frac{t}{N} > r$.

For the third relevant case, i.e. $p^* = r$ we must implement a direct proof since the best response function of firm i is not well-defined at this point. In general, we have to prove that playing r is a best response of the firm i, whenever all other firms play symmetrically $p = r$. Suppose that all firms, with the exception of the firm i, play a price $p = r$ that satisfies $r^* \leq r \leq c + \frac{t}{N}$. There are two cases to be analyzed: $p_i > r$ and $p_i < r$. It is clear that any of the two cases the price of firm i can be written as $p_i = r + \varepsilon$ and $p_i = r - \varepsilon$, for a proper $\varepsilon > 0$, respectively. Consider the first case, it is clear that profits of firm i can be written in the following way,

$$\begin{aligned}(r+\varepsilon-c)\left(\frac{1}{N}-\frac{\varepsilon}{t}-\frac{\theta\varepsilon}{tr}\right) &= (r-c)\frac{1}{N} \\ &+ \left[1-(r-c)\left(\frac{(r+\theta)N}{tr}\right)\right]\frac{\varepsilon}{N}-\left(\frac{r+\theta}{tr}\right)\varepsilon^2 \quad (17)\end{aligned}$$

Where $(r-c)\frac{1}{N}$ is the profit that is obtained by following the strategy $p_i = r$. Since $r^* \leq r \leq c + \frac{t}{N}$, there must exist a $\bar{r} \geq r$ such that $r = c + \frac{t\bar{r}}{N(\bar{r}+\theta)}$. By substituting in equation (17) and by noting that $\frac{tr}{N(r+\theta)}$ is a continuous and strictly increasing function in r, we find that,

$$(r-c)\frac{1}{N}+\left[1-\frac{\frac{t\bar{r}}{N(\bar{r}+\theta)}}{\frac{tr}{N(r+\theta)}}\right]\frac{\varepsilon}{N}-\left(\frac{r+\theta}{tr}\right)\varepsilon^2 < (r-c)\frac{1}{N} \quad (18)$$

By following a similar argument, in the case where $p_i < r$ profits of firm i can be written in the following way,

$$(r-\varepsilon-c)\left(\frac{1}{N}+\frac{\varepsilon}{t}\right) = (r-c)\frac{1}{N}+\left[(r-c)\frac{N}{t}-1\right]\frac{\varepsilon}{N}-\frac{\varepsilon^2}{t} \quad (19)$$

As before, by substituting $\bar{r} \geq r$ such that $r = c + \frac{t\bar{r}}{N(\bar{r}+\theta)}$ in equation 19 we find that,

$$(r-c)\frac{1}{N}+\left[\frac{\bar{r}}{\bar{r}+\theta}-1\right]\frac{\varepsilon}{N}-\frac{\varepsilon^2}{t} < (r-c)\frac{1}{N} \quad (20)$$

Hence, playing $p_i = r$ is a best response for firm i whenever all other firms are playing $p = r$. Then the equilibrium market price is given by the following function:

$$p^*(r) = \begin{cases} c + \frac{tr}{N(r+\theta)} & if \quad 0 \leq r < r^* \\ r & if \quad r^* \leq r \leq c + \frac{t}{N} \\ c + \frac{t}{N} & if \quad c + \frac{t}{N} < r \end{cases} \quad (21)$$

This completes the proof. ■

Proof of Proposition 2

Proof. Assume that $0 < r < r^{**}$, then the derivative of $CS(r)$ satisfies,

$$\frac{\partial CS\left(r\right)}{\partial r}=-\frac{\theta}{r^{2}}\left[\frac{1}{2}\sqrt{\frac{r+\theta}{r}}\left(\frac{\theta r}{\left(r+\theta\right)^{2}}\right)\sqrt{tF}-c-\sqrt{\frac{r}{r+\theta}}\sqrt{tF}\right]$$
$$-\frac{1}{2}\sqrt{\frac{r+\theta}{r}}\left(\frac{\theta}{\left(r+\theta\right)^{2}}\right)\sqrt{tF}+\frac{1}{8}\sqrt{\frac{r}{r+\theta}}\left(\frac{\theta}{r^{2}}\right)\sqrt{tF} \quad (22)$$

By reordering the previous expression, it is possible to show that equation 22 satisfies,

$$\frac{\partial CS\left(r\right)}{\partial r}=\frac{c\theta}{r^{2}}-\frac{c\theta}{2r^{2}}\sqrt{\frac{r}{r+\theta}}\left(\frac{\theta}{r+\theta}-2\right)\sqrt{tF}-\frac{c\theta}{2r^{2}}\sqrt{\frac{r}{r+\theta}}\left(\frac{r}{r+\theta}-\frac{1}{4}\right)\sqrt{tF} \quad (23)$$

Which reduces to the following expression,

$$\frac{\partial CS\left(r\right)}{\partial r}=\frac{c\theta}{r^{2}}\left(1+\frac{5}{8}\sqrt{\frac{r}{r+\theta}}\sqrt{tF}\right)>0 \quad (24)$$

Then the consumer surplus is always strictly increasing for $0<r<r^{**}$. Now consider the case where $r^{**}\leq r\leq c+\sqrt{tF}$, in this case the derivative of consumer surplus satisfies,

$$\frac{\partial CS\left(r\right)}{\partial r}=-1+\frac{tF}{4\left(r-c\right)^{2}} \quad (25)$$

Since $r^{**}>c$ this derivative is well define and the $CS(r)$ attains a maximum at the reference price $\hat{r}=c+\frac{1}{2}\sqrt{tF}$ and $CS(\hat{r})=U-c-\sqrt{tF}>U-c-\frac{5}{4}\sqrt{tF}$. In addition, note that there are two real roots $r1>0$ and $r2>0$ that satisfies the condition: $U-r-\frac{1}{4}\left(\frac{tF}{r-c}\right)=U-c-\frac{5}{4}\sqrt{tF}$. By a direct inspection, it is possible to show that $r1=c+\frac{1}{4}\sqrt{tF}$ and $r2=c+\sqrt{tF}$. Since $CS\left(r\right)$ is a continuous function at the point r^{**}, it is satisfied that

$$U-\theta max\left\{\frac{c+\sqrt{\frac{r^{**}}{r^{**}+\theta}}\sqrt{tF}-r^{**}}{r^{**}},0\right\}-c-\sqrt{\frac{r^{**}}{r^{**}+\theta}}\sqrt{tF}-\frac{1}{4}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{tF}=$$
$$U-r^{**}-\frac{1}{4}\left(\frac{tF}{r^{**}-c}\right) \quad (26)$$

This implies that $CS(r^{**})>U-c-\frac{5}{4}\sqrt{tF}$ for all $r1<r^{**}<r2$. Hence, there are two candidates for being the global argmax of the function $CS(r)$ depending on the values of parameters. According to the previous arguments, $\hat{r}$ would be the argmax of $CS(r)$ only if it is on the right side of r^{**}, otherwise the global argmax will be r^{**}. Hence, $\hat{r}=c+\frac{1}{2}\sqrt{tF}\geq$

$c+\sqrt{\frac{r^{**}}{r^{**}+\theta}}\sqrt{tF}=r^{**}$ whenever $\frac{1}{4}\geq\frac{r^{**}}{r^{**}+\theta}$, which is equivalent to $r^{**}\leq\frac{\theta}{3}$. This completes the proof. ■

Proof of Proposition 3

Proof. For the case of $r^{**}>\frac{\theta}{3}$, previous properties are trivially satisfied, given that r^{***} takes the closed form solution, $c+\frac{1}{2}\sqrt{tF}$. Since the optimal reference price has no closed form solution when $r^{**}\leq\frac{\theta}{3}$, we can calculate the implicit derivatives from the function $r=c+\sqrt{\frac{r}{r+\theta}}\sqrt{tF}$ with respect to each parameter of interest (In order to simplify, with some abuse of notation, in this case we simply use r instead of r^{**} for the optimal reference price).

Case 1: By implicitly differentiating $r=c+\sqrt{\frac{r}{r+\theta}}\sqrt{tF}$ with respect to c, we attain the following expression,

$$\frac{\partial r}{\partial c}=1+\frac{1}{2}\sqrt{\frac{r+\theta}{r}}\sqrt{tF}\left(\frac{(r+\theta)\frac{\partial r}{\partial c}-r\frac{\partial r}{\partial c}}{(r+\theta)^2}\right)\tag{27}$$

By reordering equation 27, we obtain,

$$\frac{\partial r}{\partial c}=1+\frac{1}{2}\sqrt{\frac{r}{r+\theta}}\sqrt{tF}\left(\frac{1}{r+\theta}\right)\left(\frac{\theta}{r}\right)\frac{\partial r}{\partial c}\tag{28}$$

Since by definition $r-c=\sqrt{\frac{r}{r+\theta}}\sqrt{tF}$, equation 28 can be reduced to the following,

$$\frac{\partial r}{\partial c}=\frac{1}{1-\frac{1}{2}\left(\frac{r-c}{r}\right)\left(\frac{\theta}{r+\theta}\right)}>0\tag{29}$$

Case 2: In a similar fashion, by implicitly differentiating the reference price function with respect to t, we obtain,

$$\frac{\partial r}{\partial t}=\frac{1}{2}\sqrt{\frac{r+\theta}{r}}\sqrt{tF}\left(\frac{(r+\theta)\frac{\partial r}{\partial t}-r\frac{\partial r}{\partial t}}{(r+\theta)^2}\right)+\frac{1}{2}\sqrt{\frac{r}{r+\theta}}\sqrt{\frac{F}{t}}\tag{30}$$

By reordering equation and substituting $r-c=\sqrt{\frac{r}{r+\theta}}\sqrt{tF}$ in equation 30, we obtain,

$$\frac{\partial r}{\partial t}=\frac{\frac{1}{2}\left(\frac{r-c}{t}\right)}{1-\frac{1}{2}\left(\frac{r-c}{r}\right)\left(\frac{\theta}{r+\theta}\right)}>0\tag{31}$$

Case 3: Basically the same for t, simply exchange t by F in the previous implicit derivative.

Case 4: By implicitly differentiating the reference price function with respect to θ, we obtain the expression,

$$\frac{\partial r}{\partial \theta} = \frac{1}{2}\sqrt{\frac{r+\theta}{r}}\sqrt{tF}\left(\frac{(r+\theta)\frac{\partial r}{\partial \theta} - r\left(\frac{\partial r}{\partial \theta}+1\right)}{(r+\theta)^2}\right) \tag{32}$$

By reordering equation 31, we can express equation 33 as,

$$\frac{\partial r}{\partial \theta} = -\frac{\frac{1}{2}\left(\frac{r-c}{r+\theta}\right)}{1-\frac{1}{2}\left(\frac{r-c}{r}\right)\left(\frac{\theta}{r+\theta}\right)} < 0 \tag{33}$$

This completes the proof. ■

Proof of Proposition 4

Proof. Let´s consider the number of firms function given by the expression $NF(r) = \frac{r-c}{F}$ for $r^{**} \leq r < c + \sqrt{tF}$.

Case 1: By directly differentiating $NF(r)$ with respect to c, we obtain,

$$\frac{\partial NF(r)}{\partial c} = \frac{1}{F}\left(\frac{\partial r^{***}}{\partial c} - 1\right) \tag{34}$$

For the case of $r^{**} > \frac{\theta}{3}$, equation 34 is equivalent to

$$\frac{\partial NF(r^{***})}{\partial c} = \frac{1}{F}\left(\frac{\frac{1}{2}\left(\frac{r^{***}-c}{r^{***}}\right)\left(\frac{\theta}{r^{***}+\theta}\right)}{1-\frac{1}{2}\left(\frac{r^{***}-c}{r^{***}}\right)\left(\frac{\theta}{r^{***}+\theta}\right)}\right) \tag{35}$$

Which is positive, since $1 - \frac{1}{2}\left(\frac{r^{***}-c}{r^{***}}\right)\left(\frac{\theta}{r^{***}+\theta}\right) > 0$. For the case of $r^{**} \leq \frac{\theta}{3}$, we know that $\frac{\partial r^{***}}{\partial c} = 1$.

Case 2: In a similar way, by differentiating $N(r)$ with respect to t, we have

$$\frac{\partial NF(r^{***})}{\partial t} = \frac{1}{F}\frac{\partial r^{***}}{\partial t} \tag{36}$$

Hence, $\frac{\partial NF(r^{***})}{\partial t} > 0$ since $\frac{\partial r^{***}}{\partial t} > 0$.

Case 3: By directly differentiating $N(r)$ with respect to F, we obtain,

$$\frac{\partial NF(r)}{\partial F} = \frac{1}{F}\left(\frac{\partial r^{***}}{\partial F} - \frac{r^{***}-c}{F}\right) \tag{37}$$

We know that $\frac{\partial r^{***}}{\partial F} = \frac{\frac{1}{2}\left(\frac{r^{***}-c}{F}\right)}{1-\frac{1}{2}\left(\frac{r^{***}-c}{F}\right)\left(\frac{\theta}{r^{***}+\theta}\right)}$ for the case of $r^{**} > \frac{\theta}{3}$. Hence, equation 37 is equivalent to

$$\frac{\partial NF\left(r^{***}\right)}{\partial F}=-\frac{1}{F}\left(\frac{\frac{1}{2}-\frac{1}{2}\left(\frac{r^{***}-c}{r^{***}}\right)\left(\frac{\theta}{r^{***}+\theta}\right)}{1-\frac{1}{2}\left(\frac{r^{***}-c}{r^{***}}\right)\left(\frac{\theta}{r^{***}+\theta}\right)}\right) \tag{38}$$

It is clear that $1-\frac{1}{2}\left(\frac{r^{***}-c}{r^{***}}\right)\left(\frac{\theta}{r^{***}+\theta}\right)>\frac{1}{2}$, then $\frac{1}{2}-\frac{1}{2}\left(\frac{r^{***}-c}{r^{***}}\right)\left(\frac{\theta}{r^{***}+\theta}\right)>0$ which implies that $\frac{\partial NF(r^{***})}{\partial F}<0$. When $r^{**}\leq\frac{\theta}{3}$, the optimal reference price satisfies $r^{***}=c+\frac{1}{2}\sqrt{tF}$ which implies that $\frac{\partial r^{***}}{\partial F}=\frac{1}{4}\sqrt{\frac{t}{F}}$ and $\frac{r^{***}-c}{F}=\frac{1}{2}\sqrt{\frac{t}{F}}$, hence $\frac{\partial NF(r^{***})}{\partial F}<0$.

Case 4: For the last case, by differentiating $N\left(r\right)$ with respect to θ, we have the expression,

$$\frac{\partial NF\left(r^{***}\right)}{\partial\theta}=\frac{1}{F}\frac{\partial r^{***}}{\partial\theta} \tag{39}$$

Which directly implies the result, since we know that $\frac{\partial r^{***}}{\partial\theta}<0$ whenever $r^{**}>\frac{\theta}{3}$ and $\frac{\partial r^{***}}{\partial\theta}=0$ whenever $r^{**}\leq\frac{\theta}{3}$.

This completes the proof. ■

Proof of Proposition 5

Proof. Let's consider the transportation cost function that is relevant for the analysis given by $TC\left(r\right)=\frac{1}{4}\left(\frac{tF}{r-c}\right)$ for $r^{**}\leq r<c+\sqrt{tF}$.

Case 1: By directly differentiating $TC\left(r\right)$ with respect to c, we obtain,

$$\frac{\partial N\left(r^{***}\right)}{\partial c}=-\frac{tF}{4\left(r^{***}-c\right)^{2}}\left(\frac{\partial r^{***}}{\partial c}-1\right) \tag{40}$$

Since for the case of $r^{**}>\frac{\theta}{3}$, we know that $\frac{\partial r^{***}}{\partial c}>1$ this implies that $\frac{\partial TC(r^{***})}{\partial c}<1$. For the case of $r^{**}\leq\frac{\theta}{3}$, we know that $\frac{\partial r^{***}}{\partial c}=1$, hence $\frac{\partial TC(r^{***})}{\partial c}=0$.

Case 2: In a similar way, by differentiating $TC\left(r\right)$ with respect to t, we have

$$\frac{\partial TC\left(r^{***}\right)}{\partial t}=\frac{\left(r^{***}-c\right)F-tF\frac{\partial r^{***}}{\partial t}}{4\left(r^{***}-c\right)^{2}} \tag{41}$$

After some steps 41 reduces to

$$\frac{\partial TC\left(r^{***}\right)}{\partial t}=\frac{tF}{4\left(r^{***}-c\right)^{2}}\left(\frac{r^{***}-c}{t}-\frac{\partial r^{***}}{\partial t}\right) \tag{42}$$

Given that $\frac{\partial r^{***}}{\partial t}=\frac{\frac{1}{2}\left(\frac{r^{***}-c}{t}\right)}{1-\frac{1}{2}\left(\frac{r^{***}-c}{r^{***}}\right)\left(\frac{\theta}{r^{***}+\theta}\right)}$ for $r^{**}>\frac{\theta}{3}$, we know that equation 42 reduces to

$$\frac{\partial TC\left(r^{***}\right)}{\partial t}=\frac{F}{4\left(r^{***}-c\right)}\left(\frac{\frac{1}{2}-\frac{1}{2}\left(\frac{r^{***}-c}{r^{***}}\right)\left(\frac{\theta}{r^{***}+\theta}\right)}{1-\frac{1}{2}\left(\frac{r^{***}-c}{r^{***}}\right)\left(\frac{\theta}{r^{***}+\theta}\right)}\right)>0 \tag{43}$$

Which is positive, since $\frac{1}{2}-\frac{1}{2}\left(\frac{r^{***}-c}{r^{***}}\right)\left(\frac{\theta}{r^{***}+\theta}\right)>0$. When $r^{**}\leq\frac{\theta}{3}$, the optimal reference price satisfies $r^{***}=c+\frac{1}{2}\sqrt{tF}$ which implies that $\frac{\partial r^{***}}{\partial t}=\frac{1}{4}\sqrt{\frac{F}{t}}$ and $\frac{r^{***}-c}{t}=\frac{1}{2}\sqrt{\frac{F}{t}}$, hence $\frac{\partial N(r^{***})}{\partial F}>0$.

Case 3: Basically the same case of t, by substituting t by F and vice versa. Hence, when $r^{**}>\frac{\theta}{3}$ it is satisfied

$$\frac{\partial TC\left(r^{***}\right)}{\partial F}=\frac{t}{4\left(r^{***}-c\right)}\left(\frac{\frac{1}{2}-\frac{1}{2}\left(\frac{r^{***}-c}{r^{***}}\right)\left(\frac{\theta}{r^{***}+\theta}\right)}{1-\frac{1}{2}\left(\frac{r^{***}-c}{r^{***}}\right)\left(\frac{\theta}{r^{***}+\theta}\right)}\right)>0 \tag{44}$$

And whenever $r^{**}\leq\frac{\theta}{3}$, $\frac{\partial N(r^{***})}{\partial F}=\frac{tF}{4(r^{***}-c)^2}\left(\frac{r^{***}-c}{F}-\frac{\partial r^{***}}{\partial F}\right)>0$.

Case 4: For the last case, by differentiating $TC\left(r\right)$ with respect to θ, we have the expression,

$$\frac{\partial TC\left(r^{***}\right)}{\partial\theta}=-\frac{tF}{4\left(r^{***}-c\right)^2}\frac{\partial r^{***}}{\partial\theta} \tag{45}$$

Which directly implies the result, since we know that $\frac{\partial r^{***}}{\partial\theta}<0$ whenever $r^{**}>\frac{\theta}{3}$ and $\frac{\partial r^{***}}{\partial\theta}=0$ whenever $r^{**}\leq\frac{\theta}{3}$.

This completes the proof. ■

Proof of Proposition 6

Proof. For the case of $r^{**}>\frac{\theta}{3}$ the social welfare function is not differentiable at r^{**}, which is the argmax of the policymaker problem. In this case, it is only possible to determine the lower and the upper bounds of the variation of the value function thought the right-hand and the left-hand side partial derivatives of the objective function evaluated at the optimal reference price. Otherwise, whenever $r^{**}\leq\frac{\theta}{3}$ the objective function is differentiable at the optimal reference price r^{**} and a regular envelope theorem can be applied.

Case 1: By right-hand side differentiating the social welfare function $CS\left(r,c,t,F,\theta\right)$ with respect to c, it is obtained

$$\frac{\partial CS\left(\cdot\right)}{\partial c}=-\frac{tF}{4\left(r-c\right)^2} \tag{46}$$

Since $r^{**} = c + \sqrt{\frac{tFr^{**}}{r^{**}+\theta}}$ for $r^{**} > \frac{\theta}{3}$, after evaluating $\frac{\partial CS(\cdot)}{\partial c}$ at r^{**} we have

$$\frac{\partial CS(\cdot)}{\partial c}|_{r=r^{**}} = -\frac{r^{**}+\theta}{4r^{**}} \tag{47}$$

In a similar way, by left-hand side differentiating the social welfare function and evaluating at $r^{**} = c + \sqrt{\frac{tFr^{**}}{r^{**}+\theta}}$, we obtain

$$\frac{\partial CS(\cdot)}{\partial c}|_{r=r^{**}} = -\frac{r^{**}+\theta}{r^{**}} \tag{48}$$

Hence, the variation of the social welfare function in the face of an increase in the marginal cost $\frac{\partial CS(r^{***},c,t,F,\theta)}{\partial c}$ must be in the interval $\left[-\frac{r^{**}+\theta}{r^{**}}, -\frac{r^{**}+\theta}{4r^{**}}\right]$ whenever $r^{**} > \frac{\theta}{3}$. For the case in which $r^{**} \leq \frac{\theta}{3}$ the partial derivative of the value function satisfies the equation 34, evaluating this expression at the optimal reference price $r^{***} = c + \frac{1}{2}\sqrt{tF}$ implies that

$$\frac{\partial CS(r^{***},c,t,F,\theta)}{\partial c} = -1 \tag{49}$$

Case 2: By right-hand side differentiating the social welfare function $CS(r,c,t,F,\theta)$ with respect to t, it is obtained

$$\frac{\partial CS(\cdot)}{\partial t} = -\frac{F}{4(r-c)} \tag{50}$$

Since $r^{**} = c + \sqrt{\frac{tFr^{**}}{r^{**}+\theta}}$ for $r^{**} > \frac{\theta}{3}$, after evaluating $\frac{\partial CS(\cdot)}{\partial t}$ at r^{**} we have

$$\frac{\partial CS(\cdot)}{\partial t}|_{r=r^{**}} = -\frac{1}{4}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{\frac{F}{t}} \tag{51}$$

In a similar way, by left-hand side differentiating the social welfare function, we obtain

$$\frac{\partial CS(\cdot)}{\partial t} = -\frac{\theta}{2r}\sqrt{\frac{r}{r+\theta}}\sqrt{\frac{F}{t}} - \frac{1}{2}\sqrt{\frac{r}{r+\theta}}\sqrt{\frac{F}{t}} - \frac{1}{8}\sqrt{\frac{r+\theta}{r}}\sqrt{\frac{F}{t}} \tag{52}$$

After simplifying and evaluating at r^{**}, equation 52 reduces to

$$\frac{\partial CS(\cdot)}{\partial t}|_{r=r^{**}} = -\frac{5}{8}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{\frac{F}{t}} \tag{53}$$

Hence, the variation of the social welfare function in the face of an increase in the transportation cost $\frac{\partial CS(r^{***},c,t,F,\theta)}{\partial t}$ must be in the interval $\left[-\frac{5}{8}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{\frac{F}{t}}, -\frac{1}{4}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{\frac{F}{t}}\right]$ whenever $r^{**} > \frac{\theta}{3}$. For the case in which $r^{**} \leq \frac{\theta}{3}$ the partial derivative of the value function satisfies equation 50, evaluating this expression at the optimal reference price $r^{***} = c + \frac{1}{2}\sqrt{tF}$ implies that

$$\frac{\partial CS\left(r^{***},c,t,F,\theta\right)}{\partial t}=-\frac{1}{2}\sqrt{\frac{F}{t}} \tag{54}$$

Case 3: The case of F is identical to the one of t, it is enough to exchange t by F and vice-versa. Hence, $\frac{\partial CS(r^{***},c,t,F,\theta)}{\partial F}$ must be in the interval $\left[-\frac{5}{8}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{\frac{t}{F}},-\frac{1}{4}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{\frac{t}{F}}\right]$ whenever $r^{**}>\frac{\theta}{3}$ and $\frac{\partial CS(r^{***},c,t,F,\theta)}{\partial F}=-\frac{1}{2}\sqrt{\frac{t}{F}}$ whenever $r^{**}\leq\frac{\theta}{3}$.

Case 4: By right-hand side differentiating the social welfare function $CS\left(r,c,t,F,\theta\right)$ with respect to θ, it is obtained

$$\frac{\partial CS\left(\cdot\right)}{\partial\theta}=0 \tag{55}$$

In a similar way, by left-hand side differentiating the social welfare function with respect to θ, we obtain

$$\frac{\partial CS\left(\cdot\right)}{\partial\theta}=\frac{\theta}{2r}\sqrt{\frac{r}{r+\theta}}\left(\frac{\sqrt{tF}}{r+\theta}\right)-\left(\frac{c+\sqrt{\frac{tFr}{r+\theta}}-r}{r}\right)+\frac{1}{2}\sqrt{\frac{r}{r+\theta}}\left(\frac{\sqrt{tF}}{r+\theta}\right)-\frac{1}{8}\sqrt{\frac{r}{r+\theta}}\left(\frac{\sqrt{tF}}{r}\right) \tag{56}$$

After simplifying and evaluating at r^{**}, equation 56 reduces to

$$\frac{\partial CS\left(\cdot\right)}{\partial\theta}|_{r=r^{**}}=\frac{1}{2}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{tF}\left(\frac{1}{r^{**}+\theta}-\frac{1}{4r^{**}}\right)>0 \tag{57}$$

Hence, the variation of the social welfare function in the face of an increase in the parameter θ, $\frac{\partial CS(r^{***},c,t,F,\theta)}{\partial\theta}$ must be in the interval $\left[0,\frac{1}{2}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{tF}\left(\frac{1}{r^{**}+\theta}-\frac{1}{4r^{**}}\right)\right]$ whenever $r^{**}>\frac{\theta}{3}$. For the case in which $r^{**}\leq\frac{\theta}{3}$ the partial derivative of the value function satisfies equation 55, evaluating this expression at the optimal reference price $r^{***}=c+\frac{1}{2}\sqrt{tF}$ implies that

$$\frac{\partial CS\left(r^{***},c,t,F,\theta\right)}{\partial\theta}=0 \tag{58}$$

This completes the proof. ■

Proof of Proposition 7

Proof. We know that at the optimal reference price r^{***} the difference in social welfare function satisfies the expression,

$$\Delta CS\left(r^{***},c,t,F,\theta\right)=U-r^{***}-\frac{1}{4}\left(\frac{tF}{r^{***}-c}\right)-\left(U-c-\sqrt{tF}-\frac{1}{4}\sqrt{tF}\right) \tag{59}$$

We also know that the optimal reference price satisfies $r^{***}=r^{**}$ whenever $r^{**}>\frac{\theta}{3}$ otherwise $r^{***}=c+\frac{1}{2}\sqrt{tF}$. Hence, for the case when $r^{**}\leq\frac{\theta}{3}$ equation 59 reduces to $\Delta CS\left(r^{***},c,t,F,\theta\right)=\frac{1}{4}\sqrt{tF}>0$. When $r^{**}>\frac{\theta}{3}$ optimal reference price satisfies $r^{***}=c+\sqrt{\frac{r^{***}}{r^{***}+\theta}}\sqrt{tF}$, hence equation 59 reduces to $\Delta CS\left(r^{***},c,t,F,\theta\right)=\left(\frac{5}{4}-\sqrt{\frac{r^{***}}{r^{***}+\theta}}-\frac{1}{4}\sqrt{\frac{r^{***}+\theta}{r^{***}}}\right)\sqrt{tF}$. It is not difficult to show that the derivative of the function $f\left(r\right)=\sqrt{\frac{r}{r+\theta}}+\frac{1}{4}\sqrt{\frac{r+\theta}{r}}$ satisfies,

$$f'\left(r\right)=\frac{\theta}{2r}\sqrt{\frac{r}{r+\theta}}\left(\frac{1}{r+\theta}-\frac{1}{r}\right) \tag{60}$$

It is clear that $f'\left(r\right)>0$ whenever $r>\frac{\theta}{3}$ and $\lim f\left(r\right)=\frac{5}{4}$ as $r\rightarrow\infty$, hence $\Delta CS\left(r^{***},c,t,F,\theta\right)>0$ whenever $r^{**}>\frac{\theta}{3}$. This completes the proof. ■

Proof of Corollary 2

Proof. Since the equilibrium consumer surplus of the model without reference price effect is independent of r and θ, the partial derivatives with respect to the parameters c,t,F and θ satisfy the following,

1. $\frac{\partial CS_{nrp}(c,t,F)}{\partial c}=1$;
2. $\frac{\partial CS_{nrp}(c,t,F)}{\partial t}=\frac{5}{8}\sqrt{\frac{F}{t}}$;
3. $\frac{\partial CS_{nrp}(c,t,F)}{\partial F}\frac{5}{8}\sqrt{\frac{t}{F}}$; and
4. $\frac{\partial CS_{nrp}(c,t,F)}{\partial \theta}=0$.

The result comes directly from adjusting intervals of variation for the equilibrium social welfare of Proposition 6 with previous partial derivatives.

This completes the proof. ■